Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION

MAT 1202 APPLIED MATHEMATICS II

DATE: OCTOBER 29, 2021 TIME: 2 HOURS

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) Sketch the family of curves given by the equation $\frac{dy}{dx} = 4x$ and determine the equation of one of these curves which passes through the point (2, 3). [3 Marks]

(b) A random variable X has probability distribution as shown in the table.

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{10}$	λ	$\frac{1}{5}$	$\frac{1}{20}$

(i) Find the value of λ. [1 Marks]

(ii) Find (A) $P(X \geq 4)$, (B) $P(X < 1)$, (C) $P(2 \leq X < 4)$. [3 Marks]

(c) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,2,-1). [3 Marks]

(d) An object is thrown vertically upwards with an initial velocity u of 20 m/s. The motion of the object follows the differential equation $\frac{ds}{dt} = u - gt$, where s is the height of the object in metres at time t seconds and $g = 9.8m/s^2$. Determine the height of the object after **three seconds** if $s = 0$ when $t = 0$. [3 Marks]

(e) The discrete random variable X has probability function given by

$$p(x) = \begin{cases} (\frac{1}{2})^x, & x = 1,2,3,4,5 \\ c, & x = 6 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant. Determine the value of c and hence the mode of X. [3 Marks]

(f) $X \sim Geo(o.5)$. Find

(i) the mode, [1 Marks]

(ii) the mean of X, [1 Marks]

(iii) the standard deviation of X [2 Marks]

(g) Find the angle between the tangents to the curve

$$\overrightarrow{r} = t^2\mathbf{i} - 2t\mathbf{j} + t^3\mathbf{k}$$

at the points $t = 1$ and $t = 2$. [3 Marks]

(h) If $\overrightarrow{r} = xi + yj + zk$ and $\overrightarrow{a}$ is a constant vector, show that

(i) $\overrightarrow{r}$ is irrotational, [2 Marks]

(ii) $curl(\overrightarrow{a} \times \overrightarrow{r}) = 2\overrightarrow{a}$. [2 Marks]

(i) A Christmass draw aims to sell 5000 tickets, 50 of which will win a prize. A syndicate buys 200 tickets. Let X represent the number of these tickets that win a prize.
Calculate $P(X \leq 3)$. [3 Marks]

QUESTION TWO (15 MARKS)

(a) Determine the particular solution of $(y^2 - 1)\frac{dy}{dx} = 3y$
given that $y = 1$ when $x = 2\frac{1}{6}$ [4 Marks]

(b) The bending moment M of a beam is given by $\frac{dM}{dx} = -\omega(l - x)$, where ω and l are constants. Determine M in terms of x given: $M = \frac{1}{2}\omega l^2$ when $x = 0$. [5 Marks]

(c) The p.d., V, between the plates of a capacitor C charged by a steady voltage E through a resistor R is given by the equation

$$CR\frac{dV}{dt} + V = E$$

(i) Solve the equation for V given that at $t = 0$, $V = 0$ [4 Marks]

(ii) Calculate V, correct to 3 significant figures, when $E = 25V$, $C = 20 \times 10^{-6}F$, $R = 200 \times 10^3\Omega$ and $t = 3.0s$ [2 Marks]

QUESTION THREE (15 MARKS)

(a) The discrete random variable X has probability function

$$P(X = x) = \begin{cases} \frac{kx}{x^2+1}, & x = 2,3 \\ \frac{2kx}{x^2-1}, & x = 4,5 \\ 0, & \text{otherwise} \end{cases}$$

(i) Show that the value of k is $20/33$. [2 Marks]

(ii) Find $F(3.2)$ (iii) Find $E(X^2)$ [4 Marks]

(b) Identical independent trials of an experiment are carried out. The probability of a successful outcome is p. On average, five trials are required until a successful outcome occurs.

(i) Find the value of p. [3 Marks]

(ii) Find the probability that the first successful outcome occurs on the fifth trial.

(c) A game consists of throwing tennis balls into a bucket from a given distance. The probability that William will get the tennis ball in the bucket is 0.4. A turn consists of three attempts.

(i) Construct the probability distribution for X, the number of tennis balls that land in the bucket in a turn. **Hint**: use a tree diagram to display all the probabilities. [4 Marks]

(ii) William wins a prize if, at the end of his turn, there are two or more tennis balls in the bucket. What is the probability that William does not win a prize? [2 Marks]

QUESTION FOUR (15 MARKS)

(a) Find the directional derivative of the scalar point function $\Phi = 3e^{2x-y+z}$ at the point $A(1, 1, -1)$ in the direction towards the point $B(-3, 5, 6)$. [5 Marks]

(b) Find the value of α for which the vector $\overrightarrow{\mu} = (x + 3y)i + (y - 2z)j + (x + \alpha z)k$ is solenoidal vector. [3 Marks]

(c) If $\overrightarrow{V} = (x^2 - y^2 + 2xz)\mathbf{i} + (xz - xy - yz)\mathbf{j} + (z^2 + x^2)\mathbf{k}$ is a vector field, find curl $\overrightarrow{V}$. Show that the vectors given by curl $\overrightarrow{V}$ at $P_0(1, 2, -3)$ and $P_1(2, 3, -2)$ are orthogonal. [5 Marks]

(d) If $\overrightarrow{F} = (x + 2y + z)\mathbf{i} + \mathbf{j} + (x + y)\mathbf{k}$, show that $\overrightarrow{F} \cdot curl\, \overrightarrow{F} = 0$. [2 Marks]

QUESTION FIVE (15 MARKS)

(a) Emma plays a game in which she throws two dice. If she gets two sixes, she wins 20p, if she gets one six, she wins 10p, otherwise she wins nothing. She has to pay 5p to enter. Write out the probability distribution of X, the amount Emma gains in one turn. [4 Marks]

(b) The probability that a telephone box is occupied is 0.2. Find, to two significant figures, the probability that a person wishing to make a telephone call will find a telephone box which is not occupied only at the sixth box tried. [5 Marks]

(c) In the mass production of bolts, it is found that 5% are defective. Bolts are selected at random and put into packets of ten. A packet is selected at random. Find the probability that it contains (i) three defective bolts, (ii) less than three defective bolts. [4 Marks]

Two packets are selected at random.

(iii) Find the probability that there are no defective bolts in either packet. [2 Marks]

END OF PAPER