Strathmore UNIVERSITY

IMPACT OF LONGEVITY IMPROVEMENTS ON LIFE INSURANCE COMPANIES: A STUDY OF TAIWAN

MAINA CATHERINE WACHERA

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

CATHERINE. W. MAINA [*Name of Candidate*]

.... [*Signature*]

30/11/2017 [*Date*]

This Research Project has been submitted for examination with my approval as the Supervisor.

PRISCILLAH M. MOGAKA [*Name of Supervisor*]

.... [*Signature*]

30th November 2017 [*Date*]

Strathmore Institute of Mathematical Sciences
Strathmore University

Abstract

The whole world has noted an unmatched reduction in the mortality rates and Kenya is no exception. These on-going improvements have brought out the need of mortality forecasting for annuitants as well as pensioners in order to prevent insolvency. This has compelled academicians and actuaries to focus their interest in the particular field of mortality and longevity risk. Appropriate modelling techniques or projected life tables are needed for pricing and reserving. In particular the use of stochastic models which take into account various risk causes and components and the relevant impact on portfolio results as opposed to deterministic models that were only based on expected present value. This study extends the literature by using the Lee-Carter method to forecast mortality risk for life insurance companies. The main focus of this paper will be to determine the uncertainty associated with future mortality and how it impacts on the overall risk assessment of Life Insurance companies in developing countries. Using the Lee-Carter proposed by Lee-Carter in 1992 to fit mortality rates, I forecasted future mortality trends using the past trends in mortality rates. I then determined the impact that the mortality trends have on life insurance companies. This results show that improved longevity has a big impact on the cashflows of life insurance companies since they affect the Actuarial Present Value.

Keywords

Mortality, Longevity, Forecasting, Managing, Lee-Carter

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CHAPTER 1: INTRODUCTION

1.1 Background to the study

Risk is a natural element of the community and business life. It is a condition that raises the chance of losses/gains and the uncertain potential events which could manipulate the success of financial institutions (Crowe, 2009). Particularly, the risks that mostly affect life insurance companies are credit risk, liquidity risk, market risk, operational risk, mortality and longevity risk. Longevity risk may be defined as the risk that the actual survival rates and life expectancy will exceed expectations or pricing assumptions resulting in greater cash flow needs for the insurer. Mortality risk on the other hand refers to the risk that the survival rates and life expectancy will be lower than expected or be lower than pricing assumptions leading to adverse cash flows for the insurer. Benjamin and Soliman (1993) confirmed that unmatched improvements in the longevity have been noted all over the world. Decreasing mortality in a great way affects the level and timing of premiums of life insurance companies.

The twenty first century has seen a great improvement in mortality and particularly in adult mortality. According to, Mohajan (2014) Tuberculosis, HIV/AIDS and Malaria were previously the leading causes of morbidity and mortality in Kenya. HIV/AIDS, lower respiratory infections, diarrheal diseases, stroke, and malaria continue to be the most common illnesses in Kenya although at a decreasing rate (Budathoki, 2017). Considerably, illnesses remained the top ten causes of deaths in Taiwan were cancer, heart disease, cerebrovascular disease, diabetes, pneumonia, accidents among others. The standardized rates for these common causes of death however continued to fall. Taiwan as of 2013 had a mean life expectancy of 79.9 which was a rise from 1990 when the life expectancy was 73.89.

For a life insurance company, mortality risk and longevity risk is vital as there is a benefit payable upon death or survival or both as in the case of an endowment assurance. Therefore, the problem comes in since longevity risk is a systematic risk as it is not diversifiable and affects everyone in the same way. The different products offered by life assurers are exposed to longevity and mortality risk at different rates

(Wang et al, 2011). Therefore, this uncertainty in the products cashflows may lead to long term insolvency as well as short term liquidity threats.

Literature has proposed a few ways to help life assurers mitigate mortality and longevity risk. Capital market solutions including mortality securitization, survivor bonds and survivor swaps proposed by, Lin and Cox(2005) and (Dowd et al,2006). However, I will not be looking at derivative hedging in this paper. Another method was internal self-insurance taking place in the industry such as the natural hedging strategy (Lin and Cox, 2007). In practice, life insurance companies may have a difficult time implementing a natural hedging strategy because they are not able to allocate insurance liability accordingly. The third method is mortality projections which aim to provide accurate estimations of mortality processes (Milevsky and Promislow, 2001). I will focus on the third method in this paper. I will look at discrete time models used to predict and manage mortality and longevity risk.

Life assurers are often faced with the tough decisions in terms of modelling and managing longevity and mortality risk. The different individual mortality experience can also lead to adverse selection. Information asymmetry and mortality heterogeneity can thus severely limit the usefulness of the proposed risk management tools. The literature provided here is to model as well as manage mortality and longevity risk. Mortality risk can be divided into these distinct types: unsystematic risk, systematic risk and adverse selection. In this paper, I intend to determine the impact of mortality and longevity improvement in the risk management of life insurers companies in Kenya.

1.1.1 Mortality trends

Mortality rates are also said to be heterogeneous in nature. Every single person has a different mortality therefore the life expectancy is just an average estimation of how long the total population is expected to live. There has also been noted an increasing concentration of deaths around the average age of death. And an increase in the average age of death over time, described as expansion of the survival function.

Benjamin and Soliman (1993) confirmed that the whole world has noted unmatched mortality improvements. This was no different for Kenya as there has been an improvement of 9.33% in adult mortality from 2010 to 2015 an improvement from

41.3 deaths per 100 population aged 15 to 59 in 2010 to 37.45 deaths per 100 population aged 15 to 59 in 2015 (Knoema, 2016).

The life expectancy for an adult male as of 2015 was 61.1 years as opposed to 50.09 in 2000. The life expectancy for females on the other hand was 65.8 years in 2015 as opposed to 51.51 in 2000 (Worldbank, 2016).

