Models for Conductor Size Selection in Single Wire Earth Return Distribution Networks

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***Abstract*—The use of the ground as the current return path often presents planning and operational challenges in power distribution networks. This study presents optimization-based models for the optimal selection of conductor sizes in Single Wire Earth Return (SWER) power distribution networks. By using mixed integer non-linear programming (MINLP), models are developed for both branch-wise and primary-lateral feeder selections from a discrete set of overhead conductor sizes. The models are based on a mathematical formulation of the SWER line, where the objective function is to minimize fixed and variable costs subject to constraints specific to SWER power flow. Load growth over different time periods is considered. The practical application is tested using a case study extracted from an existing SWER distribution line in Namibia. The results were consistent for different network operating scenarios.**

I. INTRODUCTION

THE importance of electricity in enhancing economic development and social welfare is well established. Whereas the electrification of urban areas is generally widespread, electrifying rural areas has always presented technical and economic challenges. The major setback is the high cost of connecting relatively small and sparse rural loads to the medium voltage (MV) network [1].

Single Wire Earth Return (SWER) distribution systems supply single phase power to rural areas economically from the main MV grid network, whereby the earth forms the current return path [2]. The technology, initially proposed by Mandeno in [2], has proven to be cost-effective in electrifying rural areas with small and scattered loads in countries such as Australia, Namibia, etc. [3]. The lines are typically spur extensions from radial three phase feeders of 11, 22 or 33 kV supplying single phase power at 12.7 or 19.1 kV, often via an isolating transformer [4].

The planning of SWER distribution systems faces different challenges compared to conventional systems due to the earth return circuit. Solutions to the problem of earth return power flow modeling were first proposed by Carson in [5]. In Carson's model, the earth is replaced by a plane homogeneous semi-infinite solid conductor and modeled accordingly [6]. Other approaches to ground modeling, mostly simplifications of the full Carson model, are presented in [7] and [8]. This study considers the aspect of conductor selection in the planning of SWER systems. The SWER line model, based on Carson's line, is used to develop optimization-based conductor size selection algorithms for SWER overhead lines. Two models are proposed: the first determines the optimum SWER line conductor size for each network branch, and the second selects the optimum overhead conductor sizes for the primary and lateral feeders respectively. Both models are formulated subject to load growth over different time periods. They are tested using a case study extracted from an existing SWER distribution line in Namibia, and subjected to a sensitivity analysis that considers different network operating scenarios.

The rest of the paper is organized as follows: Section II presents the mathematical model of the SWER line. Section III presents the summarized formulation of the SWER power flow model. Section IV introduces the two models formulated for selecting the optimal overhead conductor for a) each branch, and b) the primary and lateral feeders of a SWER network. Section V gives the numerical analysis results of the proposed models and the conclusion is given in Section VI.

II. THE SWER LINE MODEL

The model of the SWER distribution line is based on Carson's model for overhead transmission and distribution lines that include the effects of earth return [5]. Carson's line is used to determine the self and mutual impedances of overhead conductors with earth return. In the model, Carson considers a single overhead conductor of unit length parallel to the earth and carrying a current with the return path underneath the earth's surface.

The earth itself is modeled as a single return conductor of infinite length and uniform resistivity with a geometric mean radius (GMR) of 1 m [5], [6]. The distance, D_{ag}, between the overhead conductor and earth return path is a function of soil resistivity: higher soil resistivity causes return current to flow deeper from the earth surface increasing D_{ag}, and vice versa [6]. Details of the full development of the Carson line model are given in [5]–[7]. For brevity, only the relevant impedance equations for single wire earth return networks are given here. The SWER overhead line impedance, Z_{aa}, is given by (1).

$$Z_{aa} = \bar{z}_{aa} + \bar{z}_{gg} - 2\bar{z}_{ag} \tag{1}$$

where $\bar{z}_{aa}$ is the line self-impedance, $\bar{z}_{gg}$ the ground self-impedance, and $\bar{z}_{ag}$ the mutual impedance between the line and earth, all of which are defined by (2) through (4) respectively. The factor ($\bar{z}_{gg} - 2\bar{z}_{ag}$) in (1) represents the impedance correction due to the earth presence [6].

