Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

MASTER OF SCIENCE IN STATISTICAL SCIENCE

END OF SEMESTER EXAMINATION

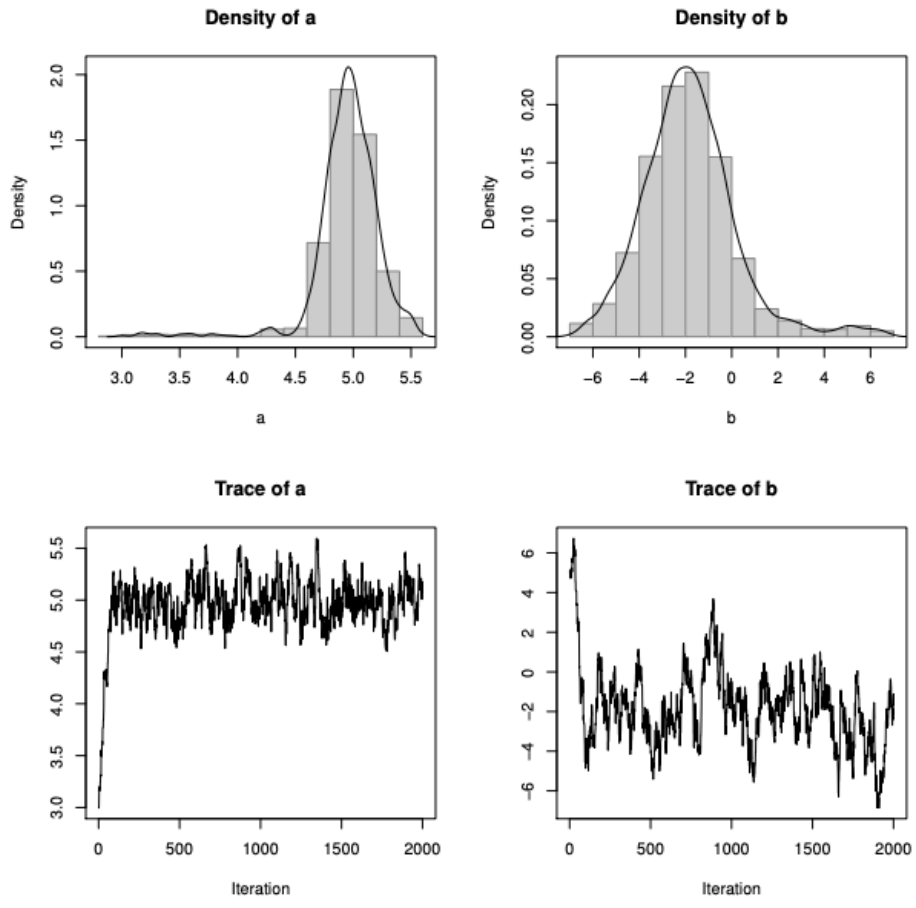
STA 8201 BAYESIAN MODELLING AND DATA ANALYSIS

Date: 2nd May 2023 **Duration:** 3 Hours

Answer Question ONE and any other Two questions:

Question 1 (30 marks)

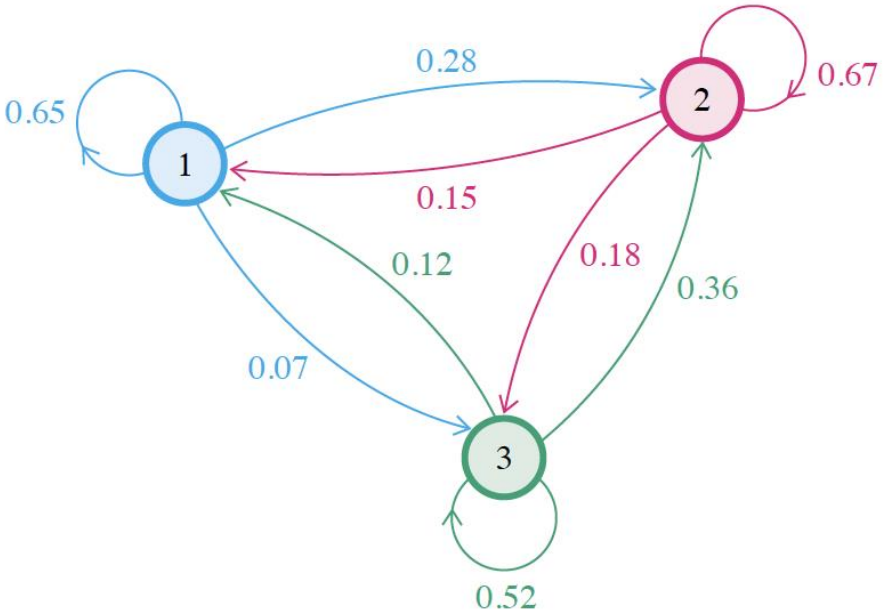
a. Explain the terms "thinning" and "burn-in" and the purpose of each in the context of the following plots of output from a Gibbs sampler. What thinning and burn-in would you recommend for this problem? (6 marks)



