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Frontier Stock Market Linkages: An African Perspective

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A Dissertation Submitted to the Strathmore Institute of Mathematical Sciences (SIMS) In Partial Fulfillment of the Requirement for the Degree of Masters of Science in Statistical Sciences at the Strathmore University.

June 6, 2018

DECLARATION
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Abstract

Volatility modelling in the multivariate case is becoming an important area of study as the world becomes increasingly more integrated and as barriers to entry in frontier markets come down. Understanding how frontier markets in the African region behave in contrast to those in developed markets is vital in driving portfolio allocation decisions as well as regulatory interventions.

In this study we investigate the co-movements of the stock indices of four African countries, Nigeria, Morocco, Mauritius and Kenya using various multi-variate volatility models in relation to those of South Africa and the United Kingdom. We also fit a Kalman filter to the data set and examine the goodness of fit of the two approaches. For the Multi-variate models we fit an Exponentially Weighted Moving Average (EWMA) model, two specifications of Dynamic Conditional Correlation (DCC) models as well as a multivariate volatility model based on Cholesky Decomposition. We use a dynamic linear specification of the Kalman filter to allow for time-varying variances, and generate forecasts.

Empirical results show that the diagnostic tests with upper tail trimming reject the EWMA model while both specifications of the DCC model as well as a multivariate model based on Cholesky decomposition is found to be adequate. Kalman filters also provide adequate modelling for each return series on the basis of assessment of residuals.

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Chapter 1

Introduction

1.1 Background

Globalisation has had the effects of making the world a smaller place, while upending the traditional view of trade and finance along country lines. Investors are now freer than ever before to seek returns across the globe as barriers to entry such as limitations on foreign investment and high transaction costs, especially in frontier markets, come down. Frontier markets have emerged as the last port of call to generate better than average returns. It has been a long held view that they are significantly decoupled from more developed markets and are thus also able to offer significant diversification benefits.

Frontier markets is a term coined in the early 1990's under the auspices of the International Finance Corporation to describe smaller equity markets that are characterized by higher barriers to entry, but are nevertheless still investable. Typically, investors expect frontier markets to generate higher returns over the long term, while being less correlated to developed markets. The MSCI Frontier Index, a reference index commonly used by market practitioners, admits companies from twenty one different countries, out of which Kenya, Nigeria, Morocco, Tunisia and Mauritius represent the African continent.

While many studies exist on volatility transmission between emerging and developed markets across asset classes i.e. equities, bonds and currency, empirical literature that examines frontier markets, and in particular markets in Sub Saharan Africa, remains thin. Studies by Nordine, Hacibedel, and Nkusu (2016) indicate that capital flows to low income developing countries have grown six fold over the period 2000-2014, largely at the expense of emerging markets. This raises many questions around what this means for capital markets in frontier economies in terms of integration with the rest of the world.

In today's world, it has become easier for investors to construct portfolios that span the globe. It is a long held view in Finance that for one to reduce the overall volatility of a portfolio, it is better to diversify a portfolio across securities that have low correlation with each other. As markets become ever more integrated, it is intuitive that diversification effects should weaken.

In this era, it is more important than ever to understand the interdependence between markets, more so in how shocks are transmitted, in order for investors to come up with suitable portfolio construction and hedging. Understanding how events affect volatility is important given the rise of unprecedented political and economic shocks. Volatility transmission studies are also important from a regulator's perspective as it plays a role in determining appropriate market interventions through monetary policy.

In it's regional outlook Outlook (2013), The International Monetary Fund finds that the volume of foreign investment in frontier markets has been steadily growing, and projects that improved business environments amid better policy making should see growth continue in the Sub-Saharan African space. Understanding the interdependencies between these markets and the investor's home markets is vital for portfolio construction purposes, to allocate funds efficiently and for proper risk management. A brief description of the markets we will consider in this study is given below.

The Nairobi Securities Exchange
Set up in 1953 in Kenya, the Nairobi Securities Exchange (NSE) is the most active East African exchange, with a market capitalisation of approximately USD 20B in 2016. Tradeable securities are hosted on four main segments, the Main Investment Market Segment (MIMS), the Alternative Investment Market Segment (AIMS), the Fixed Income Securities Market Segment (FISM) and the Growth Enterprises Market Segment (GEMS). The Nairobi All Share Index is the main index for this exchange. The NSE is home to 65 listings on the equity segment, and 76 bond listings 16 of which are Corporate bonds while the rest are government debt issues. The Nairobi Securities Exchange completed it's self listing in 2014, become the second AFrican country after South Africa to be demutualised. Plans are under way to introduce a derivatives market on the exchange, starting with futures products on indices, currency and single stocks.

The Casablanca Stock Exchange
The Casablanca stock exchange was established in 1929 in Morocco, 75 different equity instruments are traded on this exchange. In 2016 market capitalisation closed at approximately 61.5 billion dollars, cementing it's place as one of North Africa's most important markets. Electronic based trading was introduced for

all brokers in 2001. The equities market is divided into a Main Market segment, Development Market Segment and Growth Market segment. There is also a provision for a Bonds market segment on which 95 corporate debt instruments and 65 government debt instruments trade. In 2015, laws were passed allowing for the creation of derivatives and real estate investment trusts, the exchange has since upgraded it's trading platform to allow for the introduction of these new instruments. The MASI index is the exchanges' all share index.

The Nigeria Stock Exchange
This exchange serves Africa's largest economy and is home to a 176 listed equities as of August 2017. The market is divided into the Main board segment, the Premium board segment, the Alternatives Securities Market and the Fixed Income segment The bonds market segment hosts 60 government and state bond listings and 27 corporate bond listings. It's market capitalisation at the end of 2016 stood at approximately 23.6 billion dollars. Plans are nearing completion to demutualise the exchange, with the Demutualisation bill expected to be passed in 2018. The first exchange traded fund was listed in 2011, and plans to introduce other exchange traded derivatives are at an advanced stage. The exchanges' main equity index is the Nigerian All Share Index.

The Johannesburg Stock Exchange
This is the largest an one of the oldest exchanges in Africa. Formed in 1887, the Johannesburg Stock Exchange (JSE) equity market plays host to 388 listed companies, as well as vibrant bond and derivative exchanges. The equities market is divided into the Main market board and the AltX board.The bonds market comprises 43 government listings as well as 1167 corporate bond listings. The derivatives markets lists futures, options, interest rate, currency and commodity derivatives. The JSE has a market capitalisation of 1,231 billion dollars at the end of 2016. The JSE completed the demutualisation process in 2006. As the leading exchange in Africa,the JSE plans to focus on expanding it's index and data offerings as well as enhance ties with other African stock exchanges through the Africa Exchange Linkages Project. The equity market's main index is the JSE All Share Index.

The Stock Exchange of Mauritius
The Stock Exchange of Mauritius (SEM) was incorporated in 1989, and became a publicly listed entity (demutualised) in 2008. It's equity market is home to 38 listed companies. The bonds market comprises 110 government listings and 41 corporate listings. The equities market is segmented into the Official market and the Development & Enterprise market. Mauritius is known for it's multi currency listings, and is planning to grow international listings by way of a fast track method for foreign companies that are already listed in their home markets. Market capi-

talisation stood at approximately 13.4 billion dollars at the end of 2016.Due to the absence of withholding tax and capital gains tax it has steadily become a favourite of institutional investors who account for roughly 75% of the volumes traded there. Foreign investors also account for around 45% volumes traded on average. The markets' flagship index is the SEMDEX .

The London Stock Exchange
The London Stock Exchange (LSE) counts itself among one of the world's first exchanges with a history that stretches back over 300 years. With a market capitalisation of over 6 trillion dollars at the end of 2016, it is one of the most developed exchanges in the world. The LSE is a trading venue for equities, fixed income and derivative products. 2027 companies are listed on the LSE. The equities market is segmented into the Main market and Alternative Investments Market. The bonds market segment is composed of 98 government listings and 243 corporate bond listings. Derivatives traded on the LSE include futures, options, agricultural derivatives, energy derivatives, commodity derivatives, equity and interest rate derivatives among others.The LSE merged with the Milan Stock Exchange, Borsa Italiana in 2007. Subsequent attempts to merge with the Deutsche BÃűrse were prohibited by the European Commission. The all share index commonly used is the FTSE All Share Index.

Table 1.1: Exchange Statistics of Markets as at Dec 2016

Market	**Frontier**				**Emerging**	**Developed**
Country	Kenya	Morocco	Nigeria	Mauritius	South Africa	UK
Index	NSE All Share	MASI	Nigerian All Share	SEMDEX	JSE All Share	FTSE All Share
Equity Listings	65	75	176	38	388	2027
Market Capitalization(USD Bn)	20	61.5	23.6	13.4	1231	6000
Government Debt Listings	76	65	60	110	43	98
Corporate Debt Listings	16	95	27	41	1167	243
Derivatives Market	No	No	No	No	Yes	Yes

1.2 Problem statement

Understanding the interdependence of equity markets has always been a key concern for investors looking to diversify portfolio risks and enhance portfolio returns. This is especially true for African stock markets, which have witnessed significant growth in the past decade in foreign flows to their stock markets. A better understanding of how African markets move in tandem with each other and with more developed markets is needed to aid in investment allocation decisions, on the part of the investor and policy decisions, on the part of market regulators.

1.3 Objectives

1.3.1 Main Objective

- To examine the co-movements of African stock markets in relation to developed markets over time and to determine if the premise of diversification as one of the main reasons for investments in African equity markets is justified.

1.3.2 Secondary Objectives

These are:

i To determine if there is evidence of volatility transmission from developed equity markets to African equity markets.

ii To provide a comparison between various fitted multivariate volatility models as compared to fitting Kalman filters to the data set.

1.4 Justification

As technological advances continue to shrink the perceived distance between markets globally, it becomes ever more important for both local and international investors to understand how the dependence structure between developed and frontier market is shifting with time. This is particularly key as we enter an age where the financial markets in these frontier markets is growing rapidly, attracting a larger share of investment funds. Understanding the way these risks are evolving is crucial for portfolio optimization as well as policy review and formulation. The use of approaches such as the Kalman filter can also be used to check the validity of traditional volatility modelling techniques.

Chapter 2

Literature Review

Volatility transmission has been studied under a variety of approaches. Soriano and Climent (2005) in his review of studies on volatility transmission finds the most common models in empirical literature can be categorised under Regime Switching models, Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) Models and Stochastic Volatility models.

2.1 GARCH Models

A key stylised fact about equity market returns is that volatility does not stay constant throughout time, but varies in response to various factors. De Santis and Imrohoroglu (1997) have shown that emerging markets' conditional volatility tends to be high. Senbet and Otchere (2008) attribute this to political and economic factors in the African case. This builds the case for using GARCH modelling that allows for this changing volatility.

Hamao, Masulis, and Ng (1990) through the use of univariate GARCH models, found evidence of volatility spillovers from New York to London and Tokyo, as well as from London to Tokyo.The approach consists of fitting a univariate GARCH model to each market and using the squared residuals from one fitted model of one market in the variance equation of another market.This approach, while simple and easy to implement ignores causal effects in the different series of volatilities and covariances between them.

This led to further research on joint estimation of GARCH models, an example of which is work done by Engle and Kroner (1995) who laid the framework for the BEKK multivariate model. This model works well for cases where the number of return series being investigated is small, typically less than four. This is because of the number of parameters generated by the model specification. Computationally it can be difficult to estimate but tends to fit data well. Diagnostic tests often fail to reject it.

Ogotseng (2017) investigated ten African stock markets and found evidence of volatility transmission to these markets from international markets by application of the E-GARCH framework. Otieno (2017) further extended the univariate GARCH approach, by using a copula GARCH approach to model the dependence structure between indices of BRICS countries, (namely Brazil, Russia, India China and South Africa) and Kenya. He first fit a univariate GARCH model for each return series, then using the residuals generated copula parameters, using the parametric class of copulas. He was able to show that the student t copula provided the best fit for BRICS and Kenya returns in keeping with earlier results.
On the multivariate GARCH front, Frank and Hesse (2009) examined data from developing and emerging markets during the financial crisis of 2008. They used DCC GARCH models to examine the interlinkages between US Libor spread and sovereign bonds as well as sovereign CDS spreads of emerging countries represented by Brazil, Russia, Turkey, Mexico and South Africa. They reached the overall conclusion that the long held idea that developed markets and emerging markets were decoupled was not well founded. In fact during the crisis, volatility tended to spill over rapidly from developed markets into emerging markets.

Bichi, Dikko, and Nagwai (2016) investigate intra market volatility transmission between the Nigerian stock exchange and the Nigerian bond market using two of the most widely used multivariate GARCH models, Baba Engle Kraft Kroner (BEKK) GARCH model, and The Dynamic Conditional Correlation GARCH model. They found that the DCC model fit their data set better and also found a weak negative relationship between the Nigerian stock and bond markets.

2.2 Regime Switching Models

In cases where the volatility changes can be shown to be structural in nature, models such as Hamilton and Susmel (1994)'s SWARCH model i.e Switching Autoregressive Conditional Heteroscedasticity model that allows for changes in regimes.In this case, the parameters of the SWARCH model are dependent on a transition probaility matrix that describes the probability of moving from one regime to the next. It allows us to incorporate economic events that caused a shift in a particular market.

Marcucci (2005) compared the Markov Regime Switching GARCH (MRS GARCH) to other standard GARCH models, GARCH (1,1), E-GARCH(1,1) and GJR(1,1), using data on the Standard and Poor 100 Index. The parameters of the MRS

GARCH were allowed to depend on the probabilities of two regimes, a low and a high probability regime. Marcucci was trying to solve the problem of GARCH volatility forecasts that were too high and too smooth. The author found that at short horizons, the MRS- GARCH outperformed all other GARCH models in terms of volatility forecasts.

While this approach is more precise, regime switching models tend to be very difficult to estimate because it raises questions around how to properly determine the regime changes, for which a number of algorithms have been proposed. We also have to grapple with how to come up with the transition probabilities as well as how to extend the univariate regime switching case to a multi-variate one.

2.3 Stochastic Volatility Models

Stochastic volatility models consider volatility as being unobservable, the log of the volatility is then modelled as a stochastic linear model. Harvey, Ruiz, and Shephard (1994) illustrated how this can be extended to the multivariate case by generating the variances using an auto regressive process of order one (AR(1)). Estimation is achieved in his model by quasi maximum likelihood. In this model the multivariate series is assumed to follow the process:

$$y_{it} = \varepsilon_{it}(\exp(h_{it}))^{\frac{1}{2}}. \qquad (2.1)$$
$$i = 1, 2.....N,$$
$$t = 1, 2......T.$$

Where $\varepsilon_t = (\varepsilon_{1t}....\varepsilon_{Nt})'$ denotes a multivariate normal vector with zero mean and a covariance matrix, Σ_t whose leading diagonal is unity. The variance of the process is assumed to be generates by an autoregressive process of order one denoted by

$$h_{it} = \gamma_i + \phi_i h_{it-1} + \eta_{it}. \qquad (2.2)$$
$$i = 1....N.$$

This specification allows us to consider the volatility of volatility i.e η_{it} is considered separately from ε_{it}. The main problem with stochastic volatility models is that they can be difficult to estimate, Harvey et al. (1994) proposed a quasi maximum likelihood approach, which has been criticised due to the assumption of normality of errors.

A key strength of a stochastic volatility models is that they can be modified to incorporate different specifications of the variance process, denoted by h_{it} in the approach above. They can also be combined with traditional factor techniques as illustrated by Lopes and Migon (2002) who combined two factor models with stochastic volatility models to study volatility transmission between Latin American stock indices and the North American stock index.

2.4 Kalman Filter

The Kalman filter provides an alternative approach to the problem of volatility modelling and forecasting of financial data. Kalman et al. (1960) attempted to tackle the problem of detection and prediction of signals in noisy processes. By developing a recursive algorithm that can be used to update the unobservable components of a series, he laid the basis for application of this filter in a wide variety of fields, ranging from navigation, to engineering and even to economics and finance.