$$\bar{z}_{aa} = r_a + j4\pi \cdot 10^{-4} f \ln \frac{2h_a}{\mathrm{GMR}_a} \tag{2}$$

$$\bar{z}_{gg} = \pi^2 \cdot 10^{-4} f - j0.0386 \cdot 8\pi \cdot 10^{-4} f + j4\pi \times 10^{-4} \cdot f \ln \frac{2}{5.6198 \cdot 10^{-3}} \tag{3}$$

$$\bar{z}_{ag} = j2\pi \cdot 10^{-4} \ln \frac{h_a}{\sqrt{\rho/f}} \tag{4}$$

where r_a is the resistance of the phase conductor a (Ω/km), f the frequency (Hz), h_a the height of the conductor a above the earth (m), GMR_a the geometric mean radius of a (m), and ρ the ground resistivity (Ω·m).

When load current is conducted into the earth, dangerous touch and step potential gradients can result for both man and beast [4]. However, careful design of the earthing system ensures that these voltages are kept within safe levels. For safe SWER system operation, the earth current should be limited to 25 A at 19.1 kV under typical conditions and to 8 A where the SWER lines are likely to interfere with open wire communications [3]. Further details on SWER earthing requirements can be found in [3].

III. SWER Power Flow Model

The SWER load flow formulation is based on the forward/backward sweep method presented in [6] for earth return networks. All nodal current injections due to loads and shunt elements are first computed based on initial voltage values for both the overhead conductor and ground return path using (5). The branch currents are then calculated using Kirchoffs Current Law (KCL) in (6) and (7) for the line and ground respectively, where the loads are represented by their equivalent current injections. According to the KCL, the sum of all branch currents entering and leaving a node is equivalent to the load current at that node. Finally, nodal voltages for both overhead line and earth return are updated using (8). This leads to an iterative procedure that ends when the difference between the specified and calculated current injections at each bus is minimum.

$$\begin{bmatrix} I_{ia,t} \\ I_{ig,t} \end{bmatrix} = \begin{bmatrix} (S_{ia,t}/V_{ia,t})^* \\ -I_{ia,t} \end{bmatrix} - \begin{bmatrix} Y_{ia} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} V_{ia,t} \\ V_{ig,t} \end{bmatrix} \quad \forall i \in N, t \in T \tag{5}$$

$$\sum_{\substack{i\in N \\ i \neq j}} J_{ija,t} - \sum_{\substack{i\in N \\ i \neq j}} J_{jia,t} = I_{ja,t} \qquad \forall j \neq 1, t \in T \tag{6}$$

$$\sum_{\substack{i\in N \\ i \neq j}} J_{ijg,t} - \sum_{\substack{i\in N \\ i \neq j}} J_{jig,t} = I_{jg,t} \qquad \forall j \neq 1, t \in T \tag{7}$$

$$\begin{bmatrix} V_{ja,t} \\ V_{jg,t} \end{bmatrix} = \begin{bmatrix} V_{ia,t} \\ V_{ig,t} \end{bmatrix} - \begin{bmatrix} Z_{aa} & Z_{ag} \\ Z_{ag} & Z_{gg} \end{bmatrix} \begin{bmatrix} J_{la,t} \\ J_{lg,t} \end{bmatrix} \quad \forall l \in M, t \in T \tag{8}$$

where $I_{ia,t}$ and $I_{ig,t}$ are the current injections at node i for the overhead line and earth return in period t respectively, $S_{ia,t}$ is the specified complex power load at node i in period t, $V_{ia,t}$ and $V_{ig,t}$ are the complex voltages at node i for the overhead conductor and earth return in period t respectively, Y_{ia} is the shunt admittance at node i, N is the set of all network nodes, T is the set of time steps in the full planning period, and i and j are the incoming and outgoing nodes of branch l respectively. The branch impedances are as calculated in (1) to (4). All parameters and variables in (5) to (8) are complex.

