



**Strathmore**  
UNIVERSITY

**Impact of Longevity Risk on Pension Systems**

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## **INTRODUCTION**

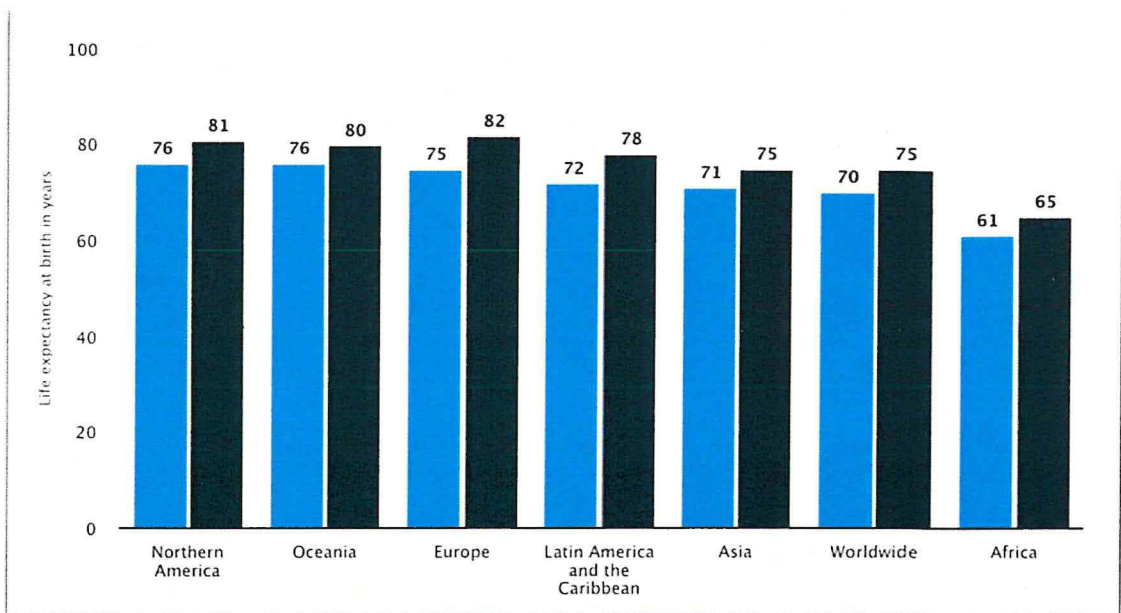
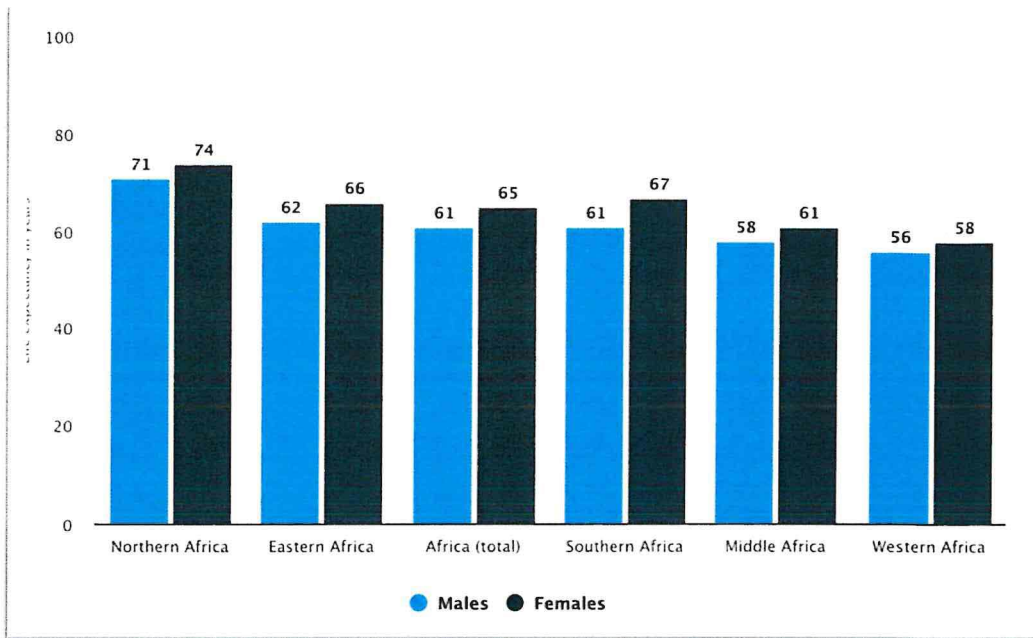
### **1.1 Background**

According to the “2019 Revision of World Population Prospects” prepared by the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat, life expectancy has risen from 60.26 in 2010 to 66.70 in 2019. This increase in life expectancy is attributed to improved healthcare facilities, proper education, diversification in agricultural production and an increase in living standards.