Figure 1: Male and Female Mortality rates in Kenya

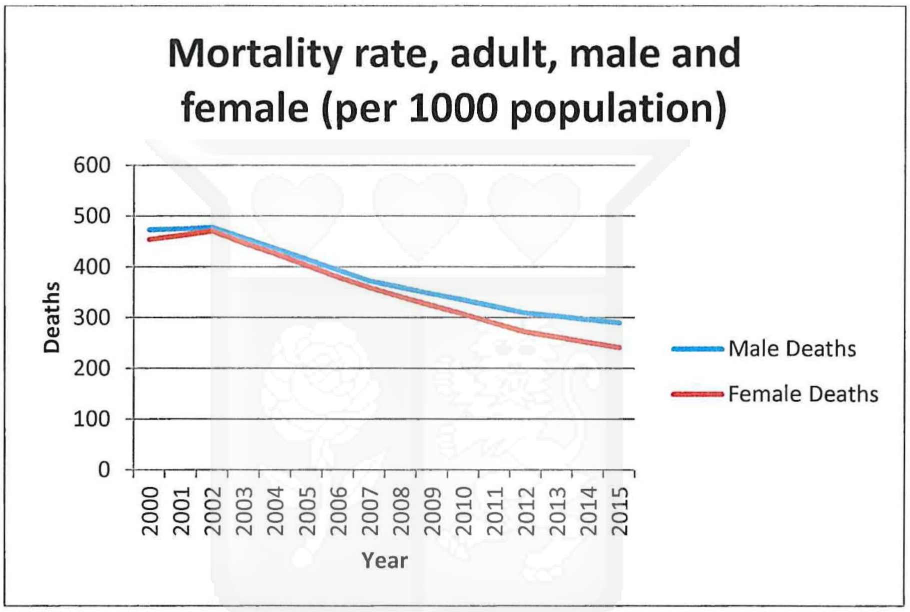
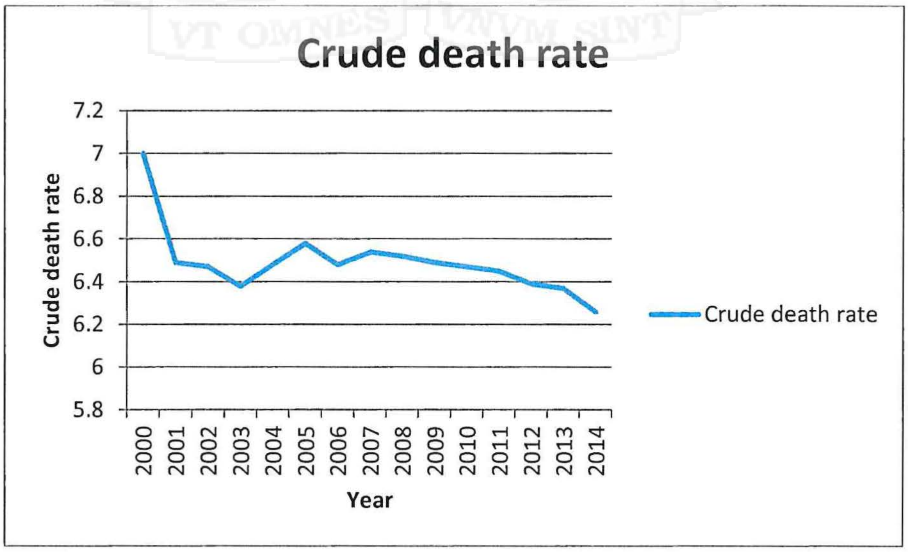


Figure 2: Total Taiwan population crude death rate



1.2 Problem Statement

Mortality rates have seen a great improvement in the twenty first century as a result of a healthier lifestyle, more people having access to health care, rising incomes and a variety of social programs. These significant increases have brought out the importance of mortality forecasting. More have been informed about the health hazards that lead to a high mortality such as HIV/AIDs, excessive consumption of alcohol and smoking. Also, the compulsory uptake of pension benefits has seen it possible for the elderly to continue with a similar lifestyle as before retirement therefore reducing lack of healthcare allowance which in turn has let to mortality improvements. Gompertz published his book in 1825, the law of Mortality and since then numerous models have been proposed. Mortality forecasting however, is more recent.

A review of literature shows that there have been several studies done on hedging of mortality risk (Wang et al, 2011). There has also been several studies done on modelling more predictable mortality risk such as the lee-carter model, Lee-Carter (1992), Multifactor age period models (Renshaw and Haberman, 2003).This model was also recreated by Cairns, Blake and Dowd (2006b) but they focused on higher ages from (60 to 89) this model will therefore not be helpful since Kenya's life expectancy though improving is merely 61.58. Most of the research done has been used to model mortality risk in Pension schemes particularly defined benefit schemes. Research has however not focused on the impact of mortality and longevity improvements in Life insurance companies in Kenya while most of the mortality forecasts have been done for pension schemes. I will also use lower ages for this model, from age 40 to 89 so at to cater and quantify early deaths as well.

I will also consider the effect of the apparent forecasting bias on Life Assurer's liabilities and the effect of aggregate mortality risk on unbiased estimates of the obligations. Finally, I will consider whether mortality risk plays a large role in relation to actual life assurer's risk assessment. To do this I will model mortality rates and forecast the mortality rates using the Lee-Carter (1992) model. I will also use the asset liability management framework to manage these risks.

1.3 Research Objectives

The objectives of this study are:

a) To forecast longevity risk using the Lee-carter model for the life insurance sector.
b) To assess the impact of mortality improvement on life insurance companies.

1.4 Research questions

The research questions that this study seeks to answer are:

a) What is the model suitable in forecasting mortality risk arising in the life insurance sector?
b) To what extent has mortality improvement affected life insurance companies?

1.5 Motivation of the study

This study seeks to benefit the management department particularly the risk department of life insurance companies who have over the years been faced with unanticipated benefits. This study will seek to benefit the management as it will empower them to come up with better models to meet their liquidity requirements in order to prevent failure to meet their commitments and to prevent insolvency as well. These models will seek to provide more predictable cashflows which will also be determined by risk profiling of the policyholders.

This study also seeks to benefit future researchers with a similar interest of mortality risk. It will help them know the impact of mortality improvement on the risk assessment of life assurers. I will also help them further their research and expand more where necessary. The shareholders in life insurance companies could also benefit from this research as it will empower them with information regarding the extent to which they are facing mortality risk and how it is affecting their mortality surplus.