IV. Conductor Selection Problem Formulation

The objective of conductor selection is to choose the conductor size with minimum investment costs and power losses subject to voltage regulation, load flow, and current carrying constraints [9]. The study considers a single-phase SWER isolating transformer supplying power from a MV three-phase network to rural loads at minimal cost with load growth in different time periods. The output terminals of the isolating transformer were considered to form the infinite bus [10]. Therefore, the transformer's installation costs and internal losses were excluded from the analysis. In addition, the following assumptions were made.

1) The network is supplied by one SWER isolating transformer located at a known point of grid extension.
2) Load data including size, location, and estimated load growth rate are known beforehand.
3) Only the peak load is considered for each year of the planning period and there is no unserved energy.
4) Equipment data including unit costs, capacities, and electrical characteristics are readily available.

A. Constraints for Branch-wise Conductor Selection

It is required to choose an optimum conductor size for each branch from a finite set of conductor options whose parameters of resistance, reactance, thermal limit, and unit fixed cost are known. Therefore, optimization techniques were formulated in (9) through (14) to select the parameters from each corresponding finite set that give the optimum conductor. The chosen parameters were then incorporated into the rest of the constraints (15) through (19) and objective function (31).

$$r'_{laa} = \sum_{c\in C_T} y_{lct} \cdot r_{laa_c} \qquad \forall l \in M, t \in T \tag{9}$$

$$x'_{laa} = \sum_{c\in C_T} y_{lct} \cdot x_{laa_c} \qquad \forall l \in M, t \in T \tag{10}$$

$$C'_{fl} = \sum_{c\in C_T} y_{lct} \cdot C_{flc} \qquad \forall l \in M, t \in T \tag{11}$$

$$\bar{J}'_l = \sum_{c\in C_T} y_{lct} \cdot \bar{J}_{lc} \qquad \forall l \in M, t \in T \tag{12}$$

$$\sum_{c\in C_T} y_{lct} = 1 \qquad \forall l \in M, t \in T \tag{13}$$

$$y_{lct} \in \{0, 1\} \qquad \forall l \in M, c \in C_T, t \in T \tag{14}$$

where y_{lct} is a binary decision variable for the optimum conductor for branch l during period t: 1 if conductor c is chosen and 0 otherwise; r'_{laa} is the total internal resistance of the optimum conductor for branch l; r_{laa_c} is the total internal resistance of conductor option c on branch l; x'_{laa} is the total internal reactance of the optimum conductor for branch l; x_{laa_c} is the total internal reactance of conductor option c on branch l; C'_{fl} is the total annualized fixed branch cost of the optimum conductor for l; C_{flc} is the total annualized fixed branch cost of conductor option c on branch l; $\bar{J}'_l$ is the current carrying limit of the optimum conductor on branch l; $\bar{J}_{lc}$ is the current carrying limit of conductor option c on branch l; and C_T is the set of all conductor options.

One optimum choice for all the variables in (9) to (12) will be made provided that constraint (13) is fulfilled as proven in [11]. The selected conductor self impedance, $\bar{z}_{laa} = r'_{laa} + jx'_{laa}$, obtained from (9) and (10) is incorporated into (1) to obtain the overall resultant SWER line impedance, Z'_{laa}. The SWER line impedance obtained thus is used in (15) to obtain the nodal voltage constraints for line and ground respectively. The ground impedance is independent of the conductor selection. The rest of the constraints were as follows.