The improvement in life expectancy is positive news but it results in increased longevity risk. Longevity risk is the risk that individuals will have longer lifetimes than expected. In pensions, this is the risk attached to the increasing life expectancy of pensioners and policyholders, which will result in higher pay-out ratios than expected for many pension funds. If gains in life expectancy could be forecasted and factored in retirement planning, then the effect of longevity risk could be minimal and thus negligible but improvement in life expectancy and mortality are uncertain. Thus, longevity risk is related to the uncertainty surrounding future mortality and life expectancy.

#### **1.1.1 Trends in Longevity risk**

The average life expectancy in Africa is 65 years for females and 61 years for males. The global average life expectancy is 75 years for females and 70 years for males as shown in the figure below.



As you can see, there's a large gap between life expectancy in other continents and in Africa. It is only in North Africa where the life expectancy is near the worldwide average. Despite this discouraging statistic, there is reason for optimism. The human development index rating for Africa has risen from 0.375 to 0.498, demonstrating an improvement in the quality of life.

This improvement can be attributed to the increased access of medical services over the years, especially in the efforts to reduce the spread of AIDS infection.

### 1.1.2 Trends in Pension Systems

A pension is a store into which an aggregate of cash is included during a worker's business years and from which installments are attracted to help the individual after retirement from work as intermittent installments. According to Caiman & Tonks (2013), a pension should ensure that the consumption of all individual should not fall and this purpose is achieved by the pension when the individual is provided an income on retirement; the income should be almost similar to the labor income.

In Kenya, over the last decade, we have seen education initiatives and improvements by the RBA to educate people on the significance of pensions. The industry has showed significant growth from both member contribution and good performances leading to the assets under management growing to Ksh 1,166.6 bn in 2018, from Ksh 287.7 bn 10-years ago, which is an annual growth rate of 14.3% over the last decade.

This growth can be attributed to:

- increase in contributions as members registered to pension schemes increased over the years.
- Actual individual contributions have also increased over the last decade.
- Increased returns on the investments.

## 1.2 Problem Statement

Longevity risk has majorly been an alarming issue in the developed countries. Many

underdeveloped nations have not experienced the risk because their mortality rates are still higher than the rates in other countries of the world.

Kenya though, has one of the fastest growing economies in Africa which has resulted in better livelihood, a constantly decreasing mortality rate and overall good standards of living way above that of third world countries.

The current trend is that of an increasing life expectancy in the country. This trend is continuing because of the constant economic, political and social developments in the country. Life expectancy may even shoot higher if the planned "Vision 2030" is completely fulfilled because that will transform the country to the level of developed nations. The ratio of life expectancy in 2010 to that of 2019 is 1.10 i.e., life expectancy has grown by approximately 10 %within 10 years. If this ratio is applied to the future, the life expectancy will be at 70 years in 2025 and 80 years in 2035. The current retirement age in Kenya is 60 years old meaning that by 2025, pension funds will be paying retirees for about 10 years and by 2035, the burden will be heavier by an additional 10 years. Clearly, longevity risk will soon catch up with life offices in Kenya and if not handled well in good time could result to a financial crisis for offices that issue pensions.

Kenyan actuaries are already raising the concern because the danger is already becoming evident. In fact, some Kenyan actuaries refer to longevity risk as an already ticking time bomb waiting to explode in good time. Furthermore, the life tables that are currently in use in the country are quite old and do not depict the current mortality situation in the country.

Longevity risk is already a risk in Kenya, and it is only a matter of time before it can become a menace. There is an urgent need to strategize on ways to handle this risk lest it will be too late in the coming 10 years. Pension schemes in Kenya need a good plan to handle longevity risk.

### **1.3 Research Objectives**

- To project future mortality rates
- To quantify longevity risk in pension systems

### **1.4 Research Questions**

- What is the trend in mortality rates?
- How can we mitigate longevity risk?

### **1.5 Significance of the Research**

This research will benefit pension schemes. It will help give a better understanding of the longevity risk and its impact to available pension funds. A better understanding will consequently help the pension schemes to incorporate longevity risk in their pension calculations. When such schemes factor in longevity risk, it will help reduce losses as a result of longevity risk.

## LITERATURE REVIEW

### 2.1 Theoretical and empirical review

Longevity risk exists when the actual life exceeds life expectancy. Many authors have come up with different theories regarding life expectancy. However, the works done fall under one of the two schools of thought which are

- life expectancy has no limits and
- life expectancy has limits

Oeppen and Vaupel (2002) argue that there are no limits to life expectancy. He argues that the leveling off of life expectancy is due to leaders falling behind. "If life expectancy were close to a maximum, then the increase in the record expectation of life should be slowing. It is not. For 160 years, best-performance life expectancy has steadily increased by a quarter of a year per year, an extraordinary constancy of human achievement" Oeppen and Vaupel (2002).

Olshansky, on the other hand contradicts Oeppen and Vaupel. He argues, based on historical trends and age paths, that there are natural limits to life expectancy. "If most humans, on average, are biologically capable of living to 100 years or more as claimed by Vaupel, then there should be little evidence of significant functional decline or pathologic anomaly among people living to the average survival times (75–80 years) that are already being attained today" Olshansky (2005).