CHAPTER 2: LITERATURE REVIEW

2.0 INTRODUCTION

This chapter reviews the literature available on theoretical foundation, concept and models used to determine and measure mortality and longevity risk. It then presents information from various sources around the world and gives solutions and methodologies approaching them.

2.1 THEORETICAL LITERATURE

This section represents some theoretical aspects related to longevity and mortality risk. I will focus on discrete time models and asset liability management in modelling and managing mortality and longevity risk.

2.1.1 Discrete Time Models

Discrete time models used in modelling mortality risk

In discrete time models, data is typically presented in the form of crude death rates.

$$m_c(t,x) = \frac{D(t,x)}{E(t,x)}$$

$$= \frac{deaths\ during\ calendar\ year\ t\ aged\ x\ last\ birthday}{average\ population\ during\ calendar\ year\ t\ aged\ x\ last\ birthday}$$

Some authors have chosen to model the death rates directly, others have chosen to model mortality rates *q(t,x),* the underlying probability that an individual aged exactly x at time t will survive until time $t+1$.

The two are linked by either of the two approximations:

$$q(t,x) = 1 - \exp[-m(t,x)]$$

or

$$q(t,x) = \frac{m(t,x)}{1+0.5m(t,x)}$$

2.1.1.1 Lee-Carter Model (1992)

Under the Lee-Carter model, annual data is subdivided into integer ages hence it is relatively easy to use rigorous statistical methods to fit discrete time models to the data. It is the earliest leading statistical model of mortality in the demographic literature and still the most popular (Deaton and Paxson, 2004). This model is a numerical algorithm used in mortality and life expectancy forecasting. The input of the model is an age specific mortality rate. In the last decade, scholars have supported this and closely related approaches, and policy analysts forecasting mortality in countries around the world have copied or altered it (Booth, Maindonald and Smith; 2002; Deaton and Paxson, 2004; Haberland and Bergmann, 1995; Lee, Carter and Tujapurkar, 1995; Lee and Rofman, 1994; Li and Boe, 2000; Wilmoth, 1996,1998a,b).

The first step of the Lee-Carter method consists of modelling these mortality rates as:

$$m_{at} = \alpha_a + \beta_a \gamma_t + \epsilon_{at}$$

Where: α_a, β_a and γ_t are parameters to be estimated and ϵ_{at} is a set of random disturbances.

$$m(t,x) = \exp\left[\beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)}\right]$$

Where $\beta_x^{(k)}$ represents the age effects and $k_t^{(2)}$ is a random period effect. $\beta_x^{(1)}$, represents an average log mortality rate over time at age x, while $\beta_x^{(2)}$represents the improvement rate at age x. $k_t^{(2)}$is often modelled as a random walk.

Different approaches have been taken to estimate the parameters. The early approach of Lee-Carter has been largely replaced by more strict statistical methods (Brouhns et al., 2002, Crado et al., 2005, and Delwarde et al., 2007) with the primary focus on goodness of fit over all the data. Lee and Miller, (2001) took a different view that the goodness of fit needs to have a greater emphasis in the final year in the dataset. They observed that the purpose of modelling is normally to project mortality rates.

Lee-Carter method has a number of advantages; it is preferred for its simplicity, the underlying model has been shown to represent a large portion of the variability in the mortality rates in developed countries (Tuljapurkar et al., 2000). The parameters of this model are easy to interpret and a simple random walk with a drift forecast has generally been appropriate for the single extrapolated parameter. A limitation of Lee-

Carter model is that it assumes that the ratio of the rates of mortality change at different ages remains constant over time where as evidence of substantial age time interaction has been found. Lee-Carter forecast rates lack across-age smoothness and become increasingly jagged over time (Girosi & King, 2006) which is non-intuitive and may be problematic in practical applications.

Given all of the mentioned models, after reviewing the Lee-Carter model and despite the limitations, Tuljapuktar and Boe (1998), recommended using the Lee-Carter model to modelling mortality rates. Dowd et al (2010) agreed that the Lee-Carter model performs well most of the time. It provides a good fit to historical data as the age function of the Lee-Carter model gives an allowance to model across all ages.

In this model, Lee and Carter used mortality data classified by the age of death and the year of death and then modelled the force of mortality in terms of those two variables. Forecasts were then obtained by treating the year of death or period parameters as a time series then forecasting the estimates of these parameters.

2.2 EMPIRICAL LITERATURE

Life Insurance companies are facing unmatched pace of change in mortality and it presents a challenge in pricing premiums. The companies therefore need to keep adjusting to the trends in mortality in order to remain in existence and maintain their profitability.

How good these companies will be in creating customer value at low premiums will depend on the skills and resourcefulness of the business. How well these companies forecast the future mortality rates will go a long way towards mitigating losses created by an unanticipated number of deaths which could lead to more benefits being paid out therefore affecting the company's profitability and liquidity.

2.2.1 Model suitable for measuring mortality

The Lee-Carter model, Lee & Carter (1992) is one of the oldest models used in measuring mortality risk. It is a discrete time stochastic model. It was originally created to forecast life expectancy; however, it can also be used to forecast mortality rates at each age. This paper will present some recent developments in the fitting of the Lee-Carter model, using smoothing methods to reduce the number of effective parameters in the model. This will also show the financial impact of these projections on the insurance companies. Model risk is an important source of uncertainty for actuaries pricing and reserving for annuities.

The Lee-Carter framework is as shown below:

$$log\mu_{x.y} = \alpha_x + \beta_x K_y \tag{1}$$

Where $\mu_{x.y}$ denotes the force of mortality at age x in year y, α_x is the effect of age x, Ky is the effect of calendar year y, and β_x is the age specific response to the calendar year effect. It is convenient to rewrite equation (1) in matrix form since we are dealing with a two-dimensional data set. This will be defined as:

M=($\mu_{x.y}$), the matrix of the mortality forces indexed by age and time.

$\alpha' = (\alpha_1, \dots, \alpha_{n_a})$, the vector of age effects (n_a is the number of ages and ' denotes the transpose of a vector and $K' = (K_1, \dots, K_{n_y})$ being the vector of calendar year effects and where n_y is the number of years and $\beta' = (\beta_1, \dots, \beta_{n_a})$, the vector of age-specific responses.