b. Briefly describe Gibbs sampler for parameters θ_1, θ_2, and θ_3, with joint posterior distribution $p(\theta_1,\theta_2,\theta_3|y)$ (6 marks)

c. Why do you think Food & Drug Administration (FDA) does not have much faith on the use Bayesian inference in Clinical Trials? How do you convince FDA to allow Bayesian inference in a Clinical Trial analysis?

d. Find the transition matrix of the transition diagram below (6 marks)



e. The number of the lions y=1, 2, 3 … breaking out of a Nairobi national park within the last one month follow the distribution $f(y|\theta) = \theta(1-\theta)^{y-1};\ y = 1,2,3\ldots; 0 < \theta < 1.$ Find the Jeffrey's prior distribution of θ and hence or otherwise its posterior distribution. (9 marks)

f. Is a vague prior the same as a non-informative prior? (3 Marks)

Question 2 (15 marks)

a. Imagine you are a financial analyst at an investment bank. According to your research of publicly-traded companies, 60% of the companies that increased their share price by more than 5% in the last three years replaced their CEOs during the period. At the same time, only 35% of the companies that did not increase their share price by more than 5% in the same period replaced their CEOs. Knowing that the probability that the stock prices grow by more than 5% is 4%, find the probability that the shares of a company that fires its CEO will increase by more than 5%. (5 marks)

b. The WinBUGS model below concerns the proportion of seeds that germinated on each of 21 plates arranged according to a 2 by 2 factorial layout by seed and type of root extract. Let r_i and n_i respectively be the number of germinated and the total number of seeds on the *i* th plate, *i* =1,...,N.

```
model {
 for (i in 1:K) {
    for (j in 1:n) {
          Y[i, j] ~ dnorm(eta[i, j], tauC)
          eta[i, j] <- phi[i, 1] / (1 + phi[i, 2] * exp(phi[i, 3] *
    x[j]))
    }
    phi[i, 1] <- exp(theta[i, 1])
    phi[i, 2] <- exp(theta[i, 2]) - 1
    phi[i, 3] <- -exp(theta[i, 3])
    for (k in 1:3) {
          theta[i, k] ~ dnorm(mu[k], tau[k])
          }
           }
             tauC ~ dgamma(1.0E-3, 1.0E-3)
    sigmaC <- 1 / sqrt(tauC)
    varC <- 1 / tauC
          for (k in 1:3) {
          mu[k] ~ dnorm(0, 1.0E-4)
          tau[k] ~ dgamma(1.0E-3, 1.0E-3)
          sigma[k] <- 1 / sqrt(tau[k])
          }
     }
}
```

i. Describe using simple mathematical equation the likelihood and the priors from the WinBUGS code above (7 marks)

ii. Sketch the densities of parameter "tauC" under Gamma and Uniform priors? (3 marks)

Question 3 (15 marks)

a. Why do you think Food & Drug Administration (FDA) does not have much faith on the use Bayesian inference in Clinical Trials? How do you convince FDA to allow Bayesian inference in a Clinical Trial analysis? (5 Marks)

b. Consider a bivariate normal posterior distribution of the parameters θ_1 and θ_2:

$$\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix} \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right)$$

i. Determine the full conditionals of $\theta_1|\theta_2$ and $\theta_2|\theta_1$. (5 Marks)

ii. Write a **R code** for Gibbs sampling from the full conditionals in (a.) above. (5 Marks)

Question 4 (15 marks)

Suppose we have number of failures y_i for 10 pumps in a nuclear plant. We also have the times t_i at which each pump was observed. Determine the Poisson likelihood, and hence the posterior distribution of the number of failures where the expected number of failures λ_i differs for each pump. Use Gamma (α, β) prior on the λ_i.