He says that even if medical advances eliminated the threat of circulatory diseases, which is highly improbable, life expectancy will not exceed 100 years at birth. However major life expectancy technologies are likely, which will increase the life expectancy of individuals.

Whichever school of thought you subscribe to; you have to acknowledge that longevity risk is a risk that is not going anywhere any time soon and we should try and hedge against it instead of ignoring it.

In trying to quantify longevity risk, we have to first to forecast mortality rates. Models used to model mortality are as many as they are varied. They include; p-spline model introduced by Currie, Durbin and Eilers, Cairns-Blake-Dowd model, Haberman and Renshaw method, the Booth-Main Donald-Smith variant and the Lee Carter (1992) model. The common models used are the Cairns-Blake-Dowd model and the Lee Carter model.

A lot of discussions have been had regarding the above models, but authors seem to agree that the Lee Carter model performs well most of the time.

The Lee carter model provides a good fit to historical data as the age function of the lee carter model gives an allowance to model across all ages. In comparison to the other models which have been discussed, the Lee Carter model has fewer parameters and therefore in provides simplicity in fitting. Simplicity is also brought out by the singular value decomposition that is easy to put into practice. Therefore, this study will use the Lee Carter model to model future mortality rates.

## **2.2 Relevance to Study**

This study will help to quantify longevity risk and to enable companies to hedge against it. Coughlan (2007) expresses that longevity exposure in the world is over AC 15 trillion and this provides a lot of possibilities for marketing longevity derivatives.

Coughlan (2015) discusses longevity risk associated with the provision of retirement. He proposes transferring of longevity risk via capital markets from pension plans and insurers to end investors.

Many derivatives have been developed and some proposed towards hedging longevity risk. Some of them include; longevity futures, longevity index swaps and longevity bonds among others. According to Blake et al (2006), the biggest obstacle for the emergence of a market for longevity futures is the need for a suitable underlying instrument.

The use of annuities or longevity bonds as underlying instruments was suggested by Blake et al (2006). The problem is updating of prices although there is the predictability of the market because of the pricing relationship with the yield curve of the government bonds. The other problem was that the insurers will be required to reveal their prices which may not be in their best interest.

The alternative of longevity bonds is also provided. Cairns et al (2009) analyses the significant attributes of longevity bonds and indicated that longevity bonds can take an enormous assortment of structures that can change broadly in their sensitivities to shocks that result from longevity. They considered the issues arising from the shortage of long government bonds.

This literature review shows how this study will forecast the mortality rates using the Lee Carter model and how longevity risk impacts the pension system.

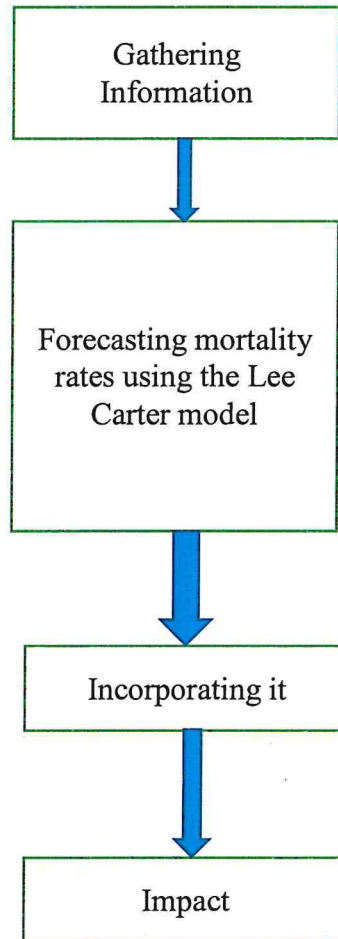
### **2.3 Knowledge gap**

Very little has been done in measuring the longevity risk and formulating a formula that can incorporate the longevity risk in pension contribution or benefit pay out so that pension schemes do not suffer great losses. This study aims to bridge that gap by trying to quantify longevity risk and its impact in the pension systems.

This kind of research has mainly focused on developed countries leaving a knowledge gap for underdeveloped or developing countries.

## 2.4 Conceptual Framework

This research first gathers the information required to undertake this research i.e. mortality rates. It then forecast the mortality rates using the lee carter model, quantifies the longevity risk and tries to incorporate it in the pension systems and showing how it impacts it.



## **RESEARCH METHODOLOGY**

### **3.1 Research design**

This study is a descriptive study. This study is descriptive as it plans to accurately and systematically describe a population, situation or phenomenon, in this case the impact of longevity risk in pension systems.

Descriptive studies can utilize a wide assortment of quantitative and subjective techniques to research at least one factor. Not at all like in trial research, the specialist doesn't control or control any of the factors, however, just watches and measures them. This particular study employs a quantitative approach. The quantitative approach will focus on obtaining the numerical findings which will be used for the qualitative analysis to show the impact of longevity risk.

