



STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)
BACHELOR OF SCIENCE IN STATISTICS AND DATA SCIENCE
MAT 1102 DISCRETE MATHEMATICS
END OF SEMESTER EXAMINATION

Date: 1st November 2024

Time: 1:00 pm to 3:00 pm

- Answer **QUESTION ONE** and any other **TWO** questions.
- Show all your workings in the booklet provided.

QUESTION ONE (30 Marks)[COMPULSORY]

- Q1).** (a) State if each of the following statements are **TRUE** or **FALSE**. Provide a brief justification for your answer.
- (i) The number of ways in which a set with 3 elements can be partitioned into 2 non-empty subsets is **3**. (2 marks)
 - (ii) In a group of 13 children, at least 2 of the children have their birthdays in the same month. (2 marks)
 - (iii) $C(10^5, 10^4) = C(10^5, 90,000)$ (2 marks)
 - (iv) The chromatic number $\chi(K_{5,5}) = 2$. (2 marks)
- (b) Simplify $\frac{(t^3 - 1)!}{(t - 1)(t^3 - 2)!}$. (3 marks)
- (c) How many different even 4-digit numbers can be formed with the digits of the number 80,697? (3 marks)
- (d) Find the number of ways to sit n married couples around a table if every woman is sitting next to her husband. (3 marks)

- (e) Using Binomial theorem, compute $(0.98)^5$ to two decimal places. (4 marks)
- (f) How many edges are there in a graph with 100 vertices each of degree 3? (2 marks)
- (g) Using Euler's formula show that the graph K_5 is not planar. (3 marks)
- (h) Give the recurrence formula for computing Bell numbers. Hence, find B_3 . Interpret your answer. (4 marks)

QUESTION TWO (20 Marks)[OPTIONAL]

- Q2).** (a) In how many ways can 5 students be arranged in a line such that
- (i) Two particular students among them are always together? (3 marks)
- (ii) Two particular students among them are never together? (3 marks)
- (b) A committee of 12 people is to be formed from a group of 10 gentlemen and 12 ladies. In how many ways can the committee be formed if:
- (i) at most 5 ladies are to be part of the committee? (3 marks)
- (ii) two thirds gender rule is satisfied in forming the committee? (That is, there should be not more than two-thirds of either gender in the committee). (3 marks)
- (c) Suppose m men and n women are to be seated in a row so that no two women sit together. If $m > n$, show that the number of ways in which they can be seated is
- $$\frac{m!(m+1)!}{(m-n+1)!} .$$
- (4 marks)
- (d) The government of Kenya is planning to restart the registration of the number plates of government vehicles. If the number plate is supposed to have 6 characters with the first two characters as G and K , then followed by any three letters of the English Alphabet and the last character as given by any digit.
- (i) How many number plates can be produced using this method? (3 marks)
- (ii) How much should the government budget with to produce all the number plates if the cost of producing 5 number plates is KES 15,000. (1 mark)

QUESTION THREE (20 Marks)[OPTIONAL]

Q3). (a) Find the coefficient of x^5 in the expansion of (5 marks)

$$\left(2x^2 - \frac{1}{3x}\right)^{10}.$$

(b) Give the recurrence formula for computing Stirling numbers of the second kind and hence find $S(4, 2)$. Interpret your answer. (6 marks)

(c) Find the middle term of

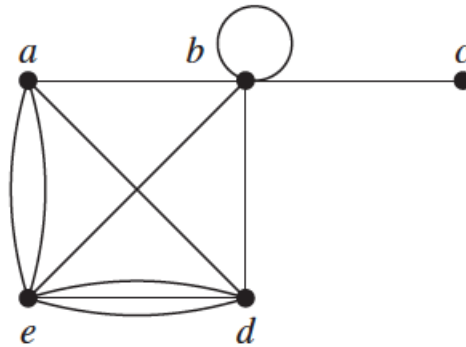
$$\left(1 - \frac{x^2}{2}\right)^{14}.$$

(4 marks)

(d) Consider a set A of size 3 defined as $A = \{Ken, Ben, Jane\}$. Give the number of ways in which A can be partitioned into non-empty subsets. (5 marks)

QUESTION FOUR (20 Marks)[OPTIONAL]

Q4). (a) Considering the following graph H .



H

(i) Find the degree of each vertex. (3 marks)