Constraints (5), (6), and (7),

$$\begin{bmatrix} V_{ja,t} \\ V_{jg,t} \end{bmatrix} = \begin{bmatrix} V_{ia,t} \\ V_{ig,t} \end{bmatrix} - \begin{bmatrix} Z'_{laa} & Z_{lag} \\ Z_{lag} & Z_{lgg} \end{bmatrix} \begin{bmatrix} J_{la,t} \\ J_{lg,t} \end{bmatrix} \quad \forall l \in M, t \in T \tag{15}$$

$$\underline{V}_a \le |V_{ia,t}| \le \bar{V}_a \quad \forall i \in N, t \in T \tag{16}$$

$$|V_{ig,t}| \ge 0 \quad \forall i \in N, t \in T \tag{17}$$

$$|J_{la,t}| \le x_l \cdot \bar{J}'_{la} \quad \forall l \in M, t \in T \tag{18}$$

$$|J_{lg,t}| \le x_l \cdot \bar{J}_{lg} \quad \forall l \in M, t \in T \tag{19}$$

where $\underline{V}_a$ and $\bar{V}_a$ are user-defined lower and upper nodal voltage limits respectively and $\bar{J}_{lg}$ is the safe upper earth current limit (Section II).

B. Constraints for Primary/Lateral Conductor Selection

Most rural distribution lines in practice use one conductor for the primary feeder and another often smaller conductor for the laterals, which typically carry less current. The branch-wise conductor selection model above was modified to allow conductor selection in such a network.

All the constraints of Section IV-A apply in this case except (9) to (14) which were modified and replaced by (20) through (30). New constraints and binary variables corresponding to selections for the primary and secondary feeders were added.

$$r'_{paa} = \sum_{c \in C_T} y^p_{ct} \cdot r_{paa_c} \quad \forall p \in M_p, t \in T \tag{20}$$

$$r'_{saa} = \sum_{c \in C_T} y^s_{ct} \cdot r_{saa_c} \quad \forall s \in M_s, t \in T \tag{21}$$

$$x'_{paa} = \sum_{c \in C_T} y^p_{ct} \cdot x_{paa_c} \quad \forall p \in M_p, t \in T \tag{22}$$

$$x'_{saa} = \sum_{c \in C_T} y^s_{ct} \cdot x_{saa_c} \quad \forall s \in M_s, t \in T \tag{23}$$

$$C'_{fp} = \sum_{c \in C_T} y^p_{ct} \cdot C_{fpc} \quad \forall p \in M_p, t \in T \tag{24}$$

$$C'_{fs} = \sum_{c \in C_T} y^s_{ct} \cdot C_{fsc} \quad \forall s \in M_s, t \in T \tag{25}$$

$$\bar{J}'_p = \sum_{c \in C_T} y^p_{ct} \cdot \bar{J}_{pc} \quad \forall p \in M_p, t \in T \tag{26}$$

$$\bar{J}'_s = \sum_{c \in C_T} y^s_{ct} \cdot \bar{J}_{sc} \quad \forall s \in M_s, t \in T \tag{27}$$

$$\sum_{c \in C_T} y^p_{ct} = 1 \quad \forall t \in T \tag{28}$$

$$\sum_{c \in C_T} y^s_{ct} = 1 \quad \forall t \in T \tag{29}$$

$$y^p_{ct}, y^s_{ct} \in \{0, 1\} \quad \forall c \in C_T, t \in T \tag{30}$$

where the subscripts p and s represent a branch on the primary and secondary feeders respectively; M_p and M_s are sets of the primary and secondary feeder branches respectively and both are subsets of M; C'_{fp} and C'_{fs} are the total fixed branch costs of the optimum conductor for the primary and secondary branches respectively; and y^p_c and y^s_c are binary variables corresponding to 1 if conductor c is chosen for the primary or lateral feeders respectively and 0 otherwise. All other symbols have their previously defined meanings subject to the subscripts p and s.