### **3.2 Population and sampling**

The population of this study will be from Israel. I chose Israel to act as a surrogate since it is a third world country just like Kenya.

The sampling technique used here is random sampling. The sample data includes the mortality rates of males and females over a period of 50 years.

### **3.3 Data collection**

Secondary data will be used in this research. This data will be derived from findings published on the internet and relevant literatures. In this case my data is derived from the Israel mortality data from the Human Mortality Database. This is because the Kenyan regulatory body collects such data, but it is not readily available to the public nor is it annually published like other countries.

The information that can be obtained by sex, age and time in the HMD include:

- Birth tallies
- Death tallies
- Population estimates
- Population presented to risk of death
- year of death
- Death rates

### 3.4 Data analysis

We will use the lee carter model in analyzing the data. Here, we require the rates of deaths and exposure to risk which we will get from the human mortality database.

The Lee-Carter model basically portrays the age-explicit central rate of death ( $m_{x,t}$ ) as an entirety of an age-explicit segment that is autonomous of time ( $a_x$ ), and the result of a period shifting boundary ( $k_t$ , otherwise called the mortality list) that sums up the general degree of mortality and an extra age-explicit segment ( $b_x$ ) that speaks to how quickly or gradually mortality at each age differs when the mortality list changes. Mathematically,

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

$\varepsilon_{x,t}$  is the error term which includes all the age specific influences which are not included in the model.

$m_{x,t}$  is defined as  $m(x,t) = D(x,t)/E(x,t)$  where  $D(x,t)$  is the number of deaths in year  $t$  and  $E(x,t)$  is the number of lived aged  $x$  in the middle of year  $t$ .

We will first estimate the parameters  $b_x$  and  $k_t$  while assuming the following:

1.  $\varepsilon_T k_t = 0$  which means every age estimate for  $a_x$  will be equal to the average over  $t$  of the log deaths.

2.  $\varepsilon_x b_x = 1$

We will then find a modified  $k_t$  which will adjust the total number of deaths  $\varepsilon_x D(x, t)$  to the estimated number of deaths as follows

$$\varepsilon(x, t) \exp(a_x + b_x k_t).$$

Assuming  $a_x$  and  $b_x$  remain constant over time.

## ANALYSIS, RESULTS AND DISCUSSIONS

### 4.1 Description of software used

For the analysis I used the R software more specifically the Stochastic Mortality Modelling (StMoMo) package. This package is based on the framework of the Generalised-Age-Period-Cohort stochastic mortality models.

I used this package with the help of other packages such as the demography package which I used to connect to the mortality database, the gnm and the forecast package.

### 4.2 Assumptions

- The rates experienced in Israel are similar to the rates experienced in Kenya.
- The retirement age is 65.

### 4.3 Analysis

I plotted a graph of the log death rates to age to determine the trend then compared the death rates experienced in the different years for both the males and females.