In the finance world Kalman filters have been applied by Dunis and Shannon (2005) in their study on emerging markets in South East and Central Asia. They were trying to establish if these markets still offered a diversification benefit to investors from more developed markets. The results of their estimations of the time varying parameters of the Kalman filter indicated closer integration with the Japanese market, while the United States and United Kingdom markets had steady or declining levels of integration. They were able to make a case for investors from these developed markets to keep investing in Asian markets as the diversification benefits still existed.

Hee Ng (2002) chose to focus on integration within South East Asian markets alone.Estimates for the time varying parameters of the Kalman filter were obtained with respect to Indonesia, Malaysia, the Philippines, Singapore and Thailand. He finds, as one would intuitively expect, no evidence of co-integration between all markets in earlier periods, that is 1988-1997, in later periods, 1993-1997, he finds that the markets became more interlinked, particularly , Indonesia, Filipino and Thai stock markets show increasing co-movement with the Singapore stock market. He attributes this mainly to liberalisation of the markets.

While frontier markets have not attracted much attention in so far as Kalman filters are concerned, there is one study by Adam and Gyamfi (2015) that considers stock return indices across 11 African countries, over the period May 2002 to May 2011. They use an adjusted market integration index based on the capital asset pricing model. Evidence was found that supports the idea of Africa's possible segmentation from the rest of the world, with the exception of South Africa.

Chapter 3

Methodology

In this section we will outline the data set we will use as well as the various approaches we will consider to tackle the problem.

3.1 Data

We will consider the all share stock indices for Kenya, Nigeria, South Africa, Mauritius, Morocco and the United Kingdom. We will use daily data obtained from Bloomberg for the period January 2009- December 2017. Bloomberg collects market data from each of the exchanges and makes it available for users of it's Bloomberg terminals.

3.2 Review of Multivariate GARCH models

3.2.1 Exponentially Weighted Moving Average(EWMA) method

As outlined in Tsay (2013) The Exponentially Weighted Moving Average approach can be used to carry out joint estimation of volatilities for a give data set. It applies a non-uniform weighting to time series data, allowing a lot of data can be used, with heavier weightings applied on recent data. These weights are based upon the exponential function. We would begin by fitting a vector auto regressive process of order 1(VAR(1)) to the data set to remove serial correlations. Vector autoregressive models are used in multivariate time series such that the structure of each time series is expressed as a linear function of past lags of itself and past lags of the other variables.
If we take the example of three time series denoted by x_{1t}, x_{2t} and x_{3t}, then the VAR(1) process can be given by:

$$
\begin{aligned}
x_{1,t} = \quad & a_1 + b_{11}x_{1,t-1} + b_{12}x_{2,t-1} + b_{13}x_{3,t-1} + c_{1,t}, \\
x_{2,t} = \quad & a_2 + b_{21}x_{1,t-1} + b_{22}x_{2,t-1} + b_{23}x_{3,t-1} + c_{2,t}, \\
x_{3,t} = \quad & a_3 + b_{31}x_{1,t-1} + b_{32}x_{2,t-1} + b_{33}x_{3,t-1} + c_{3,t}.
\end{aligned} \tag{3.1}
$$

Where a_i, b_{ij} and c_{it} are parameters to be estimated. From this fit we can obtain residuals of the above equation. If we denote the residuals of the mean equation by $\hat{a}_t$ .The EWMA model is given by;

$$
\begin{aligned}
\hat{\boldsymbol{\Sigma}}_t = \lambda\hat{\boldsymbol{\Sigma}}_{t-1} + (1-\lambda)\hat{\mathbf{a}}_{t-1}\hat{\mathbf{a}}'_{t-1}, \\
\text{for } 0 \leq \lambda \leq 1.
\end{aligned} \tag{3.2}
$$

Where λ is the persistence parameter. A high λ gives little reaction to actual market events but great persistence in volatility, and a low λ gives highly reactive volatilities that quickly die away. It is customary to begin the recursive process with the sample covariance matrix, i.e.$\hat{\Sigma}_t = \hat{\Sigma}_0$. A necessary condition is that this matrix has to be positive definite. In this model λ can be fixed beforehand based on empirical evidence or can be estimated via the quasi maximum likelihood estimation.

While this method is quite easy to estimate as it is depends on one parameter, λ, this compact form may lead to the fit being rejected by diagnostic tests, such as Portmanteau tests, which seeks to detect the presence of heteroscedasticity after model fitting. From Tsay (2013), the test statistic applicable in this case is the familiar Ljung Box test statistic generalized to the multivariate case, which can be expressed as:

$$
Q_k(m) = T^2\Sigma_{i=1}^{m}\frac{1}{T-i}\mathbf{b}'_i(\hat{\boldsymbol{\rho}}_0^{-1} \otimes \hat{\boldsymbol{\rho}}_0^{-1})\mathbf{b}_i. \tag{3.3}
$$

Where: T denotes the sample size.
$\mathbf{a}_t$ denotes the noise process of the fitted model.
$\mathbf{b}_i = \text{vec}(\hat{\rho}_i)$.
k is the dimension of $\mathbf{a}_t$.
$\hat{\boldsymbol{\rho}}_j$ is the lag j sample cross correlation matrix of $\mathbf{a}_t^2$.

In this case, the test statistic, $Q_k(m$ can be shown, asymptotically, to have a chi square distribution, i.e. $\chi^2_{k,m}$. We can then test the null hypothesis of absence of conditional heteroscedasticity against the alternative hypothesis of presence of hetersocedasticity in the fitted model, i.e.:

$$
\begin{aligned}
H_0 :& \rho_1 = \rho_2 = ... = \rho_m = 0 \\
&\text{vs.} \\
H_a :& \rho_i \neq 0 \text{ for some } i(1 \leq i \leq m).
\end{aligned} \tag{3.4}
$$

Rejection of the null would then depend on the significance of the p-values say at 5 % significance level. Given that we are dealing with financial data that tends to exhibit heavy tails and significant departures from normality, it may be prudent to trim the data in the upper 5 % tail. Again following arguments from Tsay (2013),using the residuals after trimming of the upper tail can generate a robust test statistic that will work well in diagnostic tests. We use this test statistic across the various multivariate models in the following sections as well.

Diagnostic tests based on portmanteau tests empirically tend to reject the EWMA model. This could be due to the fact that it is highly unlikely the variation in a multivariate case can be explained by just one parameter. It is however useful in cases where accuracy is not paramount but rather a rough estimate is sufficient.

3.2.2 Cholesky Decomposition

The requirement of positive definite covariance matrices in estimation methods such as the EWMA method outlined in the previous section may pose difficulties in estimation. Cholesky decomposition becomes a helpful work around as it enables us to obtain positive-definite volatility matrices easily. Given a k-dimensional innovation a_t with volatility matrix Σ_t and let F_{t-1} denote the information available at time $t-1$. Cholesky decomposition performs linear orthogonal transformations via a system of multiple linear regressions. Let $b_{1t} = a_{1t}$ and consider the simple linear regression:

$$a_{2t} = \beta_{21,t} b_{1t} + b_{2t}. \tag{3.5}$$

where $\beta_{21,t} = Cov(a_{2t}, b_{1t}|F_{t-1})/Var(b_{1t}|F_{\text{tâĹŠ1}})$. In practice, $\beta_{21,t}$ is estimated by the ordinary least-squares method using the available data in F_{t-1}. Based on least-squares theory, b_{2t} is orthogonal to $b_{1t} = a_{1t}$ and $Var(b_{2t}|F_{\text{tâĹŠ1}}) = Var(a_{2t}|F_{\text{tâĹŠ1}})\text{âĹŠ}\beta^2_{21,t} - Var(a_{1t}|F_{\text{tâĹŠ1}})$. Next, consider the multiple linear regression:

$$a_{3t} = \gamma_{31,t} b_{1t} + \gamma_{32,t} b_{2t} + b_{3t} = \beta_{31,t} a_{1t} + \beta_{32,t} a_{2t} + b_{3t}. \tag{3.6}$$

where $\beta_{3j,t}$ are linear functions of $\gamma_{3j,t}$ and $\beta_{21,t}$ for $j = 1 and 2$. Again, via the least-squares theory, b_{3t} is orthogonal to both a_{1t} and a_{2t} and, hence, to b_{1t} and b_{2t}. Repeat the prior process of multiple linear regressions until:

$$a_{kt} = \beta_{k1,t} a_{1t} + ... + \beta_{k,k-1,t} a_{k-1,t} + b_{kt}. \tag{3.7}$$

Applying the least-squares theory, we can obtain $\beta_{kj,t}$ and the conditional variance of b_{kt} given F_{t-1}. In addition, b_{kt} is orthogonal to a_{it} and, hence, b_{it} for $i =$

$1, ..., k-1$. In matrix form, we have:

$$\boldsymbol{\beta}_t \mathbf{a}_t = \mathbf{b}_t. \tag{3.8}$$

or

$$\mathbf{a}_t = \boldsymbol{\beta}_t^{-1} \mathbf{b}_t. \tag{3.9}$$

Denoting the volatility matrix of b_t by $\Sigma_{b,t}$, we obtain:

$$\boldsymbol{\Sigma}_t = \boldsymbol{\beta}_t^{-1} \boldsymbol{\Sigma}_{b,t} (\boldsymbol{\beta}_t^{-1})'. \tag{3.10}$$

where $\Sigma_{b,t}$ is diagonal. Equation (3.10) implies that $\boldsymbol{\Sigma}_t^{-1} = \boldsymbol{\beta}_t' \boldsymbol{\Sigma}_{b,t}^{-1} \boldsymbol{\beta}_t$ and $|\boldsymbol{\Sigma}_t| = \boldsymbol{\Sigma}_{b,t} = \Pi_{i=1} \sigma_{bi,t}^k$ where $\sigma_{bi,t}$ is the volatility of b_{it} given $F_{\text{tâĹŠ1}}$. Consequently, the quasi log likelihood function of $\mathbf{a}_t$ is relatively easy to compute. We can then fit univariate GARCH models to each of the return series.

3.2.3 Dynamic Conditional Correlation Models

The motivation behind DCC models is to use correlation matrices as opposed to covariance matrices, which tend to be harder to work with. This approach requires us to first model each volatility series by univariate GARCH or stochastic volatility models before modelling the dynamic dependence of the correlation matrix. If we consider the k dimensional return series say x_t, with e_t representing the innovations of x_t conditional on the information available up to time t-1, F_{t-1}.if we let the volatility matrix of e_t be denoted by:

$$\boldsymbol{\Sigma}_t = [\sigma_{ij,t}]. \tag{3.11}$$

Then the conditional correlation matrix will be given by:

$$\boldsymbol{\rho}_t = \mathbf{D}_t^{-1} \boldsymbol{\Sigma}_t \mathbf{D}_t^{-1}. \tag{3.12}$$

Where $\mathbf{D}_t = diag[\sigma_{11,t}^{0.5}, ...\sigma_{kk,t}^{0.5}]$ is the diagonal matrix of the k volatilities at time t. If we denote the standardized innovations by $\boldsymbol{\eta}_t = (\eta_{1t}, ...\eta_{kt})'$ with $\eta_{it} = e_{it}/\sqrt{\sigma_{ii,t}}$. Then $\boldsymbol{\rho}_t$ is the volatility matrix of $\boldsymbol{\eta}_t$.

There are two types of DCC model proposed in literature, one by Engle (2002) is based on the unconditional covariance matrix denoted by Q_t as per the equations below;

$$\begin{aligned} \mathbf{Q}_t &= (1-\theta_1-\theta_2)\bar{\mathbf{Q}} + \theta_1 \mathbf{Q}_{t-1} + \theta_2 \boldsymbol{\eta}_{t-1} \boldsymbol{\eta}_{t-1}', \\ \boldsymbol{\rho}_t &= \mathbf{J}_t \mathbf{Q}_t \mathbf{J}_t. \end{aligned} \tag{3.13}$$

Where $\bar{\mathbf{Q}}$ is the unconditional covariance matrix of $\boldsymbol{\eta}_t$.
θ_i denotes non negative real numbers such that $0 < \theta_1 + \theta_2 < 1$.
$\mathbf{J}_t = diag(q_{11,t}^{-0.5}, ...q_{kk,t}^{-0.5})$ where $q_{ii,t}$ denotes the (i,i)th element of $\mathbf{Q}_t$.
$\mathbf{Q}_t$ is a positive definite matrix.
$\mathbf{J}_t$ is a normalization matrix.

Tse and Tsui (2002) propose a DCC model that is not based on the standardised innovations $\boldsymbol{\eta}_t$ but rather relies on a local correlation matrix to update the unconditional correlation matrix. In this case the correlation matrix $\boldsymbol{\rho}_t$ is given by:

$$\boldsymbol{\rho}_t = (1 - \theta_1 - \theta_2)\bar{\boldsymbol{\rho}} + \theta_1 \boldsymbol{\rho}_{t-1} + \theta_2 \boldsymbol{\psi}_{t-1}. \tag{3.14}$$

Where $\bar{\boldsymbol{\rho}}$ denotes the unconditional correlation matrix of $\boldsymbol{\eta}_t$.
θ_i denotes non negative real numbers such that $0 < \theta_1 + \theta_2 < 1$.
$\boldsymbol{\psi}_{t-1}$ is a local correlation matrix depending on $(\boldsymbol{\eta}_{t-1}, ...\boldsymbol{\eta}_{t-m})$for some positive integer m. In practice $\boldsymbol{\psi}_{t-1}$ is estimated using the sample correlation matrix of $(\hat{\boldsymbol{\eta}}_{t-1}, ...\hat{\boldsymbol{\eta}}_{t-m})$.