C. Objective Function for Conductor Selection

The general objective function formulated for the conductor selection optimization problem is given by (31). It minimizes the fixed installation and power losses costs.

$$Min. \sum_{l \in M} \sum_{t \in T} [C'_{fl} + C_v |J_{la,t}|^2 R'_{laa}] \tag{31}$$

where C'_{fl} is the annualized fixed cost of investment for the selected conductor on branch l; C_v is the annualized unit cost of losses; $J_{la,t}$ is the complex current flow in branch l at peak load in time period t; and R'_{laa} is the overall SWER line resistance that includes both the chosen conductor resistance, r'_{laa}, and the ground resistance, r_{lgg}, for each branch.

The branch fixed cost, C'_{fl}, is that selected from the set of available conductor costs using (11), (24) or (25). The fixed and variable costs must be appropriately discounted for analysis in different time periods. The problem solution will consist of nodal voltages and branch currents for the network comprising only the optimum conductors.

V. Numerical Results

The proposed SWER conductor size selection models were applied to a case study of an existing SWER line in Namibia. All models and simulations were coded and run using the General Algebraic Modeling System (GAMS) on a 64-bit PC with Core™ 2.60 GHz processor and 8 GB RAM.

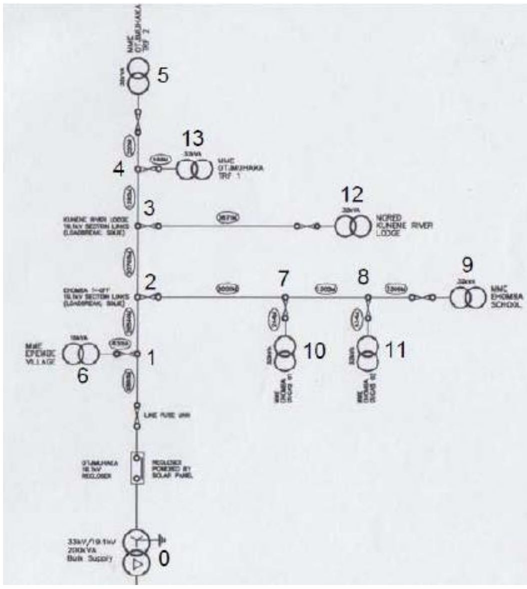

Fig. 1. Extracted case study network *(Source: Power Consult, Namibia)*

TABLE I. LOAD DATA FOR CASE STUDY NETWORK

i	j	d_{ij} (km)	S_{ja} (kVA)
0	1	6.86	0
1	2	2.65	0
2	3	2.08	0
3	4	1.28	0
4	5	0.22	32
1	6	0.53	16
2	7	8.00	0
7	8	1.30	0
8	9	1.30	32
7	10	0.24	32
8	11	0.43	32
3	12	3.67	32
4	13	0.17	32
Total demand			208

A. The Case Study Network

The case study was extracted from a rural electrification project in Opuwo, northern Namibia. The SWER line was constructed to supply rural loads using a 200 kVA isolating transformer reticulated at 19.1 kV from a 33 kV three phase backbone network. Fig. 1 shows the extracted line diagram of the SWER test network. The objective was to analyze the performance of the conductors selected using the proposed models in different operating scenarios, in relation to the existing conductors of the test network.

Table I summarizes the load data of the case study network, whereby the loads are assumed to be equivalent to the customer transformer ratings. The nodes were numbered as shown in Fig. 1, with the primary feeder numbered first.

B. Simulation Results

Five conductors were considered for selection at different load growth rates over an evaluation period of 10 years. The demand, S_t, in time period t was determined using (32).

$$S_t = S_0 \cdot (1+g)^t \tag{32}$$

TABLE II. CONDUCTOR ELECTRICAL PROPERTIES

Conductor code	R (Ω/km)	X (Ω/km)	Current rating (A)
Bantam	5.26	1.02	69
Mole	3.30	1.03	98
Magpie	3.31	0.99	92
Shrike	2.08	0.96	122
Squirrel	1.67	0.99	148

where S_0 is the peak load in the base year and g the prevailing load growth rate. Table II shows the electrical characteristics of the conductors considered. The conductor fixed costs were assumed to be proportional to the thermal limits and expressed in per unit (p.u) of the highest cost. The present worth costs for different time periods were evaluated at a discount rate of 5%. The maximum earth return current considered was 25 A. The general system data were: $\underline{V}$ = 0.95 p.u, $\overline{V}$ = 1.05 p.u, f=50 Hz, ρ = 400 Ω. m, Z_{gg} = (0.0493 + j0.3643) Ω/km, load power factor 0.9, and demand factor 1. The line shunt admittance and Z_{ag} were assumed to be negligible.