**Figure 1**

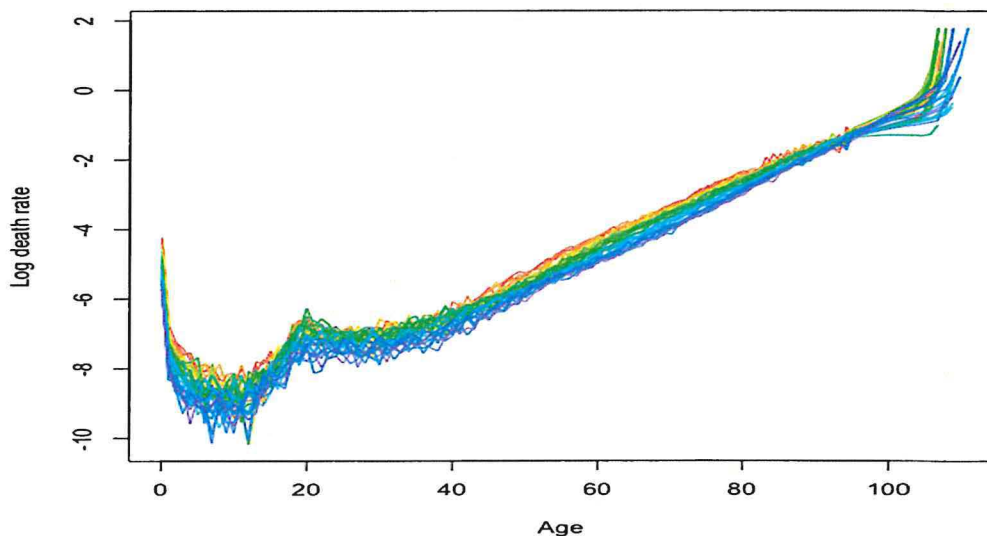
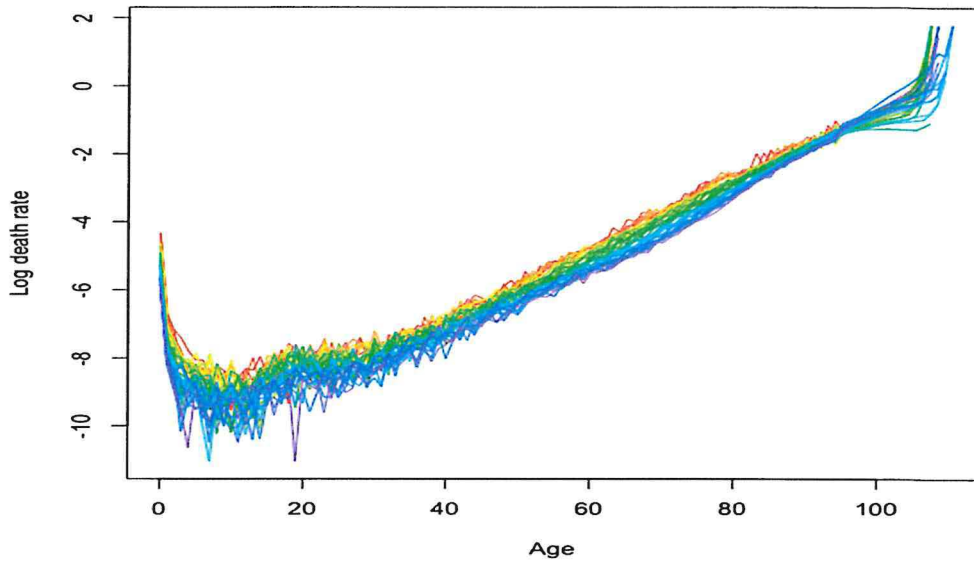
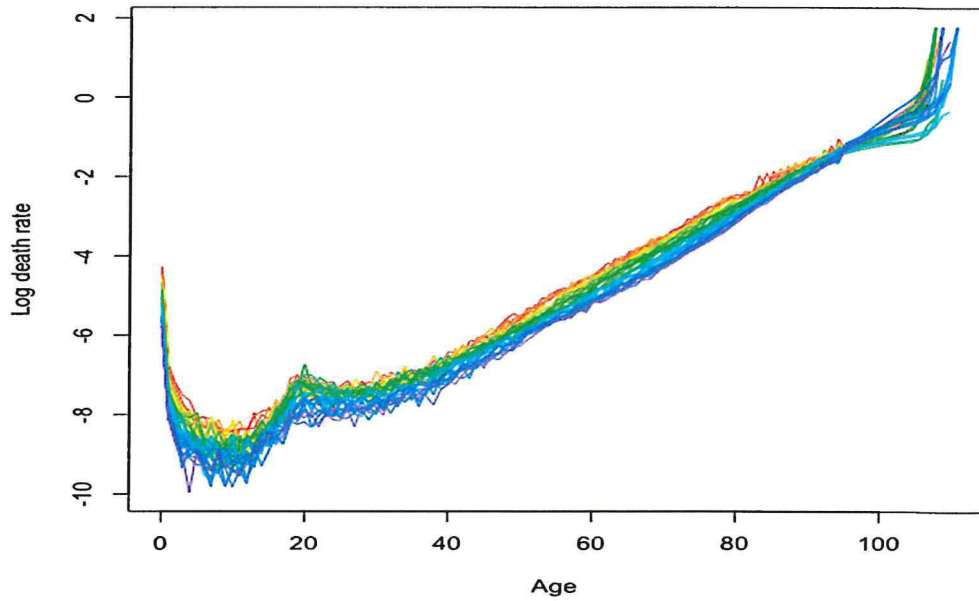