This equation can now be re-written as:

$$Log\ M = \alpha 1' + \beta K'$$

Where 1 is a vector of 1s and is of length n_y

This model like any other is not immune to model risk, parameter stability, parameter uncertainty and stochastic variation.

2.2.2 How mortality risk can affect life assurer's liabilities

This paper will also present how wrong forecasting of mortality improvement could cause severe financial consequences due to untimely claims and annuity pricing. These incorrect mortality assumptions could lead to death claims exceeding the anticipation in pricing and even a shortfall in reserve. Therefore the key protection for life insurance companies against insolvency and protection for annuity holders is capital adequacy and reserving. Finsinger and Pauly (1984) argued the need for solvency regulation for insurance companies. The risk of insolvency is quite high, the insurer simply declares bankruptcy when claims exceed the premium income and asset returns, having already invested funds equivalent to capital separately in the securities markets.

Systematic mortality and longevity risk cannot be diversified. It arises due to the stochastic and unpredictable nature of survival probabilities. Reinsurance has often been the preferred choice in managing longevity risk for annuities. However, reinsurers have a limited risk appetite and are therefore reluctant to take on this risk (Blake et al. (2006b)). The reinsurers sector is also not deep enough to absorb the immense scale of longevity risk currently undertaken by the life insurers. This therefore brings the need to forecast and manage the mortality and longevity risk associated with insurance companies in order for the company to be able to better do their core business without worrying about the mortality risk affecting the cash flows.

2.4 Research gap

A review of literature shows that there have been several studies done on hedging of mortality risk (Wang et al, 2011). There have also been several studies done on modelling more predictable mortality risk such as the Lee-Carter model (1992). This model was recreated by Cairns, Blake and Dowd (2006b) but they focused on higher ages from (60 to 89) this model will therefore not helpful since Kenya's life expectancy though improving is merely an average of 61.58. Little has been done in formulating a formula that can incorporate longevity improvements in life insurance to prevent them from making great losses.

The Insurance Regulatory Authority in 2014 adopted a Risk Based Supervision model which requires supervisors to review the manner in which they are identifying and controlling risks in order to prevent insolvency. IRA also enforced Enterprise Risk Management (ERM) in 2013 to ensure that insurance companies are managed in a sound and prudent manner by having effective systems of risk management. This therefore impacted a need for management of mortality and longevity risk as a way to managing all the risks faced by an insurance company in general.

My research aims to find the impact of mortality risk and longevity risk in the risk assessment of life insurance companies. Since the whole world is facing mortality improvements, and due to lack of availability of Kenyan data, I will use Taiwan data to represent Kenya. Most of the similar research that has been done has focused on pension schemes and in particular defined benefit schemes such as Antolin, (2007). I this research therefore, my focus will be specifically on life insurance companies. I will also use lower ages for this model, from age 40 to 89 in order to quantify and cater for early deaths as well.

2.5 Link between my research and previous work

Previous work done on longevity risk mainly focuses on longevity risk effect on pension schemes, particularly Defined Benefit pension scheme. Some of the researchers who focused on this are Antolin (2007), Currie, I.D. (2013), Cerchiara et al (2008) and Dushi et al (2006). Like Antonin (2007) and Cerchiara et al (2008), I will use the Lee-Carter model and the Generalised linear model to model the mortality and longevity risk respectively and use the asset liability management like

Dushi et al (2006) to manage the mortality and longevity risk arising in these companies. My research will however be based particularly on life insurance companies.

2.6 Conceptual Framework

Based on the research topic, surveying the impact of mortality and longevity improvement on the risk assessment of Life Insurance is time consuming which brings out the need to break down the common factors within the Life Insurance companies. This presents the independent, dependent and the intervening variables as shown.

2.6.1 Description of variables

Independent variable

The independent variable is the mortality and longevity improvement and it's relation to the Life Insurance industry.

Dependent Variable

Longevity risk in Life Insurance Industry is the dependent variable.

Intervening Variable

The intervening variable is how the mortality and longevity improvement impacts on life Insurance companies.

The diagram above shows the relationship between the independent, dependent and intervening variables. This includes the input into the model, using the model to forecast the mortality and longevity risk on the risk assessment of life insurance companies.

