



School of Computing and Engineering Sciences  
Bachelor of Science in Electrical and Electronics Engineering  
MAT 1201- Mathematics II

Date: Friday, 25<sup>th</sup> March, 2022

Time: 2 Hours

***Instruction***

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

**QUESTION ONE [30 Marks]**

- a) Evaluate the value of  $z^4 + z^3 + 2z^2 + 4z - 8$  where  $z$  is the complex number  $2i$ . [2 Marks]
- b) Given a function  $A(x)$  and a point  $a$ , when do we say that  $A(x)$  is continuous at  $a$ ? [3 Marks]
- c) For the given functions, determine whether or not the discontinuity is removable at the specified point and if so, evaluate the function.
- i.  $B(t) = (t^3 + t - 1)(t - 2)^{-1}$ ,  $t = 2$  [1 Mark]
  - ii.  $C(s) = \frac{s^2 - 4}{s - 2}$ ,  $s = 2$  [1 Mark]
  - iii.  $D(e) = (e^2 + 1)^{0.5}(e - 2)^{-1}$ ,  $e = 2$  [1 Mark]
- d) Find the Maclaurin series expansion for the function  $\text{Sin}^2(x)$  up to and including the term in  $x^6$ . (Hint:  $2\text{Sin}(x)\text{Cos}(x) = \text{Sin}(2x)$ ). [5 Marks]
- e) Find the derivative  $\frac{dy}{dx}$  for the function  $x = \text{Tan}(2t)$ ,  $y = \text{Sinh}(t)$  at the point  $t = \frac{\pi}{6}$ . [5 Marks]
- f) Find the Taylors series expansion of  $(x - 1)e^x$  about  $x = 1$ . [4 Marks]
- g) Find  $\frac{dy}{dx}$  given that  $y = \text{Tanh}(x)$  [4 Marks]
- h) Distinguish between the discontinuities: removable, jump, essential and infinite. [4 Marks]

**QUESTION TWO [15 Marks]**

- a) Find the first four terms in the Maclaurin series expansion of the function  $P(x) = \frac{x+1}{-5x+x^2+6}$  by first finding the partial fraction decomposition of the function. [6 Marks]
- b) Find the Taylor series expansion for the function  $Q(x) = e^x \cos(x)$  [5 Marks]
- c) Solve for  $x$  and  $y$  given that  $(x + iy)(3 + 4i) = 3 - 4i$  [4 Marks]

**QUESTION THREE [15 Marks]**

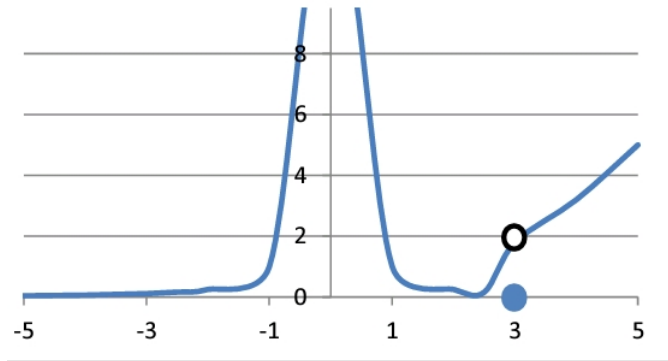
- a) Compute the absolute value and conjugate of the following:
- i.  $z = (1 + i)^6$  [3 marks]
- ii.  $w = i^{17}$  [2 Marks]
- b) Write in trigonometric form ( $k[\cos(\theta) + i\sin(\theta)]$ ) the following complex numbers:
- i. 8 [2 Marks]
- ii.  $(\cos(\frac{\pi}{3}) - i\sin(\frac{\pi}{3}))^7$ . [3 Marks]
- c) Simplify the expression  $\frac{1+i}{1-i} - (1 + 2i)(2 + 2i) + \frac{3-i}{1+i}$  [5 marks]

**QUESTION FOUR [15 Marks]**

- a) A curve  $C$  is given by the parametric equations  $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$  for  $t \in \mathbb{R}$ . Determine
- i.  $\frac{d^2y}{dx^2}$  [4 Marks]
- ii. The point(s) of intersection of the curve  $C$  and the line  $3y = 4x$  [3 Marks]
- b) Given that  $x = e^t \sin(t)$  and  $y = e^{-t} \cos(t)$ . Find  $\frac{dy}{dx}$  and simplify your answer. [5 Marks]
- c) Given that  $\frac{dy}{dx}$  at a point  $t = a$  gives the slope of the tangent line of a parametric curve, find the equation of a tangent line of a curve defined by  $x(t) = t^2 - 3$ ,  $y(t) = 2t - 1$ . Further, solving the parametric curves at  $t = a$  gives a point on the curve. Take  $t = 2$  and find the equation of the tangent line. [3 Marks]

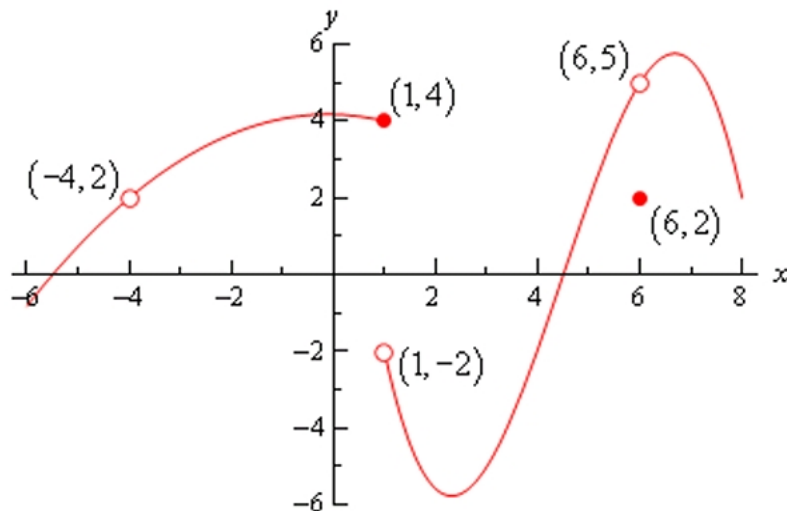
**QUESTION FIVE [15 Marks]**

- a) Determine where the function whose graph is shown below is discontinuous and identify the type of discontinuity exhibited. [5 Marks]



- b) Below is the graph of  $f(x)$ . For each of the given points determine the value of  $f(a)$ ,  $\lim_{x \rightarrow a^-} f(x)$ ,  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a} f(x)$ : [10 Marks]

- i.  $a = -4$
- ii.  $a = 1$
- iii.  $a = 6$



[10 Marks]