Figure 2



Figure



3

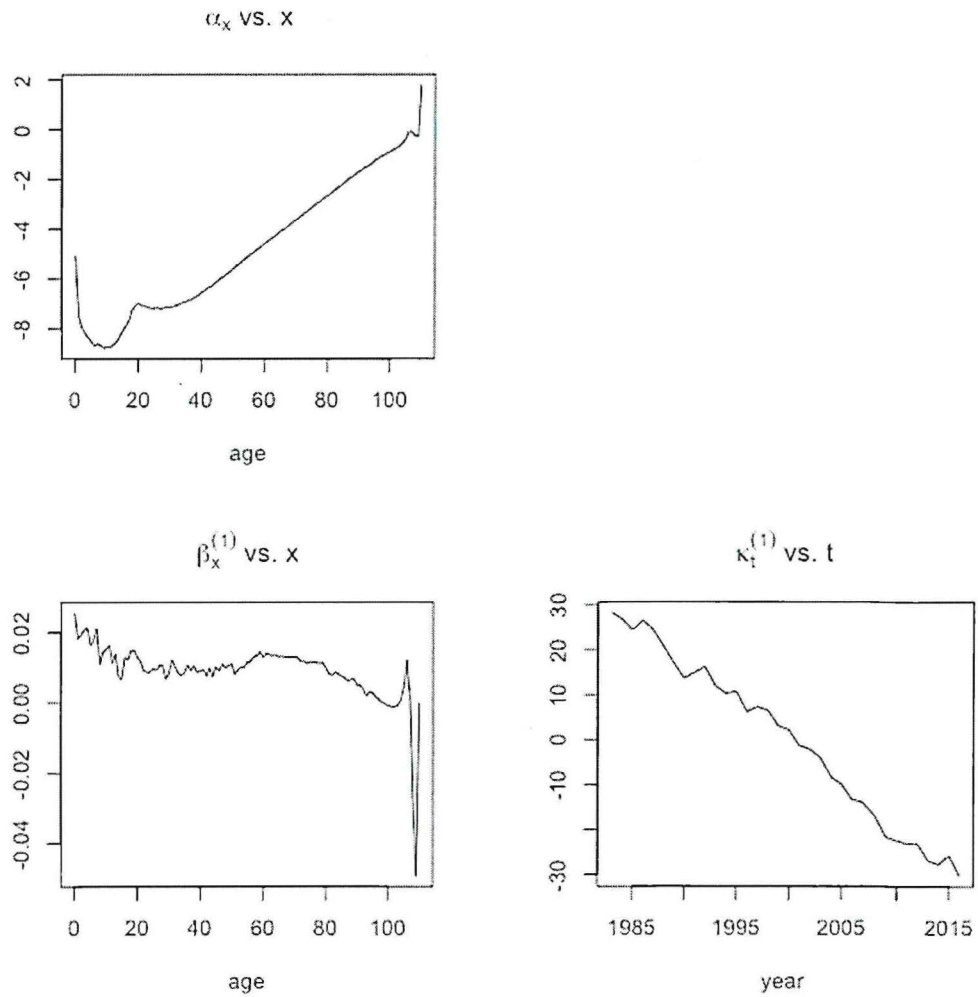
Figure 1, 2 and 3 show the death rates for the males, females and the total respectively. The rates for an individual year decrease from age 0 to the teenage years where the rates increase steadily. Mortality rates then decrease each year. The decrease is higher at some ages and lower in others.

#### 4.4 Fitting the data

In fitting the model, I used the ages between 65-90 and the years between 1970-2018.

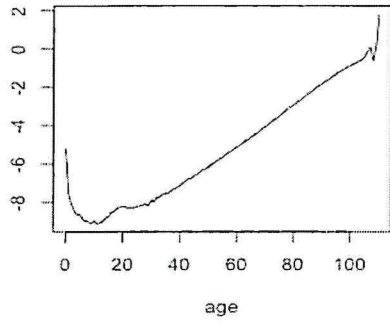
Applying Lee Carter separately between male, female and total population you get the following results.

##### Male



## Female

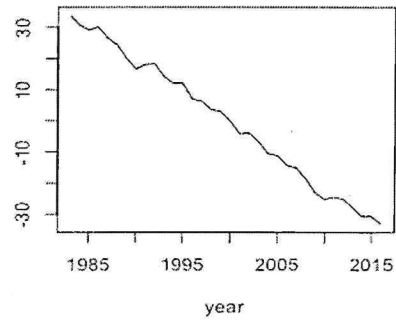
$\alpha_x$  vs. x



$\beta_x^{(1)}$  vs. x

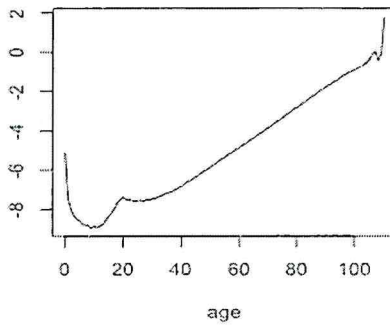


$\kappa_t^{(1)}$  vs. t



## Total

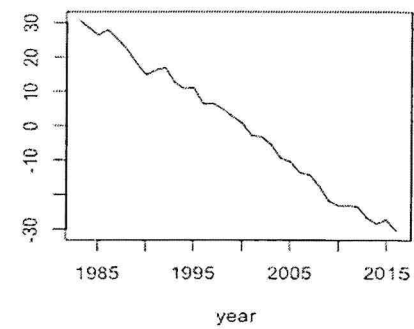
$\alpha_x$  vs. x



$\beta_x^{(1)}$  vs. x



$\kappa_t^{(1)}$  vs. t



The  $\alpha_x$  vs.  $x$  represent the general shape of the graph where we observe that the average mortality increases with age. We can also observe the mortality hump in the ages 18-25.

The  $k_t^{(1)}$  vs.  $t$  shows the change in mortality over time. We observe that mortality is decreasing linearly over time.