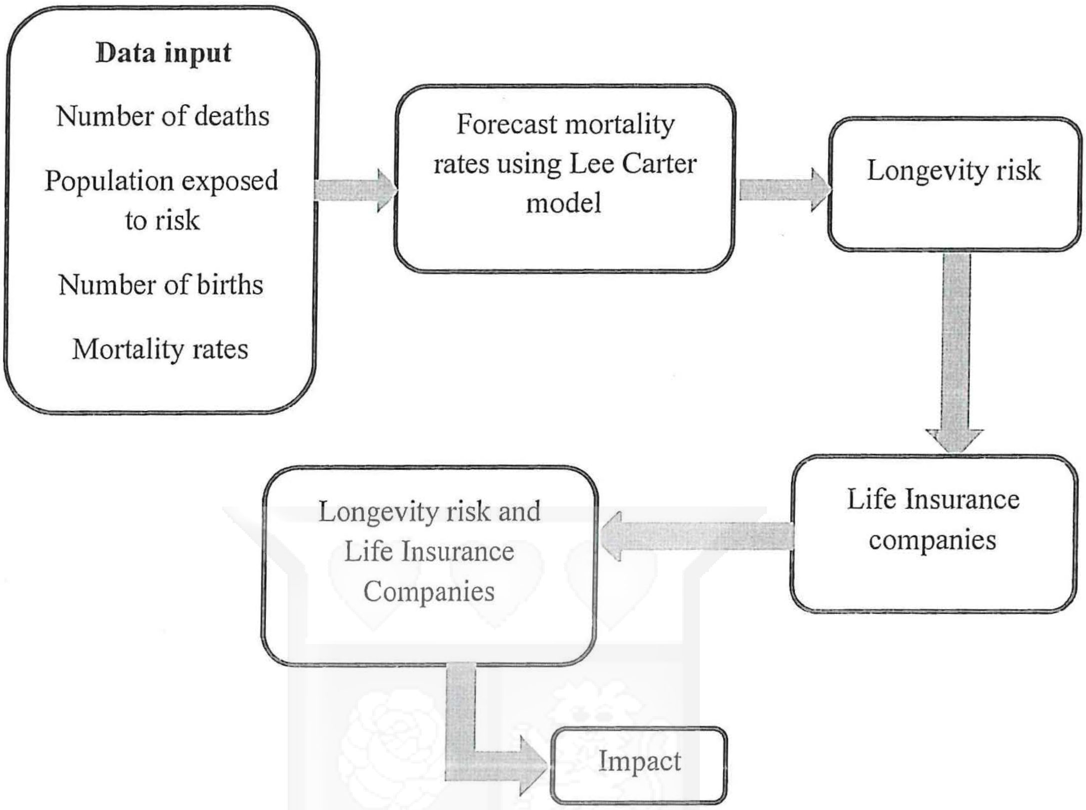


Figure 3: Description of variables

CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

This chapter sets out the various stages followed in the completion of the study. It involves the blueprint for the collection, measurement and analysis of data. This identifies the techniques used in the collection, processing and analysis of data in order to assess the impact of longevity and mortality improvements on the risk assessment of life insurance companies. The subsections included are research design, target population, data types, and the data sources.

3.2 Research design

This study was a cross sectional descriptive research. Cooper & Schindler (2006) said that a descriptive study is concerned with finding out what, who, where, when or how of a research study.

This nature of the study is suitable for this study because it concerns measurements of the same variables across the respondents (the life insurance companies) in the same industry at a particular point in time. It will answer the question 'how' to model and manage mortality improvements.

This study also employed a quantitative approach. This approach focused on obtaining the numerical findings which will be used for qualitative analysis to show the impact of mortality improvements.

3.2 Population and sample of the study

The population of the study comprised of Taiwan population. I used the data already available on the Human Mortality Database on www.mortality.org for the population of Taiwan.

The sample size comprised of the population of Taiwan for the data already available on the Human Mortality Database. This way I can get information about the impact of mortality improvements on the life insurance companies.

3.4 Data types

This study used secondary data which was derived from the findings quantified in the published literature and documents on related research.

As for the forecasting, the necessary data was obtained from the Human Mortality Database for Taiwan population.

3.5 Data Analysis

The Lee-Carter model, Lee & Carter (1992) is one of the oldest models used in measuring mortality risk. It is a discrete time stochastic model. It was originally created to forecast life expectancy; however, it can also be used to forecast mortality rates at each age. This paper will present some recent developments in the fitting of the Lee-Carter model, using smoothing methods to reduce the number of effective parameters in the model. This will also show the financial impact of these projections on the insurance companies. Model risk is an important source of uncertainty for actuaries pricing and reserving for annuities.

The Lee-Carter framework is as shown below:

$$log\mu_{x.y} = \alpha_x + \beta_x K_y + \epsilon_{xt} \qquad (1)$$

Where $\mu_{x.y}$ denotes the force of mortality at age *x* in year *y*, α_x is the effect of age *x*, *Ky* is the effect of calendar year *y*, and β_x is the age specific response to the calendar year effect. It is convenient to rewrite equation (1) in matrix form since we are dealing with a two-dimensional data set. This will be defined as:

M=($\mu_{x.y}$), the matrix of the mortality forces indexed by age and time.

$\alpha' = (\alpha_1, \ldots, \alpha_{n_a})$, the vector of age effects (n_a is the number of ages and "'" denotes the transpose of a vector while $K' = (K_1, \ldots, K_{n_y})$ being the vector of calendar year effects and where n_y is the number of years and $\beta' = (\beta_1, \ldots, \beta_{n_a})$, the vector of age-specific responses.

This equation can now be re-written as:

$$Log\ M = \alpha 1' + \beta K'$$

Where 1 is a vector of 1s and is of length n_y

This model like any other is not immune to model risk, parameter stability, parameter uncertainty and stochastic variation.

CHAPTER 4: DATA ANALYSIS AND FINDINGS

4.1 Sources of data

Mortality data is also collected and published by government agencies. National mortality data are published for a number of countries in the Human Mortality database (HMD). The Kenya Bureau of Statistics collects the data after every ten years (every census). The entire population data is the most appropriate data since it includes large number of individuals, has low sampling errors. Even though the regulatory body in Kenya collects this data, it is not easily available as it is never put in public or published every year as in other countries. In addition, the use of this kind of data can result to sampling problems as the data may not be a true representation of the entire population.

Our analysis was based on the Taiwan mortality data downloaded from the Human Mortality Databases (HMD) through demography package dedicated function. The database provides a detailed mortality and population data according to sex and year to researchers, journalists, policy analysts, students and other stakeholders. Currently, it contains data from 39 countries. The information that can be obtained by sex, age and time in the HMD includes:

- Birth counts
- Death counts
- Population size
- Exposure to risk
- Death rates (period and cohort)
- Life tables
- Life expectancy at birth

Prospects of longer life have led to concern over their implications for uncertain annuity payments. Forecasting mortality appears indeed a key issue in different fields of insurance and financial markets as pricing annuity, evaluating mortality-linked securities and quantifying longevity risk.

4.2 Assumptions

The following demographic and economic assumptions will be used:

- ✓ x will be the retirement age and will be set equal to 65 regardless of the cohort.
- ✓ m will represent the number of fractional payments per year and will be set to equal 12.
- ✓ $\ddot{a}_x^{(m)}$ will represent the actuarial present value of a yearly annuity of 1 monetary unit. The annuity will be evaluated assuming an interest rate of 4% and an inflation rate of 2%.

The formula I used for calculating mortality using the Lee-carter method is as shown:
The Lee-Carter framework is as shown below:

$$logm_{x.y} = a_x + b_x K_t + \epsilon_{xt} \quad (1)$$

Where,

a_x represents the logarithm of the geometric mean of empirical mortality rates, averaged over historical years. It describes the general shape of mortality according to different ages. While e^{a_x} measure indeed the general shape across age of the mortality schedule.

