STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MSC STATISTICAL SCIENCES
END OF YEAR EXAMINATION
STA 8205: COMPUTATIONAL STATISTICS

Date: 24th April 2024 Time:3 Hours

ANSWER QUESTION ONE AND ANY OTHER TWO

Question One: 30 Marks

a. Name any three random number generators (3 marks)
b. 10000 random numbers were obtained using Linear congruential generators are shown in the table below.

Cell	Observed
1	1000
2	960
3	980
4	850
5	1050
6	930
7	970
8	1250
9	1070
10	940

Using the Chi-square test at 0.1 significant level, perform the appropriate test for these random numbers (7 marks)

c. Suppose you have the following N =5 numbers: 0.44, 0.81, 0.14, 0.05, 0.93. Compare the CDF, $F(X)$ of the uniform distribution, with that of the empirical CDF $S_0(x)$, of the N sample observations using the Kolmogorov-Smirnov test (10 marks)
d. Assume that you have the following data set: $X_1, \dots, X_n$. Let M_n be the median. Discussing what is bootstrapping, outline how you would obtain the bootstrap estimate of the variance, MSE and confidence interval of M_n (6 marks).
e. Consider the following p.d.f.

$$f(x) = \alpha e^{-\alpha x}, x > 0, \alpha = 0.1$$

Simulate 10,000 random values from this distribution using $U(0,1)$ (4 marks)

Question Two (15 marks)

a. Suppose you want to find the integral of the following function:

$$\int_{-3}^{15} -4x^3 + 3x^2 + 15x + 35dx$$

Write an R code to evaluate the integral using Monte Carlo integration. (8 marks)

b. Consider the distribution given below:

$$f(x) = 2xexp(-x^2), x > 0$$

Using the proposal distribution: $g(x) = \exp(-x)$, evaluate $E(x)$ using importance sampling (7 marks).

Question Three (15 Marks)

Consider the following pdf: $f(x) = 6x(1-x), 0 < x < 1$. Write an R code for generating samples from f using the acceptance rejection method, by first generating samples from $g(x) = 2(1-x)$. **Show your workings as well** (15 marks)

Question Four (15 Marks)

Variance reduction a is done in such a way that one can reach the same precision with lower numbers of draws or equivalently, a higher precision with the same number of draws.
Using the following integral as an example,

$$I = \int_{0}^{1} e^x dx$$

and the following 3 draws from $U(0,1)$: 0.34, 0.66, 0.44, discuss two variance reduction methods (15 marks)

Question Five (15 marks)

a. Describe the Markov Chain Monte Carlo sampling technique (10 marks).
b. Using the following numbers, write an R function to generate 1000 random numbers, use any very simple approach to determine if they are indeed random (5 marks)

$$X_0 = 27, a = 7, c = 43, m = 100$$

Kolmogorov-Smirnov Critical Values

Degrees of Freedom (N)	$D_{0.10}$	$D_{0.05}$	$D_{0.01}$
1	0.950	0.975	0.995
2	0.776	0.842	0.929
3	0.642	0.708	0.828
4	0.564	0.624	0.733
5	0.510	0.565	0.669
6	0.470	0.521	0.618
7	0.438	0.486	0.577
8	0.411	0.457	0.543
9	0.388	0.432	0.514
10	0.368	0.410	0.490
11	0.352	0.391	0.468
12	0.338	0.375	0.450
13	0.325	0.361	0.433
14	0.314	0.349	0.418
15	0.304	0.338	0.404
16	0.295	0.328	0.392
17	0.286	0.318	0.381
18	0.278	0.309	0.371
19	0.272	0.301	0.363
20	0.264	0.294	0.356
25	0.24	0.27	0.32
30	0.22	0.24	0.29
35	0.21	0.23	0.27
Over 35	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19