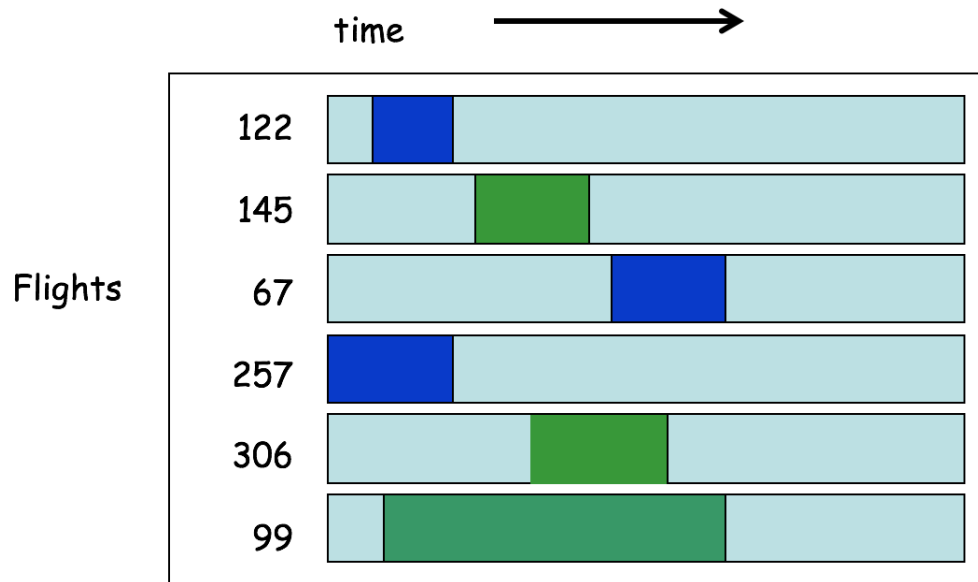
(ii) Using your answer in part (i) above, verify the Handshaking Theorem. (2 marks)

(b) Draw a directed graph represented by the following adjacency matrix.

$$\begin{pmatrix} 1 & 3 & 0 \\ 2 & 0 & 4 \\ 2 & 2 & 0 \end{pmatrix}$$

(3 marks)

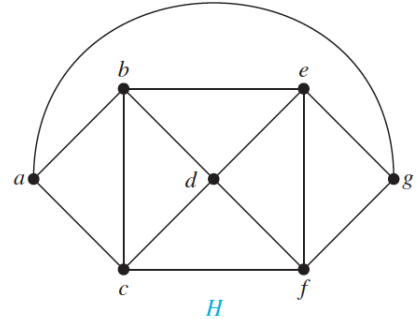
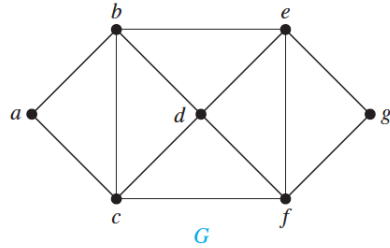
(c) Consider the following table showing various flights and corresponding times the flights need gates at an airport.



You are required to advise the airport management on the minimum number of gates that need to be allocated for each flight to avoid any conflicts that may arise when two or more flights arrive at the airport during overlapping times. Using your knowledge on Chromatic numbers, find the minimum number of gates that are needed to accommodate all the flights without any conflict. (6 marks)

(d) By clearly showing on each of the graphs given below, find the chromatic number of graphs G and H respectively. (Draw the graphs on your answer booklet).

(6 marks)



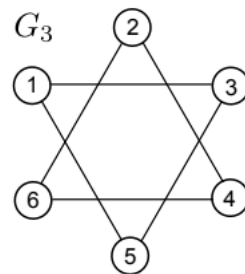
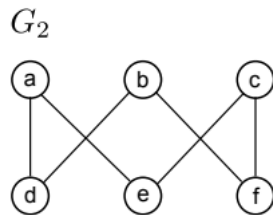
QUESTION FIVE (20 Marks)[OPTIONAL]

Q5). (a) Give an example of a graph with an Euler path but with no Euler circuit.

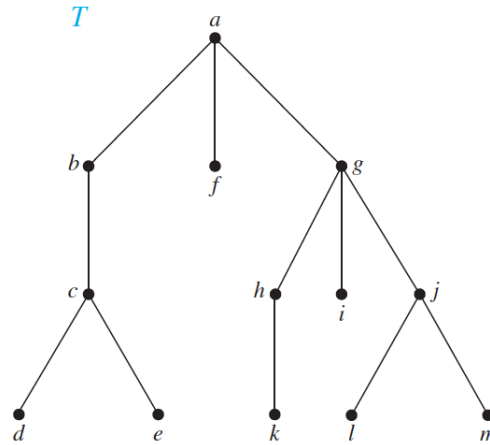
(2 marks)

(b) Check whether the following graphs G_2 and G_3 are isomorphic or not. Provide an elaborate justification for your answer.

(4 marks)



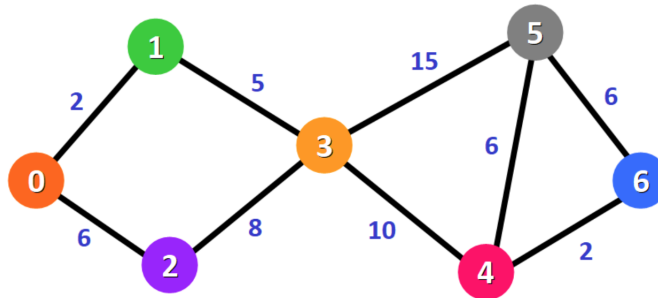
(c) Consider the following rooted tree T .



Find

- (i) The parent of c (1 mark)
- (ii) The children of g . (2 marks)
- (iii) All ancestors of e . (2 marks)
- (iv) Draw the subtree rooted at g . (2 marks)

(d) Consider the following graph.



(e) By applying the idea of Dijkstra's Algorithm compute the shortest path between vertices 0 and 6. (7 marks)

THE END