K_t reproduces the underlying time trend, while a term b_x is considered in order to take into account the different effects of time t at each age. b_x is assumed to be invariant over time and it explains how rates decline rapidly or slowly in response to change in k_t.

Finally, $\epsilon_{x,t}$ are independent and identical distributed random variables N (0, σ^2) taking into account the age and time specific trends not fully captured by the model.

4.3 Description of software used

To calculate the mortality projections, I used RStudio software. The required packages were demography, life contingencies and forecast to use and forecast Lee-Carter model.

Using a code import data from Human Mortality Database I could then use the demogdata function using the hmd.mx function. I then went ahead to plot the current mortality rates from 1983 to 2014 which were available on the Human Mortality Database website.

The resulting plots were as follows:

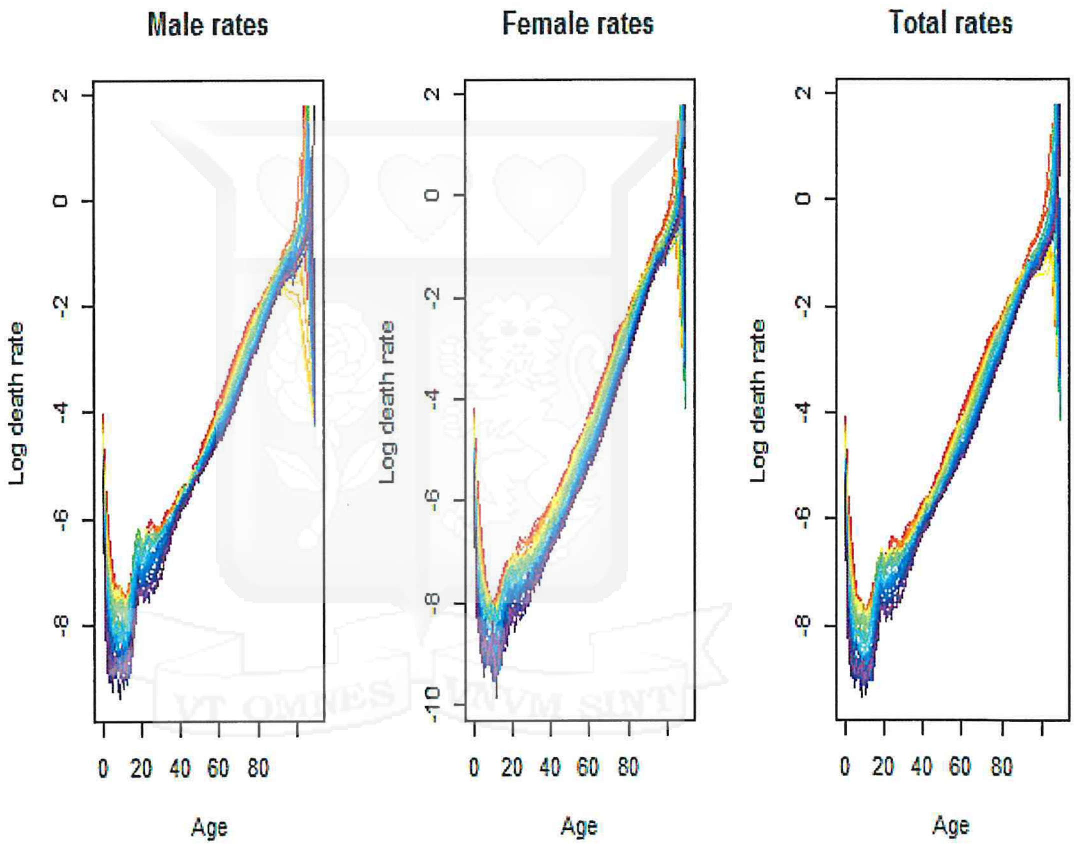


Figure 4: Log death rates against age from 0 to 110

This shows that there is improved longevity for both male and female genders as well as the total of both at all ages. This also shows a mortality hump between age 16 and 40 which could be as a result of road accidents.

To check for the mortality rates trends, See Appendix 1 for the breakdown of the steps followed.

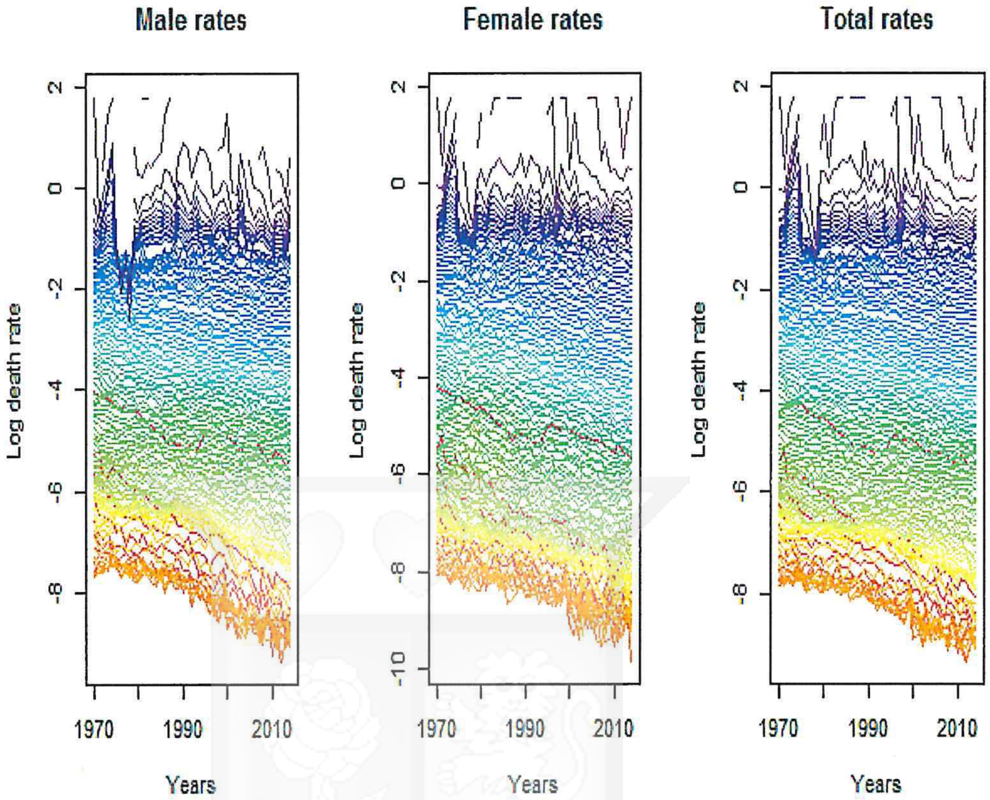


Figure 5: Log death rates against time from 1970 to 2014

Taiwan data confirms that mortality is falling at all ages with a different behaviour according to different ages from 1970 and 2014. The change in mortality however is not constant and is fluctuating over the years although in the long-run there is an observable decrease in mortality rates.