3.3 Review of Kalman Filtering

3.3.1 Outline of Kalman algorithm

In this section we will follow the arguments from Arnold, Bertus, and Godbey (2008). To illustrate the Kalman algorithm, we begin by considering the Kalman filter in the univariate case, where an observable variable denoted by X_t is related to an unobservable variable, denoted by Y_t by the measurement equation below;

$$X_t = a_t Y_t + b_t + \varepsilon_t. \tag{3.15}$$

where for simplicity's sake we can assume $b_t = 0$ and a_t is constant through time, and ε_t has mean 0 and variance h_t Thus the measurement equation can be given by;

$$X_t = aY_t + \varepsilon_t. \tag{3.16}$$

The transition equation outlined below gives the evolution of the unobserved variable through time as;

$$Y_{t+1} = c_t Y_t + d_t + \theta_t. \tag{3.17}$$

Again for simplicity we can assume $d_t = 0$ and c_t is constant hence $c_t = c$. Further to this θ_t is also assumed to have a mean of 0 and a standard deviation, s_t. Thus the transition equation reduces to:

$$Y_{t+1} = cY_t + \theta_t. \tag{3.18}$$

To initialise the transition equation we can insert an initial value say Y_0, where Y_0 has a mean μ_0 and standard deviation σ_0. Note that we also assume that ε_t,θ_t and Y_0 are uncorrelated, so we have the predicted value of Y_t being given by:

$$Y_{1p} = cY_0 + \theta_0. \tag{3.19}$$

where Y_{1p} denotes the predicted value of Y_1. We can then recursively use this value in the measurement equation to obtain the predicted value of the observable variable, which we can denote by X_{1p}, which is given by:

$$\begin{aligned} X_{1p} &= aY_{1p} + \varepsilon_1, \\ &= a(cY_0 + \theta_0) + \varepsilon_1. \end{aligned} \tag{3.20}$$

When X_1 occurs, we can obtain the error as the difference;

$$X_{1e} = X_1 - X_{1p}. \tag{3.21}$$

This error can be used to obtain a better predicted value for Y_1 which can be denoted by $Y_{(1p-adj)}$. This quantity is given by the equation:

$$\begin{aligned} Y_{(1p-adj)} &= Y_{1p} + k_1 X_{1e}, \\ &= Y_{1p} + k_1(X_1 - X_{1p}), \\ &= Y_{1p} + k_1(X_1 - aY_{1p} - \varepsilon_1), \\ &= Y_{1p}(1 - ak_1) + k_1 X_1 - k_1 \varepsilon_1. \end{aligned} \tag{3.22}$$

where k_1 is the Kalman gain. It can be shown from Arnold et al. (2008) that the Kalman gain can be given as;

$$\begin{aligned} k_1 &= \frac{g_1 * a}{g_1 * a^2 + h_1} = \frac{Cov(X_{1p}Y_{1p})}{Var(Y_{1p})}. \\ \text{where} \\ g_1 &= Var(Y_{1p}) = (c * \sigma_0^2) + s_0. \end{aligned} \tag{3.23}$$

We can now use $Y_{(1p-adj)}$ in the transition equation for Y_t to get the value at the second time step.It can also be shown that $Var(Y_{(1p-adj)})$ can be given by taking the variance of expression (3.22)above;

$$\begin{aligned} Var(Y_{(1p-adj)}) &= g_1 * (1 - ak_1)^2 + k_1^2 * h_1, \\ &= g_1 \left\{ 1 - \frac{1}{1 + \frac{h_1}{g_1 * a^2}} \right\}^2 + k_1^2 * h_1. \end{aligned} \tag{3.24}$$

Using the expressions above we have the following expressions that we can use in general for the means and variances of Y_{tp-adj} and X_{tp}:

$$E[Y_{(tp-adj)}] = E[Y_{tp} + k_t * X_{te}] = E[Y_{tp}] + k_t(X_t - E[X_{tp}]. \tag{3.25}$$

$$Var(Y_{(tp-adj)}) = g_t \left\{ 1 - \frac{1}{1 + \frac{h_t}{g_t * a^2}} \right\}^2 + k_t^2 * h_t. \tag{3.26}$$

$$E[X_{tp}] = E[a * (Y_{(tp-adj)}) + \varepsilon_t] = aE[Y_{(tp-adj)}]. \tag{3.27}$$

$$Var(X_{tp}) = Var(Y_{(tp-adj)}) * a^2 + h_t. \tag{3.28}$$

3.3.2 Maximum Likelihood Estimation of the Kalman Filter

In the outline above, it is vital to note that while we are generating estimated values of the unobserved variable i.e Y_{tp-adj}, and given that the observed variable has predicted values X_{tp}, we are yet to obtain the parameters ε_t, c and θ_t. For maximum likelihood estimation, we require a likelihood function, which can be obtained by incorporating an assumption of normality of X_{tp}, which has mean and variances described by equations (3.27) and (3.28) above. Note that these two equations incorporate the mean and variance of Y_{tp-adj}. This gives rise to the joint likelihood function below.

$$\prod_{t=1}^{t=T} \left\{ \frac{1}{\sqrt{2\pi Var(X_{tp})}} \right\}^T \exp\left\{ \frac{\sum_{t=1}^{t=T} (X_t - E(X_{tp}))^2}{2Var(X_{tp})} \right\}. \tag{3.29}$$

In keeping with usual practice we can work with the log likelihood function of equation (3.27) given by:

$$-T\frac{\ln(2\pi)}{2} - \frac{1}{2}\sum_{t=1}^{t=T} \ln[Var(X_{tp})] - \frac{1}{2}\sum_{t=1}^{t=T} \frac{(X_t - E[X_{tp}])^2}{Var(X_{tp})}. \tag{3.30}$$

We then obtain estimates of the parameters of interest by taking the partial derivatives of the log likelihood function with respect to each parameter of interest and setting it to zero, i.e the parameters ε_t, c and θ_t. We then use the Kalman filter to generate new estimates of Y_{tp-adj} and X_{tp}. The likelihood estimation is then repeated using these new estimates for Y_{tp-adj} and X_{tp}, to generate new estimates for the parameters. This is done recursively until no significant change is detected in the log likelihood equation.

Chapter 4

Results

In this section we use statistical software R and Excel for data exploration and also to achieve the objectives of the study. The R packages used for data analysis include MTS, dlm, MASS, tseries, fgarch, rugarch, forecast and pastecs.

4.1 Data Exploration

4.1.1 Summary Statistics

We consider data on the all share stock indices for the UK, Nigeria, Kenya, Morocco and Mauritius for the period January 2009-December 2017,we use the daily statistics to investigate the dependence structure between these markets. The data for each country is in the respective country's currency. The summary statistics for the return series of the data are summarised in the table below:

Table 4.1: Descriptive statistics of Index Returns

	Nigeria	UK	Morocco	South Africa	Mauritius	Kenya
min	-9.04E-02	-4.53E-02	-4.56E-02	-3.63E-02	-2.85E-02	-5.22E-02
max	1.25E-01	5.15E-02	3.35E-02	5.76E-02	4.31E-02	5.29E-02
mean	1.23E-04	3.54E-04	9.14E-05	5.34E-04	3.09E-04	4.39E-04
SE.mean	2.47E-04	1.98E-04	1.37E-04	2.16E-04	9.33E-05	1.55E-04
CI.mean.0.95	4.85E-04	3.88E-04	2.68E-04	4.23E-04	1.83E-04	3.03E-04
var	1.37E-04	8.76E-05	4.18E-05	1.04E-04	1.95E-05	5.37E-05
std.dev	1.17E-02	9.36E-03	6.46E-03	1.02E-02	4.42E-03	7.33E-03
coef.var	9.51E+01	2.64E+01	7.07E+01	1.91E+01	1.43E+01	1.67E+01

In terms of variability, South Africa surprisingly has the highest variability while the UK in keeping with expectations of a developed market has the lowest variability. We would typically expect frontier markets to have higher variability than a more established emerging market such as South Africa. From the mean values we observe that Nigeria has the highest mean while Morocco has the lowest mean value.

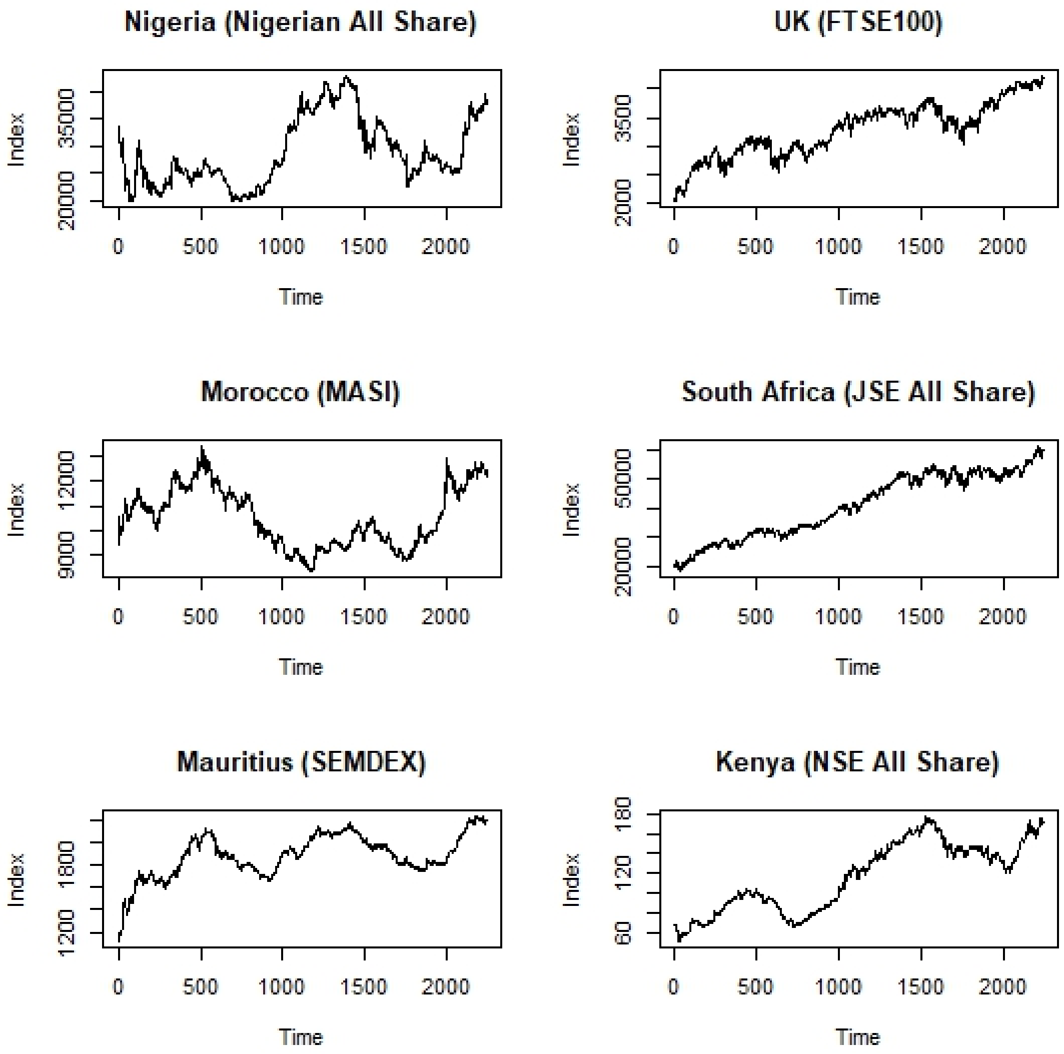

Figure 4.1: Index Time Plot

A look at the time plots of the raw data reveals that the UK and South Africa generally have a positive trend over the period with the UK exhibiting slight dips in 2011 and 2016. South Africa appears to have declines that are less sharp. Also interestingly Nigeria and Kenya both exhibited dips in 2011 and 2016 as well. Kenya's drop in 2016 may be partially ascribed to effects of the rate caps on the financial sector, 2016 was also a year ahead of the 2017 national elections, which may further explain the dip in Kenya's stock market. The dip in Nigeria's stock market in 2016 corresponded with a dip in global oil prices (Nigeria relies heavily on it's exports of oil for foreign exchange). Due to this, many investors became wary of holding Naira denominated assets, resulting in the negative performance of the stock market. Morocco's most pronounced declines came in 2013 and 2016 while that of Mauritius seems less in tune with the rest coming in 2012 and 2015. The Jarque Bera test is a statistical test designed to assess if the sample data has skewness and kurtosis matching that of the normal distribution. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero. If the sample data comes from a normal distribution we expect the skewness to be 0 and the kurtosis to be 3 (corresponding to an excess kurtosis of 0).The Jarque Bera test statistic can be shown to asymptotically follow a chi squared distribution with two degrees of freedom. The Jarque-Bera test statistic is given by:

$$J = \frac{n-k+1}{6}\left\{S^2 + \frac{1}{4}(C-3)^2\right\}. \tag{4.1}$$

where n= number of observations, k= number of regressors, S= sample skewness and C= sample kurtosis. The sample skewness and kurtosis are given by the expressions below;

$$\begin{aligned} S &= \frac{\hat{\mu}^3}{\hat{\sigma}^3}, \qquad (4.2)\\ &= \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right\}^{\frac{3}{2}}}. \\ C &= \frac{\hat{\mu}^4}{\hat{\sigma}^4}, \qquad (4.3)\\ &= \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4}{\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right\}^{2}}. \end{aligned}$$

Where $\hat{\mu}^3$ and $\hat{\mu}^4$ denote the third and fourth central moments, while $\hat{\sigma}^2$ denotes the variance. We reject the null hypothesis of normality, when the p-value is small, which happens when the skewness and kurtosis differ from their expected values under normality. The summary statistics of this test are illustrated in the table below:

Table 4.2: Jarque-Bera Statistics

Country	jacqbera.tes	df	p.value	skewness	kurtosis
Nigeria	14671.88	2.00	0.00E+00	0.568316	12.48065
UK	567.836	2.00	0.00E+00	-0.16834	2.44237
Morocco	1339.693	2.00	0.00E+00	0.088411	3.782828
South Afria	283.7993	2.00	0.00E+00	0.007092	1.742927
Mauritius	25278.29	2.00	0.00E+00	1.280981	16.24911
Kenya	3439.101	2.00	0.00E+00	-0.01231	6.067457

The Jarque-Bera statistics indicate a departure from normality that is to be expected from financial data sets, given that the this test generates highly significant test statistics, we also note the p values are zero and that all data exhibits some form of skewness, UK and Kenya are negatively skewed while Nigeria, Morocco, South Africa and Mauritius exhibit positive skew. Kurtosis co-efficients all indicate a departure from normality, particularly so for Mauritius, Nigeria and Kenya in that order.

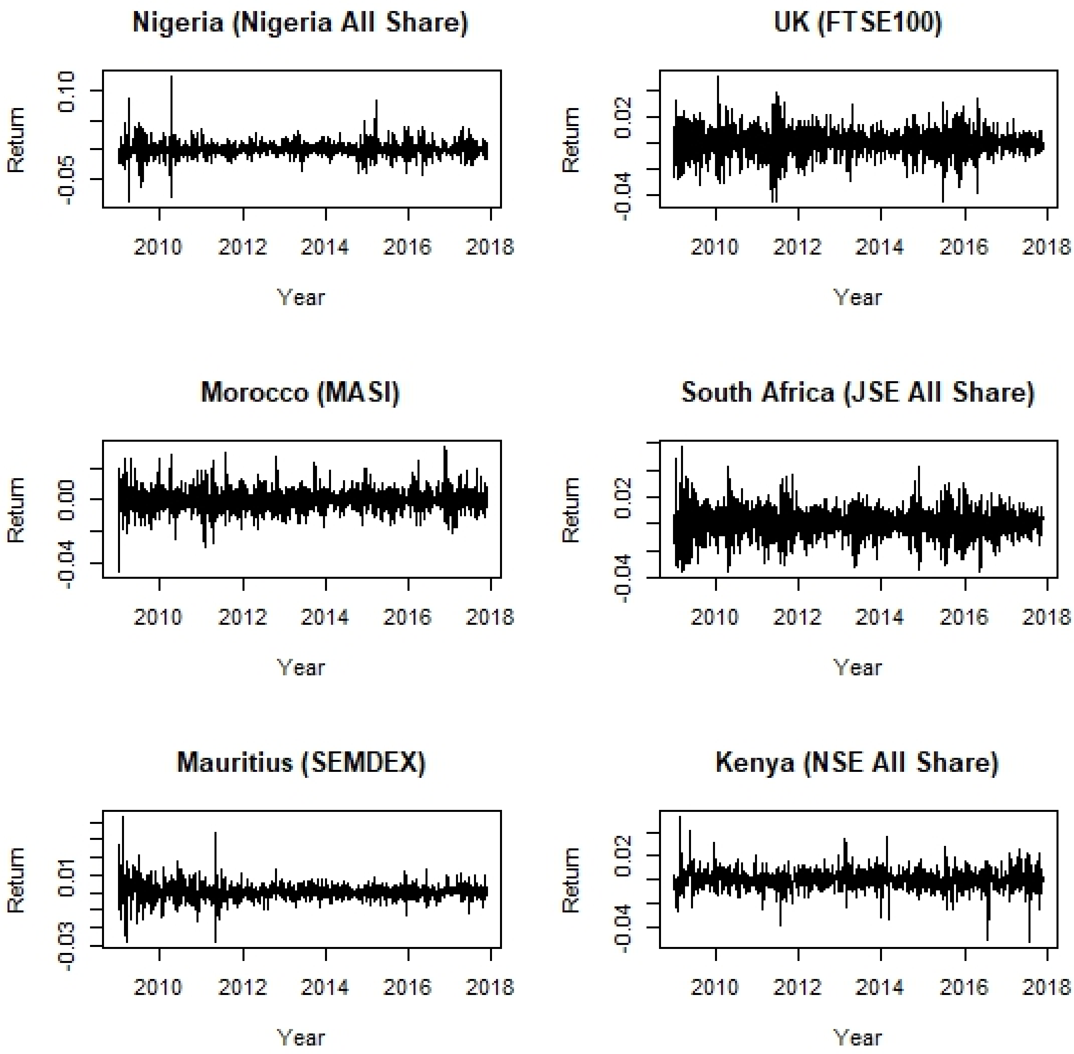

Figure 4.2: Return Time Plot

Observing the plots of the returns shows Kenya exhibiting higher volatility in 2009 and 2016.Nigeria and Mauritius also exhibit higher volatility in 2009 and 2010. UK and South Africa exhibit more consistent returns data, with volatility highest in 2009. Overall, markets appear more stable after 2010.