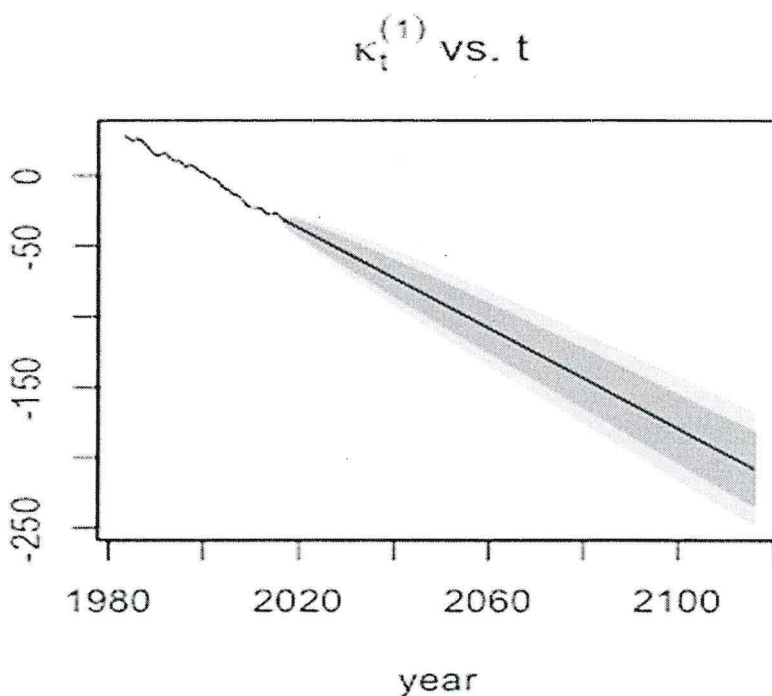
$\beta_x^{(1)}$  vs.  $x$  shows at which ages mortality has improved faster. We observe that at age 60-70 the improvement is higher compared to mortality at age 100.

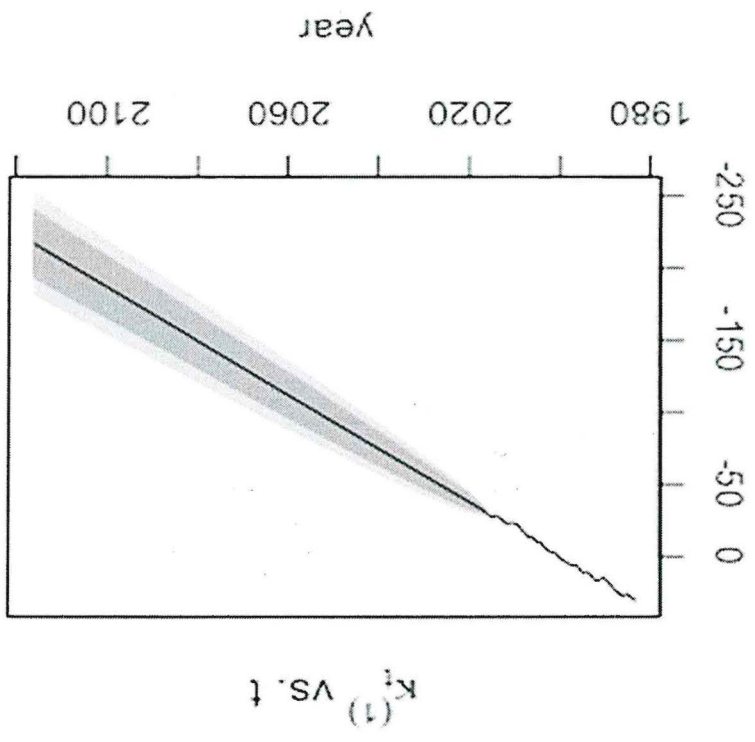
#### 4.5 Forecasting

Here we only forecast the components that depend on time. In our case we will be forecasting the  $k_t^{(1)}$  where we assume they follow a multivariate random walk with drift.

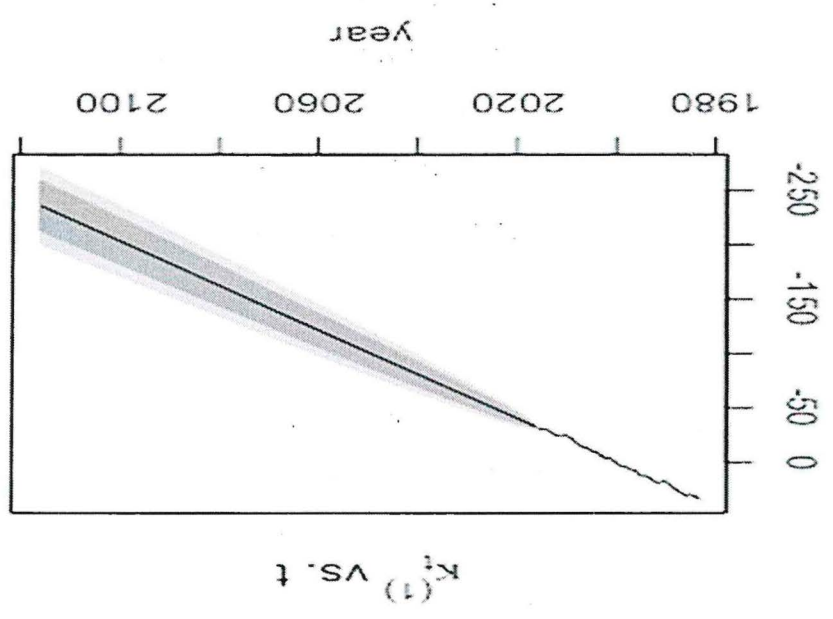
Having already fitted the model, we use the forecast function to forecast the mortality rates over the next 100 years where we clearly see the mortality rates decreasing with time.