4.4 Fitting the model

To fit Lee - Carter model (without going through logarithms) lca function in Rstudio can be used. Lee-Carter is here applied separately between male, female and total population and by considering a maximum age equal to 100.

The lca function returned object allows us to inspect a_x, b_x and k_t. The figures represent the values of the estimated parameters. To see the breakdown of the code used refer to Appendix 1.

The steps followed in estimation of the parameters using Singular Value Decomposition approach are as follows:

1. $a_x=\frac{1}{T}\sum_{t=t_1}^{t}\ln(m_{x,t})$

2. A matrix Z_x, is created for estimating b_x and k_t

3. Singular Value Decomposition(SVD) is applied to matrix Z_x, to decompose the matrix Zx, into product of three other matrices:

$ULV'=SDV\ Z_x, =L_1U_{x1}V_{t1}+ \ldots\ldots\ldots +L_1U_{xX}V_{tX}$

U represents the age component, L represents the singular values and V represents the time component.

4. The first time-component matrix and the first singular values $k_t=L_1U_{x1}$ will give the estimated values of k_t. The first vector of the age components $b_x=U_{x1}$ will give estimated values of b_x.

5. Estimation of a new matrix Z_x, using the product of the estimated parameters b_x and k_t to get $Z_{x1,1}=b_{x1}\ k_{t1}$

6. The natural logarithm of the central death rates is estimated by:

$\ln m_{x,t}=a_x+Z_{x,t}=a_x+b_xk_t$

The resulting graphs were as shown below:

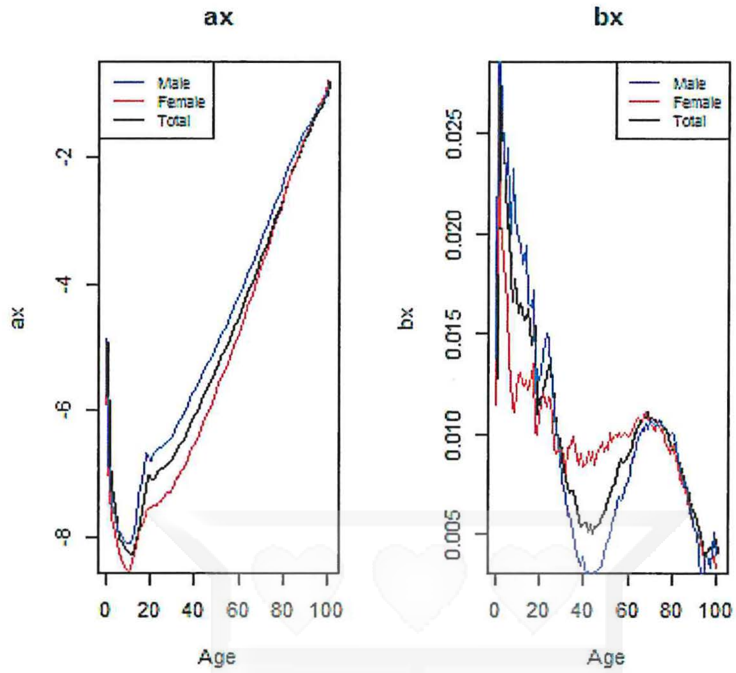


Figure 6: Parameter estimates ax,bx of the Lee-Carter model

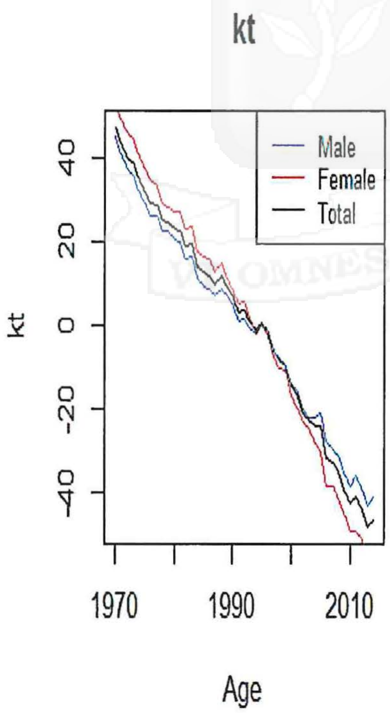


Figure 7: Parameter estimates k_t of the Lee-Carter model

4.5 Forecasting

We can therefore use forecast package to project the future ***k_ts*** (up to 110). Projection is based on ARIMA extrapolation.

The random walk drift model has been used and the model is as shown below:

$$k_t = k_{t-1} + \theta + \varepsilon_t$$

θ is the drift parameter and $\varepsilon_t$is the error term where:

$$\varepsilon_t = \frac{k_T - k_1}{T - 1}$$

To forecast 2 periods ahead in time we substitute k_{t-1} moved back in time 1 period:

$$= k_{t-2} + \theta + \varepsilon_{t-1} + \theta + \varepsilon_t$$

$$= k_{t-2} + 2\theta + \varepsilon_{t-1} + \varepsilon_t$$

The predicted values of k_t rescaled to zero in the last observed year (2014) are here reported. To refer to the breakdown of the steps followed to come up with the projected future values of k_t refer to Appendix 2