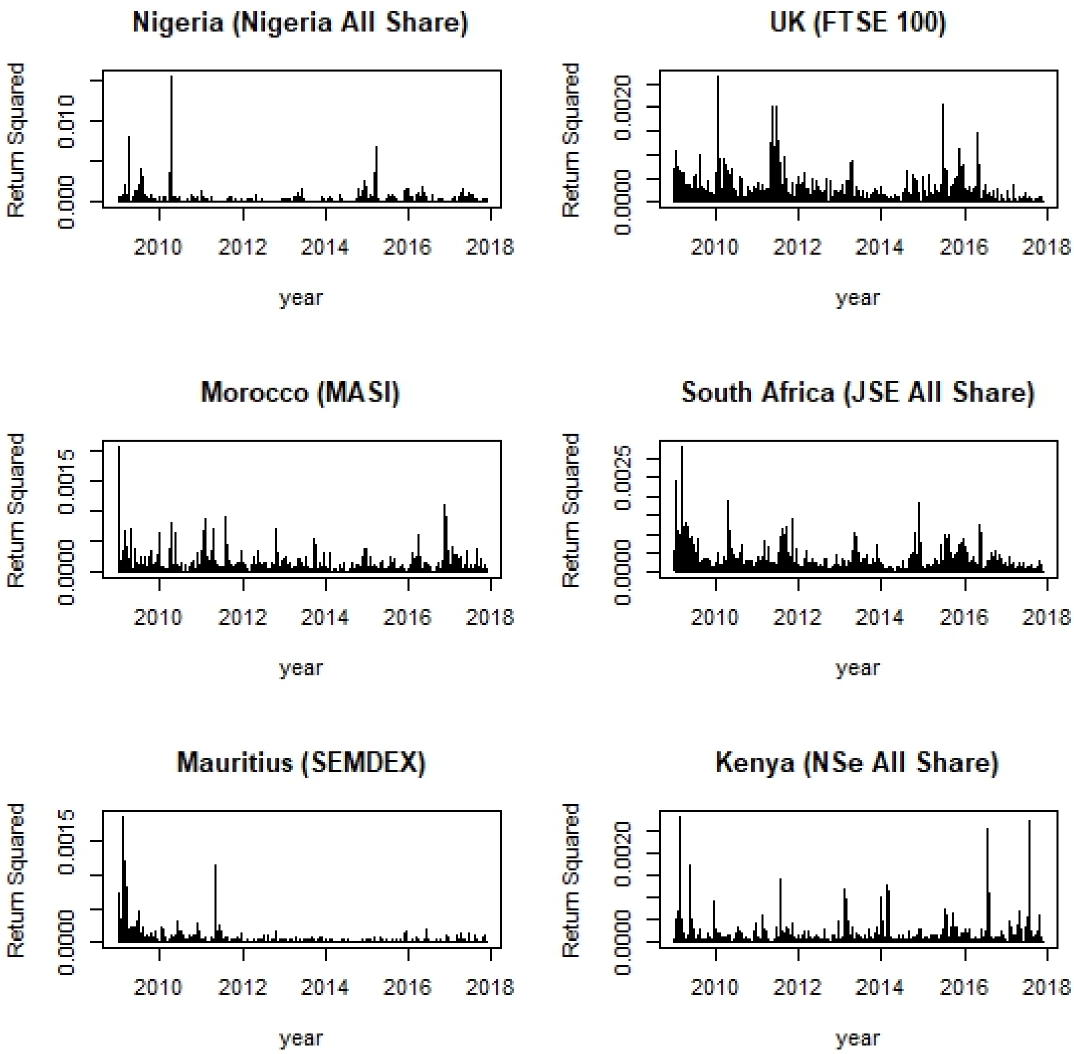

Figure 4.3: Squared Returns Time Plot

Investigating the time plot of the squared return series shows evidence of the presence of conditional heteroskedasticity in this series. This is experienced most significantly in all the countries under consideration in 2009 and 2010, while in Kenya we see the trend intensifying again in 2016-2017, in the latter part of the series.

4.1.2 Correlation Analysis

As in table 4.3 below we can see that the emerging market and the developed market, represented by South Africa and the United Kingdom indices are highly positively correlated with each other, with correlation coefficient of 0.9237, the countries with the highest correlation coefficient are however South Africa and Kenya, with a correlation co-efficient of 0.9295. When we consider the frontier markets, that is Nigeria, Morocco, Mauritius and Kenya, we see that Kenya is most highly correlated with the developed market of UK, with Morocco being the negatively correlated to this market, and indeed all other frontier markets with the exception of Mauritius, with which it has a weak positive correlation of 0.089091.

Table 4.3: Correlation of Stock Indices

	Nigeria	UK	Morocco	SouthAfrica	Mauritius	Kenya
Nigeria	1	0.59064	-0.39238	0.587061	0.634282	0.71637
UK	0.59064	1	-0.13443	0.923693	0.729555	0.862657
Morocco	-0.39238	-0.13443	1	-0.27569	0.089091	-0.27804
SouthAfrica	0.587061	0.923693	-0.27569	1	0.62357	0.929521
Mauritius	0.634282	0.729555	0.089091	0.62357	1	0.674065
Kenya	0.71637	0.862657	-0.27804	0.929521	0.674065	1

We consider the cross correlation plots of each of the frontier markets in relation to the more developed market represented by the United Kingdom. Cross correlation plots are used to gauge the similarity between two series, as well as to examine if lags of one series may be useful in predicting the other series. Looking at figure we can see there is an inverse relationship between Morocco and the UK, with the correlation decreasing with increasing lags till lag 0 from which a steady increase is noted in the positive lags. Kenya and South Africa appear to have increasing correlation that peaks at lag 0 and stabilises thereafter, in contrast to Nigeria's plots which increases, but peaks much later, closer to lag 36.

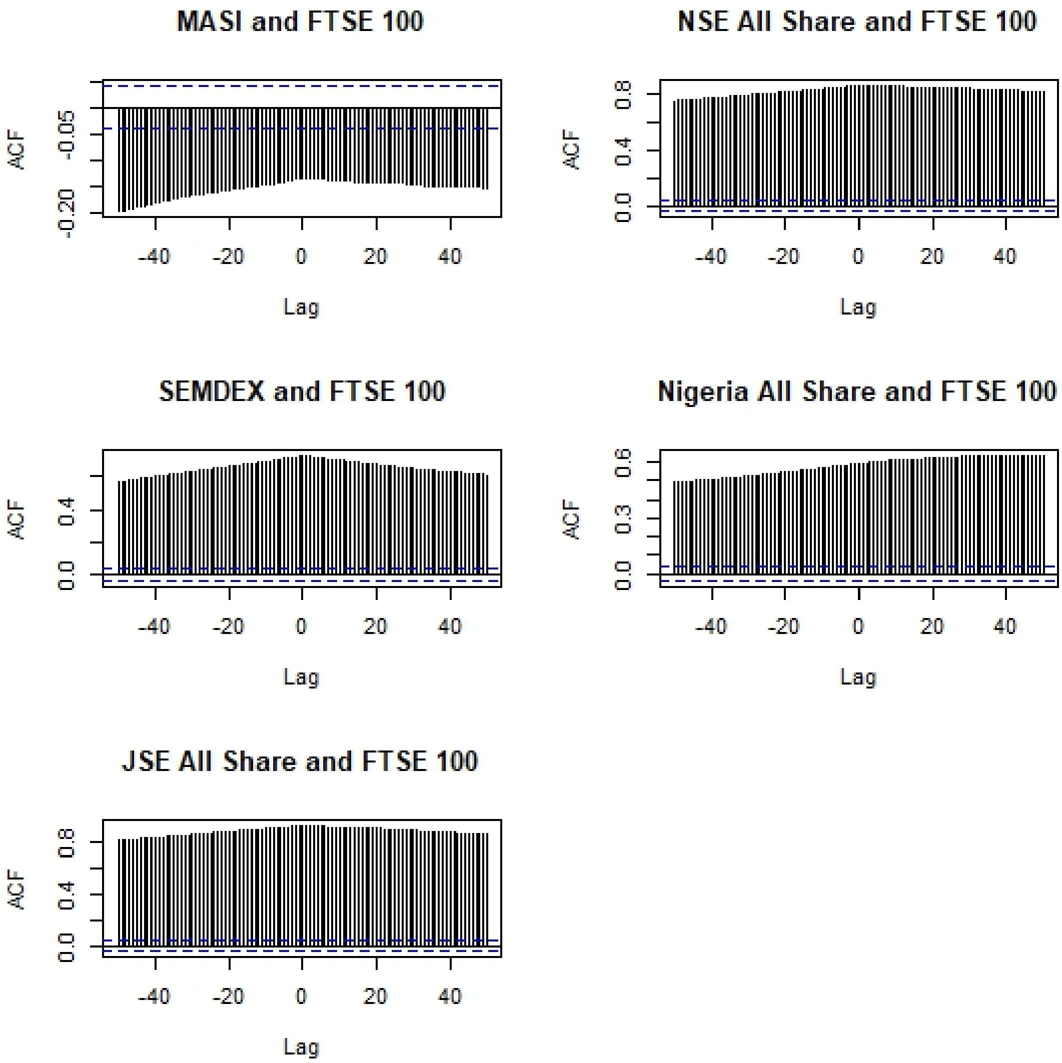

Figure 4.4: Cross Correlation Plot of Returns

We can also examine the cross correlation plots of the squared returns series of the data, as this should give us an idea of what interrelations there are in the second moments, this should inform whether or not we should proceed with a joint estimation or not.Considering figure 5 below we see that at 5% significance level, there is significant interaction in the second moments, most notably between Nigeria and South Africa, the UK and South Africa and South Africa and Mauritius

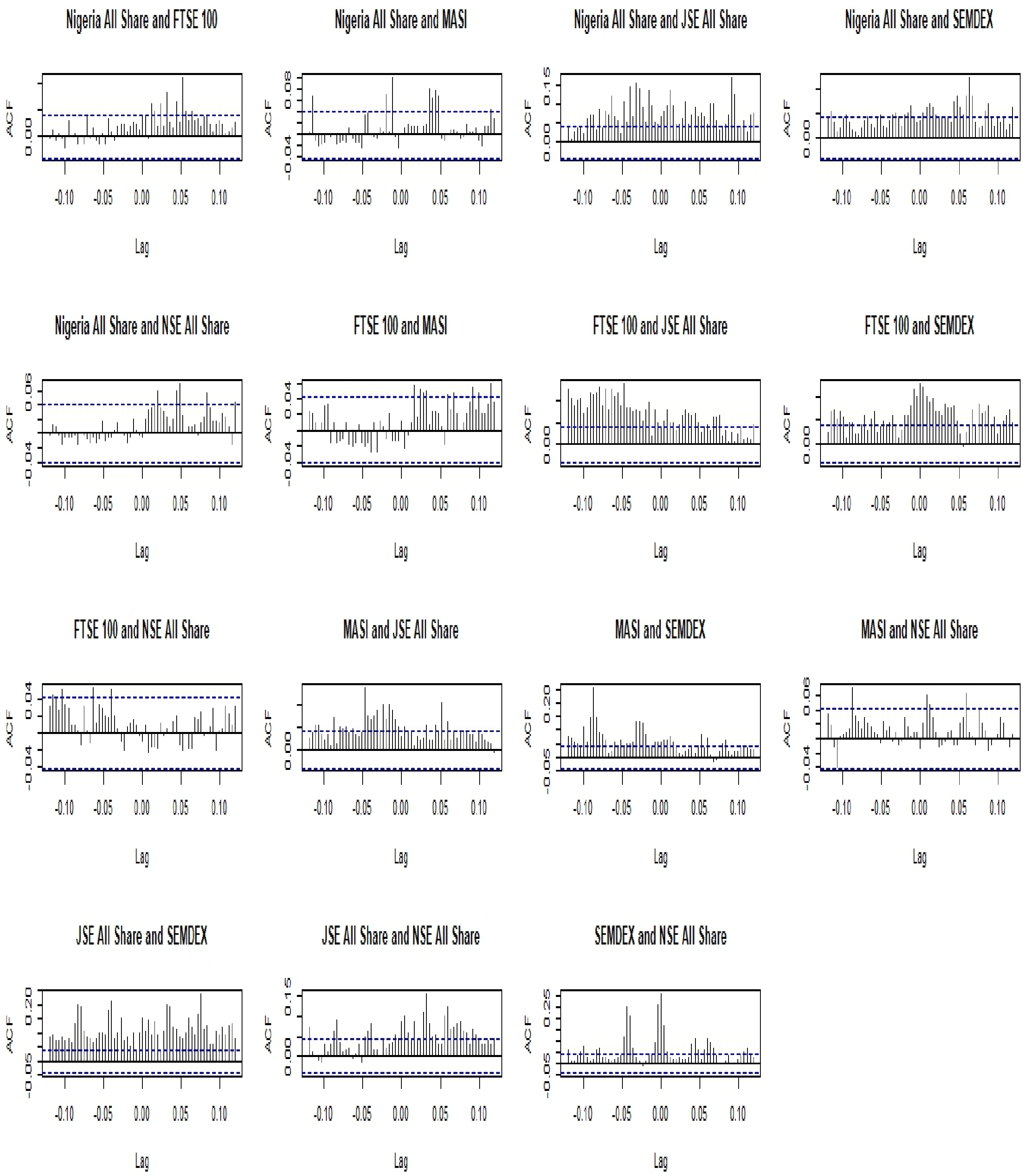

Figure 4.5: Cross Correlation Plot of Squared returns

The autocorrelation function allows us to check for linear dependence at different lags for each particular series. Considering figure we can see evidence of correlation at different lags for all stock market indices. Kenya and Morocco exhibit significant correlations with increasing lags. To further examine the autocorrelations we can use the Ljung Box test, which tests the null hypothesis that each series is independently distributed versus the alternative hypothesis that the the data set is not independently distributed, that is it exhibits serial correlation. The Ljung Box test statistic is given by:

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho_k}^2}{n-k}. \tag{4.4}$$

where n=sample size;
$\rho_k{}^2$= sample auto correlation at lag k;
h=Number of lags

We reject the null hypothesis at α level of significance if $Q > \chi^2_{1-\alpha,h}$. As seen in the table below, at 5% level of significance there is strong statistical evidence to reject the null hypothesis of randomness, hence confirming presence of auto correlation across the data set.

