Strathmore Institute of Mathematical Sciences (SIMS)
Special Examination for the Degree of Bachelor of Business Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE: 28th March 2025 **Time: 2 Hours**

Instructions

- **This examination consists of FIVE questions.**
- **Answer Question ONE (COMPULSORY) and any other TWO questions.**

1. (a) Consider a Bernoulli utility function given by:

$$u(w) = \frac{w^{1-\sigma}}{1-\sigma}, \quad \text{for } \sigma \geq 0, \sigma \neq 1$$

(i) Derive the coefficient of absolute risk aversion. **(2 marks)**

> **Solution:**
> - Absolute risk aversion: $A(w) = -\frac{u''(w)}{u'(w)}$

(ii) Derive the coefficient of relative risk aversion. **(3 marks)**

> **Solution:**
> - Relative risk aversion: $R(w) = wA(w)$

(b) An investor is comparing two assets with the following returns across three states:

State	Probability	Asset 1 Return (%)	Asset 2 Return (%)
1	$\frac{1}{3}$	10	0
2	$\frac{1}{3}$	0	10
3	$\frac{1}{3}$	10	20

(i) Does one asset dominate the other in a state-by-state comparison? **(2 marks)**

(ii) Which asset exhibits mean-variance dominance? **(3 marks)**

(iii) Which asset has the highest Sharpe ratio? **(3 marks)**

(c) Provide an intuitive explanation of the following asset pricing concepts:

(i) Two-fund theorem (separation theorem) **(6 marks)**

(ii) Complete Markets **(5 marks)**

(iii) Roll's critique (of CAPM) **(6 marks)**

2. (a) An investor with initial wealth $Y_0 = 100$ invests in stocks and a risk-free bond. - The stock returns $r_G = 16\%$ in a good state (probability 0.5) and $r_B = 2\%$ in a bad state (probability 0.5). - The risk-free rate is $r_f = 8\%$. - The investor maximizes:

$$\max_a \mathbb{E}[\ln Y]$$

Determine the optimal investment a^* in stocks. **(8 marks)**

(b) A risk-averse investor has utility:

$$u(Y) = \frac{Y^{1-\gamma}}{1-\gamma}, \quad \text{where } \gamma = 1/2.$$

- The investor faces a risky asset that pays $Z_G = 75$ (good state) and $Z_B = 25$ (bad state), each with probability 0.5.

(i) Compute the certainty equivalent $CE(\tilde{Z})$. **(8 marks)**

(ii) Compute the risk premium $\Psi(\tilde{Z})$. **(4 marks)**

3. (a) A portfolio manager forms a portfolio with three assets where: - Asset 1: $\mu_1 = 8$, $\sigma_1 = 2$ - Asset 2: $\mu_2 = 4$, $\sigma_2 = 2$ - Asset 3: $\mu_3 = 6$, $\sigma_3 = 2$ - The portfolio is constructed to achieve $\mu_p = 6$ with minimum variance.

Use the Lagrangian method to derive the optimal weights w_1^*, w_2^*. **(8 marks)**

(b) Explain the interpretation of the Capital Asset Pricing Model (CAPM) test equation:

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \epsilon_i$$

What values should γ_0 and γ_1 take if CAPM holds? **(6 marks)**

(c) explain three advantages of the Arrow-Debreu approach to asset pricing compared to CAPM. **(6 marks)**

4. Suppose a typical investor solves the following problem

$$\max_\alpha U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]$$

$$\text{subject to } c_t = e_t - \alpha p_t \text{ and } c_{t+1} = e_{t+1} + \alpha x_{t+1}$$

where c_t and c_{t+1} denotes consumption at date t and $t+1$ respectively. The other parameters in the model are; e_t (investor's endowment at period t and $t+1$ respectively), p_t (asset price at time t), α (the number of units of the asset purchased by the investor), and x_{t+1} (payoff of the asset at period $t+1$), β represents the time preference parameter of the investor ($0 \leq \beta \leq 1$). The general functional form $U(.)$ represents the investor's utility function. An often convenient utility function used in many applications is the power utility $u(c_t) = \dfrac{1}{1-\gamma} c^{1-\gamma}$

(i) Show that the price of an asset at any given time can be represented as follows: **(3 Marks)**

$$p_t = E_t[\frac{\beta u'(c_{t+1})}{u'(c_t)} x_{t+1}]$$

(ii) Provide an intuitive interpretation of the term $\frac{\beta u'(c_{t+1})}{u'(c_t)}$ and briefly discuss how this term influences the asset price p_t **(3 Marks)**

(iii) Show that a simple manipulation of the price equation in (i) can yield the following equation (ignoring time subscripts)

$$1 = E(mR)$$

where R represents the gross return of the asset, while $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$ **(2 Marks)**

(iv) Assuning the power utility function provided and the manipulations in parts (i)-(iii) above, one can show that the risk-free rate can be expressed as

$$R^f = \frac{1}{\beta}(\frac{c_{t+1}}{c_t})^{\gamma}$$

Intuitively discuss three factors that influence interest rates in an economy. **(6 Marks)**

(v) Show that the basic pricing equation ($1 = E(mR)$) can be rewritten as follows;

$$E(R) = R_f - \frac{Cov(u'(c_{t+1}), R)}{E(u'(c_{t+1}))}$$

Interpret this equation. **(6 Marks)**

5. (a) You are given the following information: Show how you can combine stock and

	Stock	Bond	up.A-D	down.A-D
upstate	1260	1050	1	0
Downstate	840	1050	0	1
Current Price	1050	1000		

bonds in such a way that you obtain an upstate and downstate pure security **(8 Marks)**

(b) There are three possible states in the future, states 1,2,and 3, and the consumption at each state is denoted by $c_1, c_2, \text{and} c_3$. There are one million homogeneous consumers with expected utility function

$$q_1 u(c_1) + q_2 u(c_2) + q^3 u(c_3)$$

, where $(q_1, q_2, q_3) = (0.5.0.3, 0.2)$, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 0.5$, and with endowment $(y_1, y_2, y_3) = (100, 81, 64)$ Treating the state 1 good as numeraire, find the general equilibrium prices for the state 2 good and for state 3 good. **(12 Marks)**

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