1) Branchwise Conductor Selection: The optimization model proposed in Section IV-A was applied to the test network. The results obtained for different load growth rates in the 10th year of the evaluation period are given in Table III.

2) Primary/Lateral Conductor Selection: The line compris- ing nodes 0 to 5 was arbitrarily chosen as the primary feeder with the laterals comprising the remaining nodes as shown in Fig. 1. The model formulated in Section IV-B was then applied to the case study network. The results for different load growthrates in the 10th year are shown in Table IV.

C. Discussion

The branch-wise conductor selections by the proposed model were consistent, with larger or smaller conductors

TABLE III. RESULTS OF BRANCHWISE CONDUCTOR SELECTION

Branch		Conductor selected at t=10		
i	j	g=3%	g=5%	g=7%
0	1	magpie	magpie	shrike
1	2	bantam	magpie	magpie
2	3	bantam	bantam	bantam
3	4	bantam	bantam	bantam
4	5	bantam	bantam	bantam
1	6	bantam	bantam	bantam
2	7	bantam	magpie	magpie
7	8	bantam	bantam	magpie
8	9	bantam	bantam	bantam
7	10	bantam	bantam	bantam
8	11	bantam	bantam	bantam
3	12	bantam	bantam	bantam
4	13	bantam	bantam	bantam
Cost (p.u)		13.8	14.8	16.7

TABLE IV. RESULTS OF PRIMARY AND LATERAL CONDUCTOR SELECTION

g	Conductor selected at t=10		Cost (p.u)
	primary	lateral	
3%	bantam	bantam	14.4
5%	magpie	magpie	15.9
7%	shrike	magpie	19.3

chosen for increasing or reducing annual load growth as shown in Table III. The results of branch-wise selection were on average 9% lower than those for primary and lateral selection due to the more detailed optimization per branch.

The existing overhead conductor of the considered case study network was *magpie* on all branches. This corresponded with the conductors selected by the primary/lateral model in the 10^{th} year at 5% annual load growth (Table IV). For the lower load growth rate of 3%, the smaller conductor *bantam* was selected for both primary and lateral feeders in the tenth year. At 7% annual load growth, the larger conductors *shrike* and *magpie* were chosen for the primary and laterals respectively. The results were consistent with the requirement to minimize investment costs while optimizing network performance to supply demand in different scenarios.

Despite the MINLP formulation, the execution times of the proposed models for the multi-period problem were less than 10 CPU seconds in all cases. This was mainly attributed to the restriction of binary variables to the selection of known conductor parameters from discrete sets in the problem formulation. Only chosen parameters were then incorporated into the voltage and current constraints. This ensured there were no direct binary manipulations in current and voltage constraints which would otherwise increase problem complexity. Hence, the models can be applied to the analysis of larger networks.

VI. Conclusion

Models for optimal conductor size selection in rural Single Wire Earth Return distribution networks were proposed in this paper. They were formulated as mixed integer non-linear programming (MINLP) problems to minimize fixed and variable costs subject to SWER load flow and load growth in different time periods. Modeling the earth return path allowed the incorporation of ground voltage, current and safety constraints into the optimization problem. The developed algorithms were tested on a rural case study in Namibia giving consistent results for different growth scenarios in reasonable execution times.

The models are applicable to SWER grid-extension planning for previously un-electrified rural areas or the analysis of conductor performance in existing networks for future upgrade. Future work will include the optimization of reactive power compensation for higher demand SWER networks where increase in conductor size may not be the best solution.

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