Table 4.4: Ljung-Box Test Statistics

Country	Box. Stats	P Value
Nigeria	223.0293	0
UK	2.600876	0
Morocco	76.9279	0
SouthAfrica	0.042302	0
Mauritius	182.9215	0
Kenya	370.5141	0

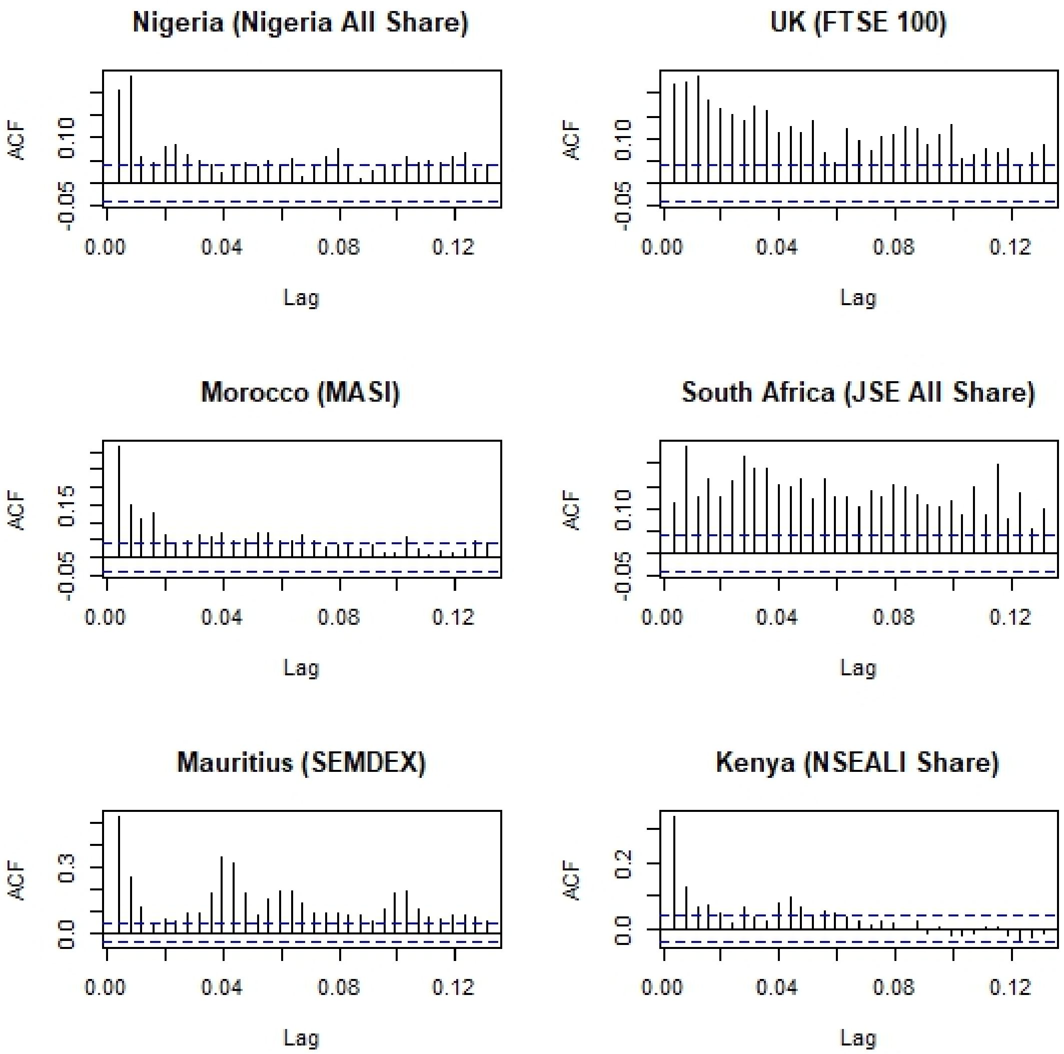

Figure 4.6: Auto Correlation Plot

4.2 Multivariate Volatility Modelling

4.2.1 Exponentially Weighted Moving Average(EWMA) model

As discussed earlier, the Exponentially Weighted Moving Average model is given by:

$$\hat{\mathbf{\Sigma}}_t = \lambda\hat{\mathbf{\Sigma}}_{t-1} + (1-\lambda)\hat{\mathbf{a}}_{t-1}\hat{\mathbf{a}}'_{t-1}. \tag{4.5}$$

where λ, such that $0 < \lambda < 1$ is the persistence parameter and $\hat{\mathbf{a}}_t$ denote the residuals of the mean equation. We begin the recursive process with the sample covariance matrix, i.e.$\hat{\mathbf{\Sigma}}_t = \hat{\mathbf{\Sigma}}_0$, and fit a Vector Auto Regressive process of order one to remove serial correlation in the data set. If we denote the mean equation by $\hat{\boldsymbol{\mu}}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{z}_{t-1}$ where $\mathbf{z}_{t-1}$ denotes the log returns from our sample. We then fit the EWMA model to the residuals, which we can denote by $\mathbf{a}_t$ such that $\hat{\mathbf{a}}_t = z_t - \hat{\boldsymbol{\mu}}_t$ The fitted mean equation has the following co-efficients:
$\mathbf{\Phi}_0$:

$$\begin{bmatrix} 1.03446e-04 & 2.8357e-04 & 6.3516e-05 & 5.4042e-04 & 2.3032e-04 & 2.2933e-04 \end{bmatrix}$$

and $\mathbf{\Phi}_1$:

$$\begin{bmatrix} 0.3145 & -0.03720 & -0.0199 & 0.0058 & -0.0164 & -0.0044 \\ -0.0332 & 0.0268 & 0.0492 & 0.0197 & 0.1927 & 0.0061 \\ -0.0091 & 0.0071 & 0.1822 & 0.0150 & 0.0775 & -0.0030 \\ 0.0209 & 0.0106 & 0.0537 & 0.0033 & -0.1224 & 0.0538 \\ 0.0049 & 0.0061 & 0.0265 & 0.0004 & 0.2826 & -0.02176 \\ 0.0159 & 0.0154 & 0.0245 & 0.0027 & 0.0607 & 0.4057 \end{bmatrix}$$

the model parameter, λ is then estimated by quasi maximum likelihood estimation and used to come up with the the covariance matrix that we can then use for forecasting the next time period. The fit generates the following parameter estimate for λ:

Table 4.5: EWMA Parameter Estimates

Coefficient(s):	Estimate	Std. error	t-Value	Pr(> |t|)
lambda	0.977019	0.001137	859.4	<2e-16 ***
Signif. codes: 0:***	0.001:**	0.0:*	0.05: .	0.1:

However diagnostic checking of the model, based on portmanteau test statistics, still detects the presence of heteroscedasticity, indicating the model is not a good fit, this is in line with our expectations as it is unlikely that the relationship between all six markets can be explained by just one parameter, λ. A summary of the portmanteau test statistics when $\lambda = 0.96$ and when λ is estimated is given below;

Table 4.6: EWMA Diagnostic Tests Results

EWMA Test results:	lambda=0.96	
Qk(m) of epsilon_t:		
Test and p-value:	958.896	0.00E+00
Robust Qk(m):		
Test and p-value:	613.2372	0
EWMA Test results:	lambda (estimated)	
Qk(m) of epsilon_t:		
Test and p-value:	1388.496	0.00E+00
Robust Qk(m):		
Test and p-value:	882.5282	0

4.2.2 Dynamic Conditional Correlation Models

The two types of Dynamic Conditional Correlation models are given by Engle (2002) and Tse and Tsui (2002). Engle's representation is given by;

$$\begin{aligned} \mathbf{Q}_t &= (1-\theta_1-\theta_2)\bar{\mathbf{Q}} + \theta_1 \mathbf{Q}_{t-1} + \theta_2 \boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}, \\ \boldsymbol{\rho}_t &= \mathbf{J}_t \mathbf{Q}_t \mathbf{J}_t. \end{aligned} \quad (4.6)$$

where $\bar{\mathbf{Q}}$ is the unconditional covariance matrix of η_t.
θ_i denotes non negative real numbers such that $0 < \theta_1 + \theta_2 < 1$.
$\mathbf{J}_t = diag(q_{11,t}^{-0.5}, ...q_{kk,t}^{-0.5})$ where $q_{ii,t}$ denotes the (i,i)th element of $\mathbf{Q}_t$.
$\mathbf{Q}_t$ is a positive definite matrix.
$\mathbf{J}_t$ is a normalization matrix.

While Tse and Tsui (2002) propose their DCC model based on correlations where the correlation matrix ρ_t is given by:

$$\boldsymbol{\rho}_t = (1-\theta_1-\theta_2)\bar{\boldsymbol{\rho}} + \theta_1 \boldsymbol{\rho}_{t-1} + \theta_2 \boldsymbol{\psi}_{t-1}. \quad (4.7)$$

where $\bar{\boldsymbol{\rho}}$ denotes the unconditional correlation matrix of $\boldsymbol{\eta}_t$.
θ_i denotes non negative real numbers such that $0 < \theta_1 + \theta_2 < 1$.

$\boldsymbol{\psi}_{t-1}$ is a local correlation matrix depending on $(\boldsymbol{\eta}_{t-1}, ...\boldsymbol{\eta}_{t-m})$for some positive integer m.
The main difference between equations (4.6) and (4.7) is that the Tse and Tsui (2002) representation, updates the conditional correlation matrix using local correlations, and as such will largely depend on the choice of the positive integer m.The larger m is, the soother the resulting correlations. It does not require normalization of the correlation matrix, as is the case with the Engle (2002) representation, which relies on the normalization matrix $\mathbf{J}_t$. It is shown in Tsay (2013) that the difference between the two representations, can be illustrated by considering the expressions for the correlation between the first two innovations. For Tse and Tsui (2002) we have:

$$\rho_{12,t} = \theta^* \bar{\rho_{12}} + \theta_1 \rho_{12,t-1} + \theta_2 \frac{\sum_{i=1}^{m} \eta_{1,t-i} \epsilon_{2,t-i}}{\sqrt{\left\{\sum_{i=1}^{m} \eta_{1,t-i}^2\right\} \left\{\sum_{i=1}^{m} \eta_{2,t-i}^2\right\}}}. \quad (4.8)$$

While for Engle (2002) we have:

$$\rho_{12,t} = \frac{\theta^* \bar{\rho_{12}} + \theta_1 q_{12,t-1} + \theta_2 \eta_{1,t-1} \eta_{2,t-1}}{\sqrt{\left\{\theta^* + \theta_1 q_{11,t-1} + \theta_2 \eta_{1,t-1}^2\right\} \left\{\theta^* + \theta_1 q_{22,t-1} + \theta_2 \eta_{2,t-1}^2\right\}}}. \quad (4.9)$$

Where $\theta^* = 1 - \theta_1 - \theta_2$ and $\bar{\mathbf{Q}} = \bar{\boldsymbol{\rho}}$ is the unconditional covariance matrix of $\boldsymbol{\eta}_t$. DCC models are largely explained by two parameters, θ_1 and θ_2. This makes them relatively easier to implement but again this simplicity may come at the expense of accuracy. Diagnostic tests may show an improvement though from the EWMA approach.

The dccpre command in the MTS package in R does preliminary fitting of a VAR process to the time series data, from here a univariate GARCH models to the residuals of each series from the VAR fit before using the marginally standardised series for the actual multi-variate estimation. Fitting the univariate GARCH(1,1) to each series generates the parameter estimates in Table 8:

Table 4.7: GARCH(1,1) Parameter Estimates

Sample mean of the returns:	Countries		
0.000123	Nigeria		
0.000354	UK		
0.000091	Morocco		
0.000534	South Africa		
0.000309	Mauritius		
0.000439	Kenya		
Component: 1			
Estimates:	0.00001	0.24282	0.71924
se.coef :	0.028778	0.02793	0.02878
t-value :	5.79703	8.69503	24.99283
Component: 2			
Estimates:	2.00E-06	0.11867	0.856217
se.coef :	1.00E-06	0.017808	0.021018
t-value :	3.784917	6.663983	40.78727
Component: 3			
Estimates:	1.00E-05	0.257021	0.495804
se.coef :	2.00E-06	0.036366	0.065983
t-value :	5.661476	7.067523	7.514098
Component: 4			
Estimates:	2.00E-06	0.080434	0.903231
se.coef :	0	0.011428	0.013379
t-value :	3.462996	7.038548	67.51162
Component: 5			
Estimates:	1.00E-06	0.212403	0.737871
se.coef :	0	0.029328	0.033335
t-value :	4.931897	7.242237	22.13477
Component: 6			
Estimates:	7.00E-06	0.290893	0.602203
se.coef :	0.033716	0.033716	0.043982
t-value :	5.536977	8.627637	13.69189

Fitting tse2002multivariate and DCC model generates the parameter estimates below for Tse and Tsui's specification.

$$\boldsymbol{\rho}_t = (1 - 0.4 - 0.007392517)\bar{\boldsymbol{\rho}} + 0.4\boldsymbol{\rho}_{t-1} + 0.007392517\boldsymbol{\psi}_{t-1}. \tag{4.10}$$

while Engle's specification is given by:

$$\begin{aligned} \mathbf{Q}_t = (1 - 0.4 - 0.02649769)\bar{\mathbf{Q}} + 0.4\mathbf{Q}_{t-1} + 0.02649769\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}, \\ \boldsymbol{\rho}_t = \mathbf{J}_t\mathbf{Q}_t\mathbf{J}_t. \end{aligned} \tag{4.11}$$

Observing the diagnostic tests to check for model adequacy we observe that the regular Portmanteau test statistic denoted by Qk(m) in the table does not perform very well at 5% level of significance, this may be attributable to the fact that we are dealing with financial data which has a tendency to be thick tailed. When we consider the robust Portmanteau test statistic, which is modified by trimming away data in the upper 5% tail, we see that this works well for purposes of model testing.

Comparing results of both types of DCC models we see that there is minimal difference in terms of efficacy of the model. Looking at the summary statistics of the time varying correlations of both DCC models we notice that there is more variability in the Engle DCC model indicating that there may be stronger persistence in the time varying correlations. At 5% level of significance using the robust Portmanteau test statistic we can conclude that both models provide an adequate fit for the data, as qualitatively, there isn't much difference between them.

Table 4.8: DCC Fit and Tests of Goodness of Fit

Tse and Tsui DCC			
Estimates:	0.4	0.007393	12.02299
st.errors:	NaN	NaN	0.851129
t-values:	NaN	NaN	14.12594
Engle DCC			
Estimates:	0.400000	0.026498	12.12368
st.errors:	0.156369	0.006949	0.860745
t-values:	2.558054	3.813118	14.0851
Diagnostic Test results:			
Tse and Tsui DCC			
Qk(m) of epsilon_t:			
Test and p-value:	480.0851	2.24E-05	
Robust Qk(m):			
Test and p-value:	383.4456	0.189375	
Engle DCC			
Qk(m) of epsilon_t:			
Test and p-value:	478.2423	2.86E-05	
Robust Qk(m):			
Test and p-value:	378.6712	0.239194	

4.2.3 Cholesky Decomposition

In this section we use the fgarch and mts packages in R, to carry out the Cholesky decomposition.We carry out the multi-variate volatility modelling by first fitting a recursive least squares estimation to equation(8) and obtain the estimates of b_{it} which we can denote by $\hat{\beta_{it}}$ for $i = 1, 2...k$ (a_{it} is a k dimensional innovation). Next we use the Exponentially Weighted Moving Average(EWMA) approach to obtain smoothed estimates of $\hat{\beta_{it}}$. If we denote the sample mean of the estimates by $\hat{\mu_i}$, and the deviations from the mean can be denoted by $\hat{\beta_{it}^*}$ then $\hat{\beta_{it}^*} = \hat{\beta_{it}} - \mu_i$ The smoothed estimates say $\tilde{\beta_{it}} = \tilde{\beta_{it}^*} + \mu_i$ where,

$$\tilde{\beta_{it}^*} = \lambda \tilde{\beta_{it-1}^*} + (1 - \lambda)\hat{\beta_{it}^*}.$$

Given that we had the orthogonal transformation $\hat{b_{1t}} = a_{1t}$, the function MCholV computes the residuals series $\hat{b_{it}} = a_{it} - a_{it}^T\hat{\beta}_{it}$. We then fit a univariate GARCH process to each $\hat{\beta_{it}}$and obtain the conditional variances denoted by σ^2_{bit}. From here we end up generating the fitted variance covariance matrix, say $\hat{\Sigma}_t$ using σ^2_{bit} and $\tilde{\beta_{it}}$.