Male





Total

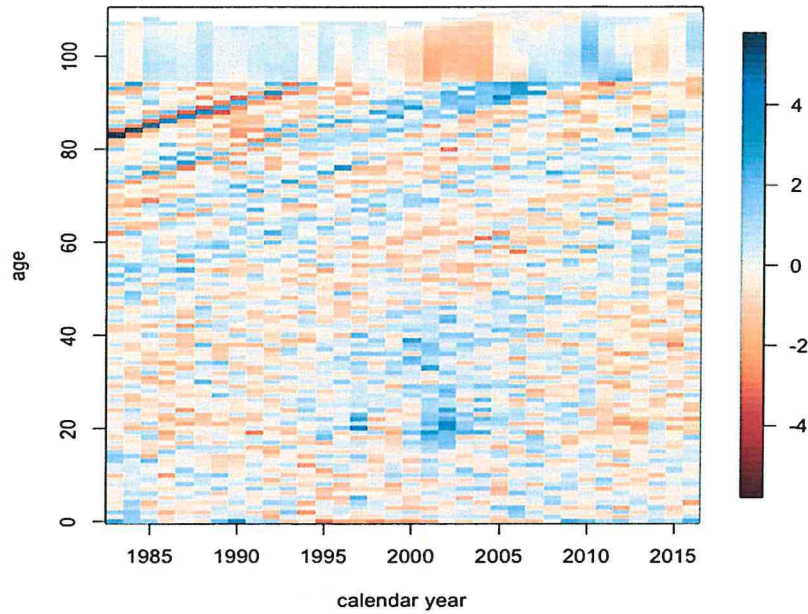


Female

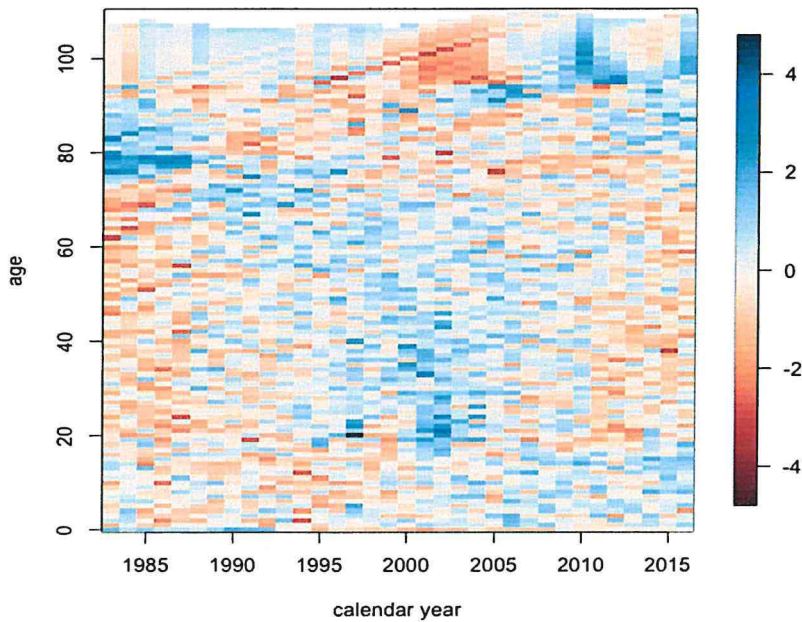
## 4.6 Comparison of the model

I compared the Lee-Carter model to the Cairns, Blake and Dowd (CBD) model to see if the Lee-Carter is more accurate. I did this by plotting the residuals. We know that the better the model the more random the residuals will be.

Lee-Carter



CBD



We can see that the graphs are almost similar with both having some congestion at age 100.

## **CONCLUSIONS**

### **5.1 Conclusion**

From the above analysis it is clear that mortality rates are decreasing with time and there is an increase in life expectancy. We can therefore conclude that the presence of longevity risk is undeniable, and pensioners will more likely live longer than expected.

If pension funds choose to ignore the changes in mortality rates and not put them into consideration they will end up overpaying annuities and suffer the risk of loss. Therefore, pension funds should reserve for longevity risk.

### **5.2 Limitations**

- The assumptions made may provide inaccurate results. We assumed a fixed retirement age for everyone. This can be costly especially if a large number retire earlier than age 65.
- The model does not put cohort effects into consideration.

### **5.3 Recommendation**

We have clearly proven the existence of longevity risk in pension schemes. Therefore, we recommend the study and implementation of longevity risk management techniques.

Also, the limitation of not putting the cohort effect into consideration provides room for further study on the Lee Carter model.

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