The fitted GARCH(1,1) series for the return series of Nigeria, United Kingdom, Morocco, South Africa, Mauritius and Kenya respectively is given by volatility models.

$$
\begin{aligned}
\sigma^2_{1t} &= 8 * 10^{-6} + 0.24546b^2_{1t-1} + 0.713745\sigma^2_{1t-1}, \\
\sigma^2_{2t} &= 2 * 10^{-6} + 0.120616b^2_{2t-1} + 0.853532\sigma^2_{2t-1}, \\
\sigma^2_{3t} &= 9 * 10^{-6} + 0.216796b^2_{3t-1} + 0.565339\sigma^2_{3t-1}, \\
\sigma^2_{4t} &= 2 * 10^{-6} + 0.082265b^2_{4t-1} + 0.89972\sigma^2_{4t-1}, \\
\sigma^2_{5t} &= 1 * 10^{-6} + 0.198171b^2_{5t-1} + 0.74506\sigma^2_{5t-1}, \\
\sigma^2_{6t} &= 8 * 10^{-6} + 0.298605b^2_{16-1} + 0.565936\sigma^2_{6t-1}.
\end{aligned} \tag{4.12}
$$

Model checking by way way of evaluation of the robust portmanteau test statistic again fails to detect the presence of conditional heteroscedasticity in the data. Again the difference in test results as compared to the unmodified test statistic at 5% level of significance may be due to non-normality of the raw data. Trimming the upper tail is a viable solution to this challenge. Summary of the test statistics is provided in Table 10 below.

Table 4.9: Portmanteau Test Statistics

Cholesky Decomposition Test results:		
Qk(m) of epsilon_t:		
Test and p-value:	473.9152	5.02E-05
Robust Qk(m):		
Test and p-value:	383.4547	0.189287

4.3 Kalman Filters

In this section we use the dlm package in R to fit a Kalman filter to the data and to perform a 10 day forecast using the fitted filter. The acronym dlm here refers to dynamic linear models of which the Kalman filter is an example. In general we can define a a state space model as one that is made up of two states, the observed state, which is conditionally independent and can be denoted by $y_o, y_1......, y_t$ and the unobserved state, denoted by $x_0, x_1....., x_t$. These states are dependent on the probabilities below:

1. $p(x_0)$ - The initial state probabilities.
2. $p(x_t|x_{t-1})$ - The Transition probability matrix
3. $p(y_t|x_t)$ - Conditional probability of the observed variable given the unobserved variable

A simple representation of a dynamic linear model can be formulated as :

$$\mathbf{y_t} = \mathbf{A_t x_t} + \mathbf{b_t}. \tag{4.13}$$

$$\mathbf{x_t} = \mathbf{C_t x_{t-1}} + \mathbf{d_t}. \tag{4.14}$$

Where b_t is normally distributed, with distribution $N_m(0, B_t)$ and d_t is also normally distributed, with distribution $N_p(0, D_t)$. This model can be initialized by setting x_0 $N(\mu_0, E_0)$ Equations (4.13) and (4.14) correspond to the measurement and transition equation sets of our Kalman filter as specified in the methodology section, with non constant co-efficients.

$\mathbf{y_t}$ represents the observations of our state space system, while the vector $\mathbf{x_t}$ corresponds to the unobserved states of the system that are assumed to evolve in time according to an m x m matrix, C_t, which can be thought of as a linear system operator. It is important to note that the assumption of linearity implies that A_t and C_T are linear operators, though they may be allowed to change through time. In terms of statistical distributions, we are interested in the probabilities $p(y_t|x_t, \theta), p(x_t|\theta)$ and $p(\theta)$. The dlm package in R allows us to take a Bayesian approach, where we have full prior probabilities, that are then estimated by maximum likelihood and plugged back in the equations.

The dlmFilter and dlmSmooth function uses the recursive Kalman filter formulas to estimate the marginal distributions of DLM states given the observations. We assume that the initial distributions at $t = 1$ are available. First, we perform

Kalman filter forward recursion for the predicted states:

$$\begin{aligned} p(\mathbf{x_{t+1}}|\mathbf{x_t}, \mathbf{y_{1:t}}, \theta &= \mathbf{N}(\mathbf{\hat{x}_{t+1}}, \mathbf{F_{t+1}}). \\ t &= 1, 2, ...n-1. \end{aligned} \quad (4.15)$$

Where

$$\begin{aligned} \mathbf{v_t} &= \mathbf{y_t} - \mathbf{A_t}\mathbf{\hat{x_t}}. \\ \mathbf{F_t^y} &= \mathbf{A_t}\mathbf{\hat{F}_t}\mathbf{A^T} + \mathbf{B_t}. \\ \mathbf{K_t} &= \mathbf{C_t}\mathbf{\hat{F}_t}\mathbf{A^T}\mathbf{F_t^{y-1}}. \\ \mathbf{\hat{x}_{t+1}} &= \mathbf{C}\mathbf{\hat{x}_t} + \mathbf{K_t}\mathbf{b_t}. \\ \mathbf{\hat{F}_{t+1}} &= \mathbf{C_t}\mathbf{\hat{F}_t}(\mathbf{C_t} - \mathbf{K_t}\mathbf{A_t}) + \mathbf{D}. \end{aligned} \quad (4.16)$$

Where $\mathbf{v_t}, \mathbf{F_t^y}, \mathbf{K_t}, \mathbf{\hat{x}_{t+1}}$ and $\mathbf{\hat{F}_{t+1}}$ denot the prediction error, the prediction error covariance, the Kalman gain, the next state prior mean and the next state prior covariance respectively. Then, we apply Kalman smoother backward recursion to obtain the smoothed states through the dlmSmooth function:

$$\begin{aligned} p(\mathbf{x_t}|\mathbf{y_{1:n}}, \theta) &= \mathbf{N}(\mathbf{x_t}, \mathbf{\tilde{F}_t}). \\ t &= n, n-1, ...2, 1. \end{aligned} \quad (4.17)$$

Where

$$\begin{aligned} \mathbf{L} &= \mathbf{C_t} - \mathbf{K_t}\mathbf{F_t}. \\ \mathbf{r} &= \mathbf{A_t^T}\mathbf{F_t^{y-1}}\mathbf{b_t} + \mathbf{L^T}\mathbf{r}. \\ \mathbf{N} &= \mathbf{A_t^T}\mathbf{F_t^{y-1}}\mathbf{A_t} + \mathbf{L^T}\mathbf{N}\mathbf{L}. \\ \mathbf{\tilde{x}_t} &= \mathbf{\hat{x}_t} + \mathbf{\hat{F}_t}\mathbf{r}. \\ \mathbf{\tilde{F}_t} &= \mathbf{\hat{F}_t} - \mathbf{\hat{F}_t}\mathbf{N}\mathbf{\hat{F}_t}. \end{aligned} \quad (4.18)$$

Where $\mathbf{\tilde{x}_t}$ and $\mathbf{\tilde{F}_t}$ denote the smoothed state mean and smoothed state covariance respectively.

The maximum likelihood estimation for the Kalman filter is done using the function dlmMLE which returns the unknown parameters of the state space model as specified above This estimation is based on the fitting of a random walk plus noise model to the underlying time series data. From this we obtain the variance of the observation noise of each time series denoted by V and the variance of the state vector, denoted by W. A summary of these values is given by the table below:

Table 4.10: Kalman Maximum Likelihood Estimation of Parameters

	Nigeria	UK	Morocco	South Africa	Mauritius	Kenya
V	0.2681121	0.0009260	0.0001163	0.918896	1.04E-06	2.79E-09
W	109,033.90	832.11	4,922.14	155022.3	57.45487	0.774253

These parameters are then used to specify the Random Walk plus Noise model that is then used to fit the Kalman filter and generate the Kalman filter results, using the dlmFilter function. We obtain the filtered values of the state vectors as well as the variance covariance matrices of the same. A plot of the time series data of the indices (in yellow) against their filtered state estimates (in blue) is shown below:

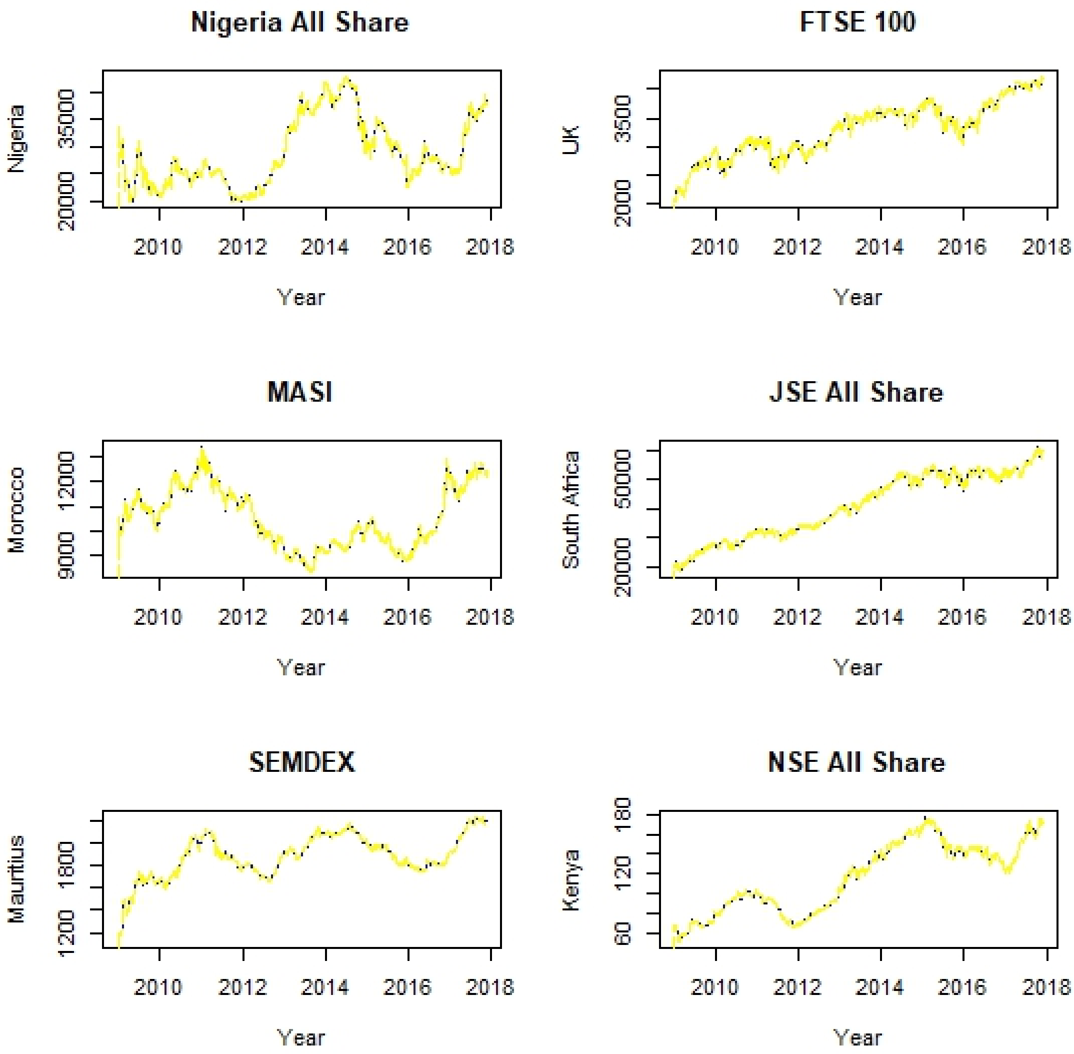

Figure 4.7: Kalman Filter Plot

The function dlmSMooth is used to obtain smoothed values of the state space vector as well as it's associated variance-covariance matrix, the plot of the time series data of the indices (in red) against their smoothed state estimates (in green) is shown in figure 8 below:

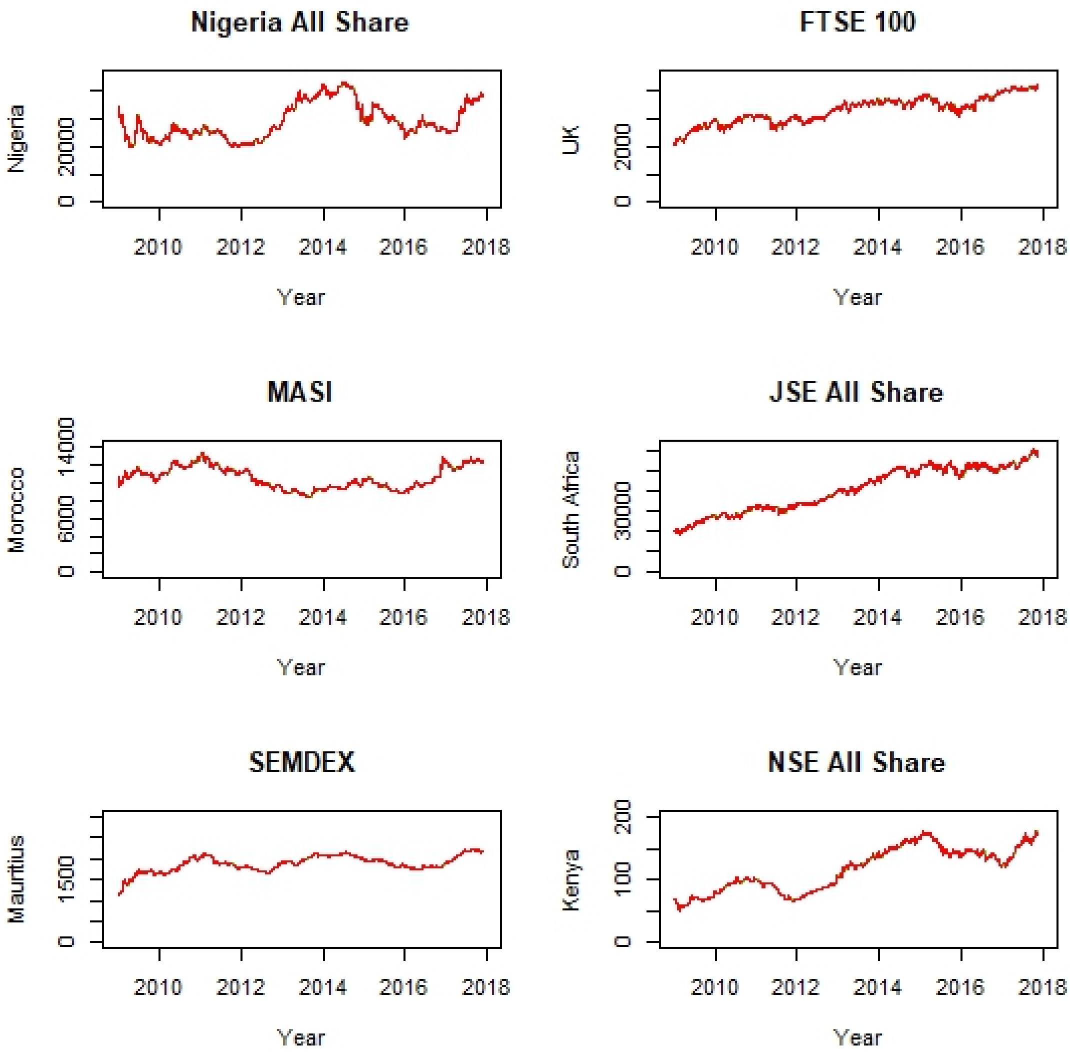

Figure 4.8: Kalman Smoothed Values Plot

The dlmForecast function can now be used to predict future observed values as well as system states, samples can also be generated to show the possible evolution of the stock path indices. In figure 9 below we can see how increasing the number of sample sizes improves the accuracy of the mean level.

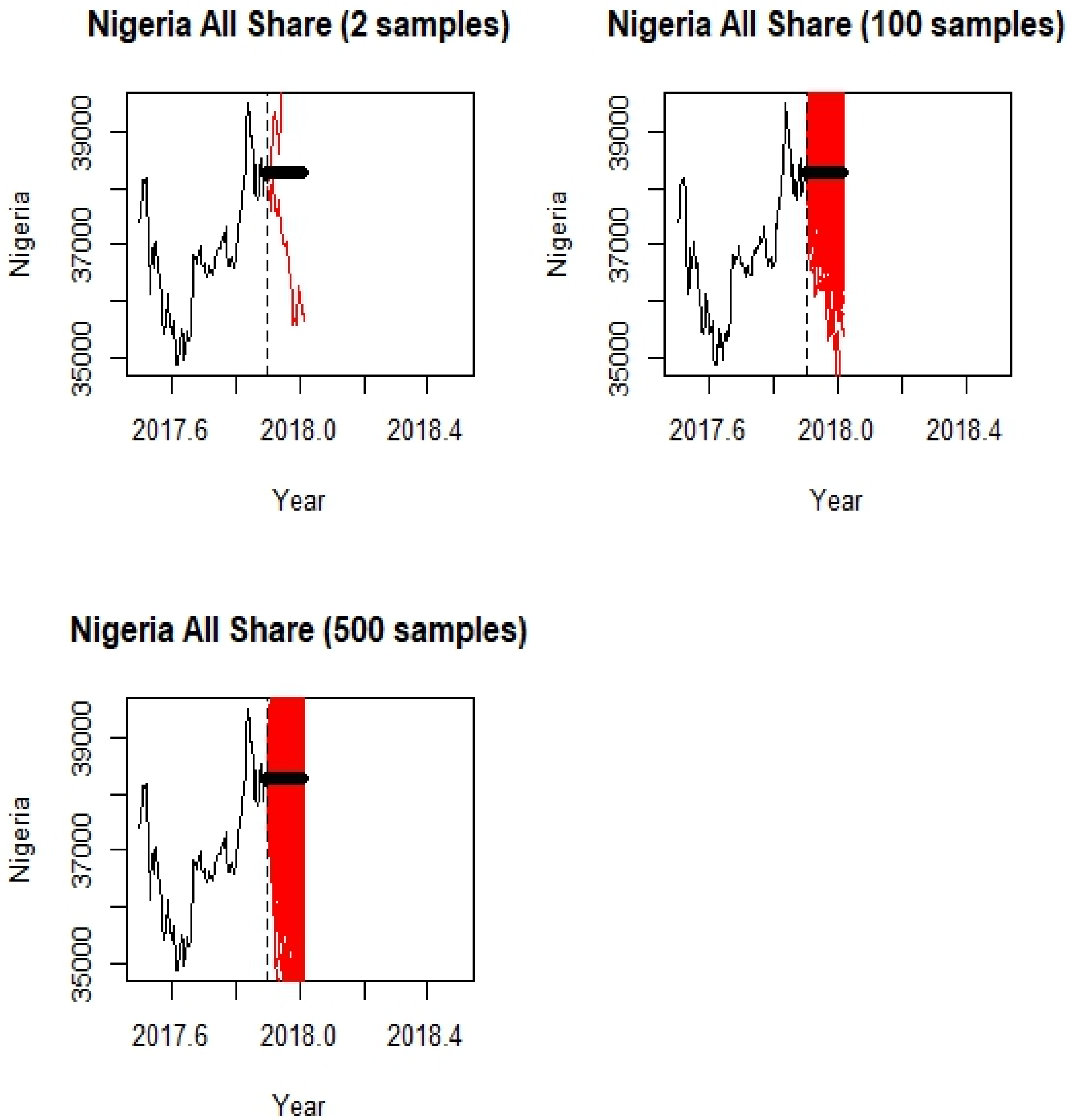

Figure 4.9: Kalman Forecasted Values Plot:Nigeria

We can also do the same for the other markets, to illustrate we will use two paths as in figure 10 below, but for the sake of accuracy, it is advised to use a higher number of samples when generating forecasts.

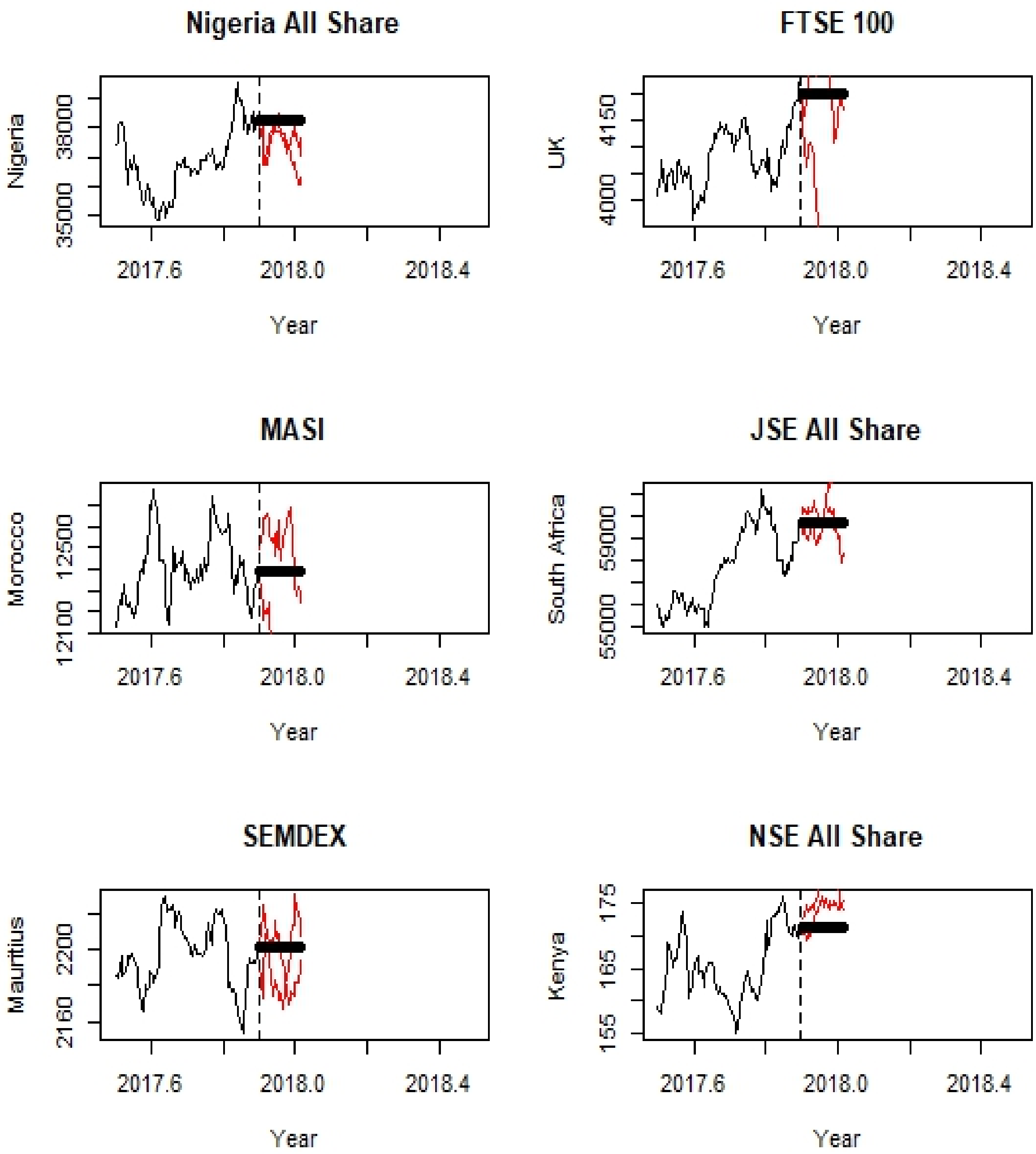

Figure 4.10: Kalman Forecasted Values Plot

Chapter 5

Discussion and Conclusion

Understanding how markets move in tandem with one another is increasingly more important in today's era of globalisation and market integration.Removal of barriers to trade through technology have made it possible to build ever more geographically diversified portfolios with ease. With these developments come the risk of loss to investors through a lack of awareness of how volatility is transmitted between these markets. This is important not only for construction and rebalancing of portfolios on the part of investors but also for market regulators as they design policy interventions following volatility shocks on a regional or global scale.

In this study, we use various multivariate volatility models to examine the volatility structure between the stock indices of four frontier markets, namely Nigeria, Morocco, Mauritius and Kenya, an emerging market represented by South Africa and a developed market represented by the UK.The data set is drawn from the period January 2009 to December 2017. Initial exploration of the data reveals deviations from normality and heavy tails as seen in the Ljung Box test statistics which is line with key stylised facts about financial data.Interestingly Morocco stands out as being less integrated with other markets considered which may provide some protection in terms of diversification. Kenya moves quite closely with South African and the UK markets.

Among the multivariate models, the EWMA model is rejected by diagnostic tests based on the robust Portmanteau test statistic, leaving DCC models and volatility models based on Cholesky decomposition as being most suitable for the data set. DCC model estimation however is the most time consuming of the three models fitted. Of the two specifications empirical results show that the Engle model has time varying correlations of higher variability which is in line with expectations as it does not use a local correlation matrix in estimation like the Tse and Tsui specification.

We also demonstrate how a Kalman filter may be fitted to the data set and used for purposes of forecasting. The main weakness of this approach is that it does not consider the co-dependencies of the indices but rather models each market separately. Looking at the residuals, the Kalman filter provides an adequate fit to each series, but is less informative on how they evolve together. It is however the most useful for generating forecasts as these can be easily generated with greater accuracy arising out of using a higher number of sample paths.

In this study, no attempt was made to convert the stock index returns to one base currency in order to provide a more accurate picture of the volatility movements. Using a base currency however could introduce the added problem of considering fluctuations in the corresponding exchange rates, a problem that is not the main focus of this study. It is a recommendation for further research that a base currency be explored for purposes of standardised comparison.

Another area in which further research may be necessary is that of contagion, or the study of volatility spillover effects from one country to others due to unforeseen, country specific shocks. An example could be the effect of United Kingdom's decision to leave the European Union in 2016 on frontier markets, or the effect of the European debt crisis of 2011-2012 on the frontier markets in this study. Studies of this particular events could hold lessons for policy interventions as well as shed light on the degree of integration witnessed in periods of market stress.

It is also notable that we use a dynamic linear filter in this study which makes assumptions on normality and linearity of the data that may not be accurate, we recommend that future studies consider extended Kalman filters to account for data that shows evidence of non linearities, such as may be the case where the transition equation (50) may be reformulated as function of it's value in the previous state as illustrated below;

$$\mathbf{x_t} = \mathbf{f_{t-1}}(\mathbf{x_{t-1}}) + \mathbf{d_t} \tag{5.1}$$

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Appendices

.1 Data Preliminaries

```
data2=read.csv("PD2.csv")
ni<-ts(data2$Nigeria)
uk<-ts(data2$UK)
mo<-ts(data2$Morocco)
sa<-ts(data2$SA)
ma<-ts(data2$Mauritius)
ke<-ts(data2$Kenya)
```

.2 Time series Plots

```
#Time series data plot
par(mfrow=c(2,3))
plot(ni,type="l",xlab="Time",ylab="Index",main="Nigeria")
plot(uk,type="l",xlab="Time",ylab="Index",main="UK")
plot(mo,type="l",xlab="Time",ylab="Index",main="Morocco")
plot(sa,type="l",xlab="Time",ylab="Index",main="South Africa")
plot(ma,type="l",xlab="Time",ylab="Index",main="Mauritius")
plot(ke,type="l",xlab="Time",ylab="Index",main="Kenya")

#time plot returns
par(mfrow = c(3, 2))
plot(rtnn,type="l",xlab="Year",ylab="Return",main="Nigeria (Nigeria All Share)")
plot(rtnu,type="l",xlab="Year",ylab="Return",main="UK (FTSE100)")
plot(rtnm,type="l",xlab="Year",ylab="Return",main="Morocco (MASI)")
plot(rtns,type="l",xlab="Year",ylab="Return",main="South Africa (JSE All Share)")
plot(rtnma,type="l",xlab="Year",ylab="Return",main="Mauritius (SEMDEX)")
plot(rtnk,type="l",xlab="Year",ylab="Return",main="Kenya (NSE All Share)")

#Time plot for squared returns
par(mfrow = c(3,2))
plot(n_squared,type="l",xlab="year",ylab="Return Squared",main="Nigeria (Nigeria All Share)")
plot(u_squared,type="l",xlab="year",ylab="Return Squared",main="UK (FTSE 100)")
plot(m_squared,type="l",xlab="year",ylab="Return Squared",main="Morocco (MASI)")
plot(s_squared,type="l",xlab="year",ylab="Return Squared",main="South Africa (JSE All Share)")
plot(ma_squared,type="l",xlab="year",ylab="Return Squared",main="Mauritius (SEMDEX)")
plot(k_squared,type="l",xlab="year",ylab="Return Squared",main="Kenya (NSe All Share)")
```

.3 Returns

```
#returns
rtn=diff(ni)/ni[-length(ni)]
rtu=diff(uk)/uk[-length(uk)]
rtm=diff(mo)/mo[-length(mo)]
rts=diff(sa)/sa[-length(sa)]
rtma=diff(ma)/ma[-length(ma)]
rtk=diff(ke)/ke[-length(ke)]

#returns_time series
rtnn=ts(rtn,frequency=252,start=c(2009,1))
rtnu=ts(rtu,frequency=252,start=c(2009,1))
rtnm=ts(rtm,frequency=252,start=c(2009,1))
rtns=ts(rts,frequency=252,start=c(2009,1))
rtnma=ts(rtma,frequency=252,start=c(2009,1))
rtnk=ts(rtk,frequency=252,start=c(2009,1))

#Squared Daily returns
```

```
n_squared=rtnn^2
u_squared=rtnu^2
m_squared=rtnm^2
s_squared=rtns^2
ma_squared=rtnma^2
k_squared=rtnk^2
```

.4 Jarque Bera Tests

```
#Skewness
sn=skewness(rtn)
su=skewness(rtu)
sm=skewness(rtm)
ss=skewness(rts)
sma=skewness(rtma)
sk=skewness(rtk)
skewness=c(sn,su,sm,ss,sma,sk)

#kurtosis
kn=kurtosis(rtn)
ku=kurtosis(rtu)
km=kurtosis(rtm)
ks=kurtosis(rts)
kma=kurtosis(rtma)
kk=kurtosis(rtk)
kurtosis=c(kn,ku,km,ks,kma,kk)

#jacque bera test of normality
jbn=jarque.bera.test(rtn)
jbu=jarque.bera.test(rtu)
jbm=jarque.bera.test(rtm)
jbs=jarque.bera.test(rts)
jbma=jarque.bera.test(rtma)
jbk=jarque.bera.test(rtk)

jacq.bera=c(jbn,jbu,jbm,jbs,jbma,jbk)

stock=c("Nigeria","UK","Morocco","South Africa","Mauritius","Kenya")
jacqbera.test=c(jbn$statistic,jbu$statistic,jbm$statistic,
jbs$statistic,jbma$statistic,jbk$statistic)
df=c(jbn$parameter,jbu$parameter,jbm$parameter,jbs$parameter,
jbma$parameter,jbk$parameter)
p.value=c(jbn$p.value,jbu$p.value,jbm$p.value,jbs$p.value,jbma$p.value,jbk$p.value)
norm.test=data.frame(stock,jacqbera.test,df,p.value,skewness,kurtosis)

norm.test
```

.5 Correlation Analysis

```
#Correlation
cor(ni,uk)
cor(ni,mo)
cor(ni,sa)
cor(ni,ma)
cor(ni,ke)
cor(uk,mo)
cor(uk,sa)
cor(uk,ma)
cor(uk,ke)
```

```
cor(mo,sa)
cor(mo,ma)
cor(mo,ke)
cor(sa,ma)
cor(sa,ke)
cor(ma,ke)


#Autocorrelation function
par(mfrow=c(3,2))
acf(rtnn,main="Nigeria (Nigeria All Share)")
acf(rtnu,main="UK (FTSE100)")
acf(rtnm,main="Morocco (MASI)")
acf(rtns,main="South Africa (JSE ALL Share)")
acf(rtnma,main="Mauritius (SEMDEX)")
acf(rtnk,main="Kenya (NSE All Share)")
#cross-correlation plot
par(mfrow=c(4,4))
ccf(n_squared,u_squared,main="Nigeria All Share and FTSE 100")
ccf(n_squared,m_squared, main="Nigeria All Share and MASI")
ccf(n_squared,s_squared, main="Nigeria All Share and JSE All Share")
ccf(n_squared, ma_squared, main="Nigeria All Share and SEMDEX")
ccf(n_squared,k_squared, main="Nigeria All Share and NSE All Share")
ccf(u_squared,m_squared, main="FTSE 100 and MASI")
ccf(u_squared,s_squared, main="FTSE 100 and JSE All Share")
ccf(u_squared,ma_squared, main="FTSE 100 and SEMDEX")
ccf(u_squared,k_squared, main="FTSE 100 and NSE All Share")
ccf(m_squared,s_squared, main="MASI and JSE All Share")
ccf(m_squared,ma_squared, main="MASI and SEMDEX")
ccf(m_squared,k_squared, main="MASI and NSE All Share")
ccf(s_squared,ma_squared, main="JSE All Share and SEMDEX")
ccf(s_squared,k_squared, main="JSE All Share and NSE All Share")
ccf(ma_squared,k_squared, main="SEMDEX and NSE All Share")

#crosscorrelation plot
par(mfrow=c(3,2))
ccf(data2$Morocco,data2$UK ,50, main="MASI and FTSE 100")
ccf(data2$Kenya,data2$UK,50,main="NSE All Share and FTSE 100")
ccf(data2$Mauritius,data2$UK,50, main="SEMDEX and FTSE 100")
ccf(data2$Nigeria,data2$UK,50,main="Nigeria All Share and FTSE 100")
ccf(data2$SA,data2$UK,50, main="JSE All Share and FTSE 100")
```

.6 Ljung Box Test Statistics

```
#ljung box test statistics
boxn=Box.test(rtnn,type="Ljung",lag=1,fitdf=1)
boxu=Box.test(rtnu,type="Ljung",lag=1,fitdf=1)
boxm=Box.test(rtnm,type="Ljung",lag=1,fitdf=1)
boxs=Box.test(rtns,type="Ljung",lag=1,fitdf=1)
boxma=Box.test(rtnma,type="Ljung",lag=1,fitdf=1)
boxk=Box.test(rtnk,type="Ljung",lag=1,fitdf=1)
box=c(boxn,boxu,boxm,boxs,boxma,boxk)

stock=c("Nigeria","UK","Morocco","South Africa","Mauritius","Kenya")
box.stats=c(boxn$statistic,boxu$statistic,boxm$statistic,
boxs$statistic,boxma$statistic,boxk$statistic)
df=c(boxn$parameter,boxu$parameter,boxm$parameter,boxs$parameter,
boxma$parameter,boxk$parameter)
p.value=c(boxn$p.value,boxu$p.value,boxm$p.value,boxs$p.value,
boxma$p.value,boxk$p.value)
box.test=data.frame(stock,box.stats,df,p.value)
```

.7 Multivariate Models and Goodness of fit tests

```
##### Exponentially Weighted Moving Average Approach
m1=VAR(data3,1) ## Fit VAR(1) model to remove serial correlations
at=m1$residuals ## ARCH test
MarchTest(at)
m2=EWMAvol(at,lambda=0.96)
Sigma.t=m2$Sigma.t ### Volatility matrices
summary(Sigma.t)
m3=MCHdiag(at,Sigma.t)
m4=EWMAvol(at,lambda=-0.1) ### Estimation of decaying rate
Sigma.t=m4$Sigma.t ### Volatility matrices
m5=MCHdiag(at,Sigma.t)
m6<-EWMATVC(m2)

#### DCC Models
lreturn=read.csv("lrtn.csv")
m1=dccPre(lreturn,include.mean=T,p=0)
names(m1)
rtn1=m1$sresi
Vol=m1$marVol
m2=dccFit(rtn1) ### Use Tse and Tsui model
names(m2)
S2.t=m2$rho.t
m3=dccFit(rtn1,type="Engle") ## Use Engle model
S3.t=m3$rho.t
MCHdiag(rtn1,S2.t) ### Model checking
MCHdiag(rtn1,S3.t)
m4=dccPre(data3,include.mean=T,p=0)
dat2=m4$sresi
Vol2=m4$marVol
m5=dccFit(dat2)
names(m5)
S4.t=m5$rho.t
m6=dccFit(dat2,type="Engle") ## Use Engle model
S5.t=m6$rho.t

##### Cholesky Decomposition
require(fGarch)
m4=MCholV(lreturn)
names(m4)
at=scale(lreturn[37:2242,],center=T,scale=F)
Sigma.t=m4$Sigma.t
MCHdiag(at,Sigma.t)
```

.8 Kalman Filter fit, smoothing and forecasts

```
###Kalman Filters
library(dlm)
nig<-ts(data2$Nigeria,start=c(2009,1),frequency=252)
unk<-ts(data2$UK,start=c(2009,1),frequency=252)
mor<-ts(data2$Morocco,start=c(2009,1),frequency=252)
soa<-ts(data2$SA,start=c(2009,1),frequency=252)
mau<-ts(data2$Mauritius,start=c(2009,1),frequency=252)
ken<-ts(data2$Kenya,start=c(2009,1),frequency=252)

kalmanBuild<-function(par)
{
dlmModPoly(1,dV=exp(par[1]),dW = exp(par[2]))
}
```

```
kalmanMLE1<-dlmMLE(nig,rep(0,2),kalmanBuild)
kalmanMLE2<-dlmMLE(unk,rep(0,2),kalmanBuild)
kalmanMLE3<-dlmMLE(mor,rep(0,2),kalmanBuild)
kalmanMLE4<-dlmMLE(soa,rep(0,2),kalmanBuild)
kalmanMLE5<-dlmMLE(mau,rep(0,2),kalmanBuild)
kalmanMLE6<-dlmMLE(ken,rep(0,2),kalmanBuild)

kalmanMLE1$conv
kalmanMLE2$conv
kalmanMLE3$conv
kalmanMLE4$conv
kalmanMLE5$conv
kalmanMLE6$conv

kalmanMod1 <- kalmanBuild(kalmanMLE1$par)
V(kalmanMod1)
W(kalmanMod1)
kalmanMod2 <- kalmanBuild(kalmanMLE2$par)
V(kalmanMod2)
W(kalmanMod2)
kalmanMod3 <- kalmanBuild(kalmanMLE3$par)
V(kalmanMod3)
W(kalmanMod3)
kalmanMod4 <- kalmanBuild(kalmanMLE4$par)
V(kalmanMod4)
W(kalmanMod4)
kalmanMod5 <- kalmanBuild(kalmanMLE5$par)
V(kalmanMod5)
W(kalmanMod5)
kalmanMod6 <- kalmanBuild(kalmanMLE6$par)
V(kalmanMod6)
W(kalmanMod6)

kalmanPoly1<- dlmModPoly(order = 1, dV = 0.2681121, dW = 109033.9)
kalmanPoly2<- dlmModPoly(order = 1, dV = 0.0009260116, dW = 832.1146)
kalmanPoly3<- dlmModPoly(order = 1, dV = 0.0001162985, dW = 4922.138)
kalmanPoly4<- dlmModPoly(order = 1, dV = 0.9188961, dW = 155022.3 )
kalmanPoly5<- dlmModPoly(order = 1, dV = 0.000001044, dW = 57.45487)
kalmanPoly6<- dlmModPoly(order = 1, dV = 0.000000002789, dW = 0.7742532)

unlist(kalmanPoly1)
unlist(kalmanPoly2)
unlist(kalmanPoly3)
unlist(kalmanPoly4)
unlist(kalmanPoly5)
unlist(kalmanPoly6)

kalmanFilt1<-dlmFilter(nig,kalmanPoly1)
str(kalmanFilt1,1)
kalmanFilt2<-dlmFilter(unk,kalmanPoly2)
str(kalmanFilt2,1)
kalmanFilt3<-dlmFilter(mor,kalmanPoly3)
str(kalmanFilt3,1)
kalmanFilt4<-dlmFilter(soa,kalmanPoly4)
str(kalmanFilt4,1)
kalmanFilt5<-dlmFilter(mau,kalmanPoly5)
str(kalmanFilt5,1)
kalmanFilt6<-dlmFilter(ken,kalmanPoly6)
str(kalmanFilt6,1)

n1 <- length(nig)
attach(kalmanFilt1)
```

```
dlmSvd2var(U.C[[n1 + 1]], D.C[n1 + 1,])
n2 <- length(unk)
attach(kalmanFilt2)
dlmSvd2var(U.C[[n2 + 1]], D.C[n2 + 1,])
n3 <- length(mor)
attach(kalmanFilt3)
dlmSvd2var(U.C[[n3 + 1]], D.C[n3 + 1,])
n4 <- length(soa)
attach(kalmanFilt4)
dlmSvd2var(U.C[[n4 + 1]], D.C[n4 + 1,])
n5 <- length(mau)
attach(kalmanFilt5)
dlmSvd2var(U.C[[n5 + 1]], D.C[n5 + 1,])
n6 <- length(ken)
attach(kalmanFilt6)
dlmSvd2var(U.C[[n6 + 1]], D.C[n6 + 1,])

kalmanSmooth1 <- dlmSmooth(kalmanFilt1)
str(kalmanSmooth1, 1)
attach(kalmanSmooth1)
drop(dlmSvd2var(U.S[[n1 + 1]], D.S[n1 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth21<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))
kalmanSmooth2 <- dlmSmooth(kalmanFilt2)
str(kalmanSmooth2, 1)
attach(kalmanSmooth2)
drop(dlmSvd2var(U.S[[n2 + 1]], D.S[n2 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth22<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))
kalmanSmooth3 <- dlmSmooth(kalmanFilt3)
str(kalmanSmooth3, 1)
attach(kalmanSmooth3)
drop(dlmSvd2var(U.S[[n3 + 1]], D.S[n3 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth23<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))
kalmanSmooth4 <- dlmSmooth(kalmanFilt4)
str(kalmanSmooth4, 1)
attach(kalmanSmooth4)
drop(dlmSvd2var(U.S[[n4 + 1]], D.S[n4 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth24<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))
kalmanSmooth5 <- dlmSmooth(kalmanFilt5)
str(kalmanSmooth5, 1)
attach(kalmanSmooth5)
drop(dlmSvd2var(U.S[[n5 + 1]], D.S[n5 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth25<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))
kalmanSmooth6 <- dlmSmooth(kalmanFilt6)
str(kalmanSmooth6, 1)
attach(kalmanSmooth6)
drop(dlmSvd2var(U.S[[n6 + 1]], D.S[n6 + 1,]))
hwid <- qnorm(0.025, lower = FALSE)*sqrt(unlist(dlmSvd2var(U.S, D.S)))
smooth26<- cbind(s, as.vector(s) + hwid %o% c(-1, 1))

#Kalman Filter Plot
par(mfrow = c(3,2))
plot(nig, xlab="Year", ylab="Nigeria", type="l", col=c("blue"), main="Nigeria All Share")
lines((kalmanFilt1$m), lty = "longdash", col=c("yellow"))
plot(unk, xlab="Year", ylab="UK", type="l", col=c("blue"), main="FTSE 100")
lines((kalmanFilt2$m), lty = "longdash", col=c("yellow"))
plot(mor, xlab="Year", ylab="Morocco", type="l", col=c("blue"), main="MASI")
lines((kalmanFilt3$m), lty = "longdash", col=c("yellow"))
```

```
plot(soa, xlab="Year", ylab="South Africa", type="l", col=c("blue"), main="JSE All Share")
lines((kalmanFilt4$m), lty = "longdash", col=c("yellow"))
plot(mau, xlab="Year", ylab="Mauritius", type="l", col=c("blue"), main="SEMDEX")
lines((kalmanFilt5$m), lty = "longdash", col=c("yellow"))
plot(ken, xlab="Year", ylab="Kenya", type="l", col=c("blue"), main="NSE All Share")
lines((kalmanFilt6$m), lty = "longdash", col=c("yellow"))

#Smoothed Kalman plot
par(mfrow = c(3,2))
plot(nig, xlab="Year", ylab="Nigeria", type="s", col=c("green"),
xlim = c(2009,2018), ylim=c(0,45000),main="Nigeria All Share" )
lines((kalmanSmooth1$s), col=c("red"))
plot(unk, xlab="Year", ylab="UK", type="s", col=c("green"),
xlim = c(2009,2018),
ylim=c(0,4500),main="FTSE 100")
lines((kalmanSmooth2$s), col=c("red"))
plot(mor, xlab="Year", ylab="Morocco", type="s", col=c("green"),
xlim = c(2009,2018),
ylim=c(0,14000), main="MASI")
lines((kalmanSmooth3$s), col=c("red"))
plot(soa, xlab="Year", ylab="South Africa", type="s", col=c("green"),
xlim = c(2009,2018),
ylim=c(0,62000), main="JSE All Share")
lines((kalmanSmooth4$s), col=c("red"))
plot(mau, xlab="Year", ylab="Mauritius", type="s", col=c("green"),
xlim = c(2009,2018),
ylim=c(0,3000), main="SEMDEX")
lines((kalmanSmooth5$s), col=c("red"))
plot(ken, xlab="Year", ylab="Kenya", type="s", col=c("green"),
xlim = c(2009,2018), ylim=c(0,200), main="NSE All Share")
lines((kalmanSmooth6$s), col=c("red"))

forecast1<- window(cbind(nig, kalmanFilt1$f), xlim = c(20009,2019))
forecast2<- window(cbind(unk, kalmanFilt2$f), xlim = c(20009,2019))
forecast3<- window(cbind(mor, kalmanFilt3$f), xlim = c(20009,2019))
forecast4<- window(cbind(soa, kalmanFilt4$f), xlim = c(20009,2019))
forecast5<- window(cbind(mau, kalmanFilt5$f), xlim = c(20009,2019))
forecast6<- window(cbind(ken, kalmanFilt6$f), xlim = c(20009,2019))

par(mfrow = c(3,2))
plot(forecast1[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="Nigeria All Share")
lines(forecast1[, 2], lty = "longdash",col="green",xlim = c(20009,2019))
plot(forecast2[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="FTSE 100")
lines(forecast2[, 2], lty = "longdash",col="green",xlim = c(20009,2019))
plot(forecast3[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="MASI")
lines(forecast3[, 2], lty = "longdash",col="green",xlim = c(20009,2019))
plot(forecast4[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="JSE All Share")
lines(forecast4[, 2], lty = "longdash",col="green",xlim = c(20009,2019))
plot(forecast5[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="SEMDEX")
lines(forecast5[, 2], lty = "longdash",col="green",xlim = c(20009,2019))
plot(forecast6[, 1], type = "l", col = "red", xlab = "", ylab = "Level",
xlim = c(20009,2019), main="NSE All Share")
lines(forecast6[, 2], lty = "longdash",col="green",xlim = c(20009,2019))

future1 <- dlmForecast(kalmanFilt1, nAhead = 30, sampleNew = 2)
future11 <- dlmForecast(kalmanFilt1, nAhead = 30, sampleNew = 100)
future21 <- dlmForecast(kalmanFilt1, nAhead = 30, sampleNew = 500)
```

```
future2 <- dlmForecast(kalmanFilt2, nAhead = 30, sampleNew = 2)
future3 <- dlmForecast(kalmanFilt3, nAhead = 30, sampleNew = 2)
future4 <- dlmForecast(kalmanFilt4, nAhead = 30, sampleNew = 2)
future5 <- dlmForecast(kalmanFilt5, nAhead = 30, sampleNew = 2)
future6 <- dlmForecast(kalmanFilt6, nAhead = 30, sampleNew = 2)
```