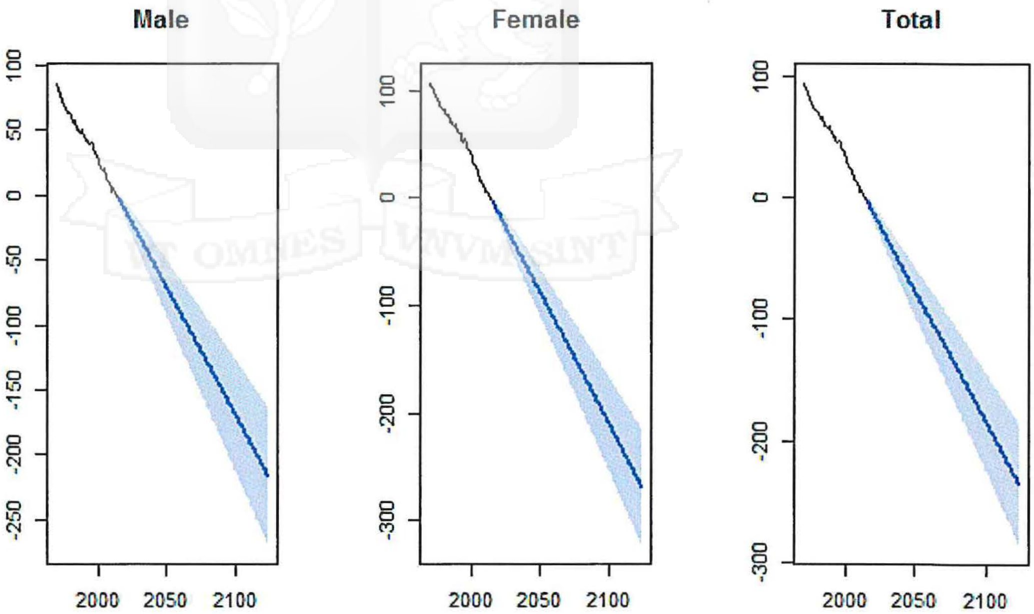


Figure 8: Projected values of kt

To derive the full pattern of rates, past and forecasted rates are here bound in the same matrix.

We report here the pattern of past and forecasted rates according to different population for people aged 65. The expected improvement is clearly visible in the figure. It is showing the past and future trend with smoothed future values.

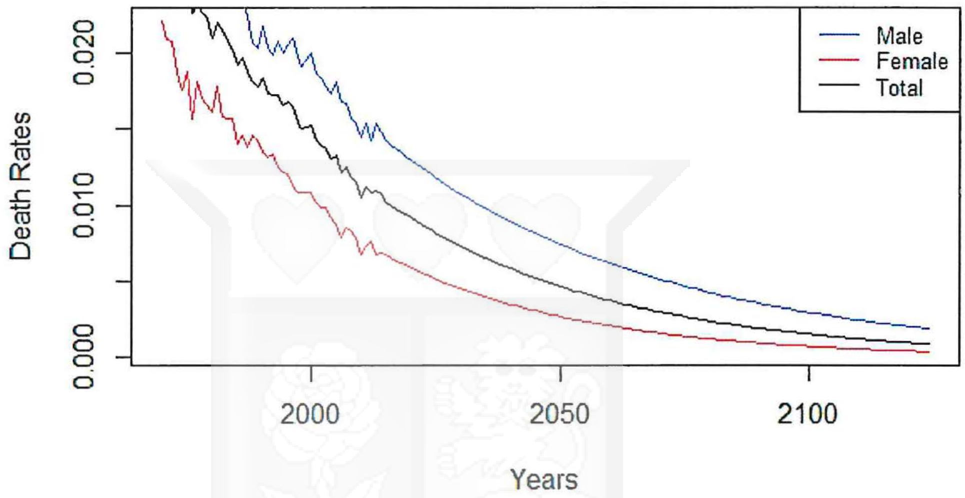


Figure 9: Past and forecasted mortality trends

4.6 Performing Actuarial Projections

Our aim is to create a function to project life table depending by year of birth, using results from Lee-Carter model. In particular, for ages 0 to τ on which Lee-Carter model has been fit, while for extreme ages, τ +1; …; ω on which no data were provided.

We use the following to obtain the life expectancy and the actuarial present value:

$$ln\mu_{x,t} = \hat{a}_x + b_x k_t$$

$$p_{x,t} = \exp(-\mu_{x,t})$$

To see the lines of code run to calculate the APV refer to Appendix 3.

We can therefore calculate the APV of $\ddot{a}_{65}{}^{(12)}$ for the selected cohorts. The values have been derived separately between males and females and by using directly the total population.

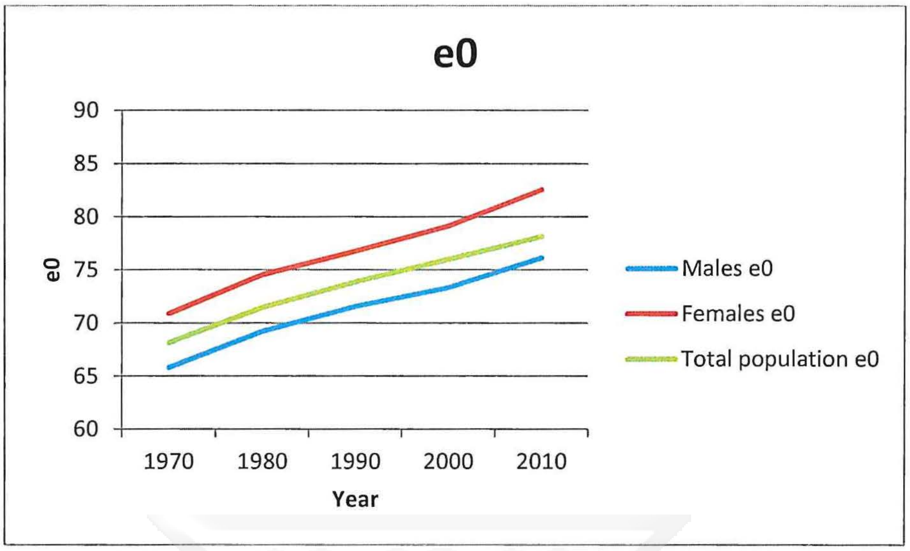


Figure 10: Males, Females, Total population e0 for Taiwan population

Figure 10 shows an increase in life expectancy since 1970.

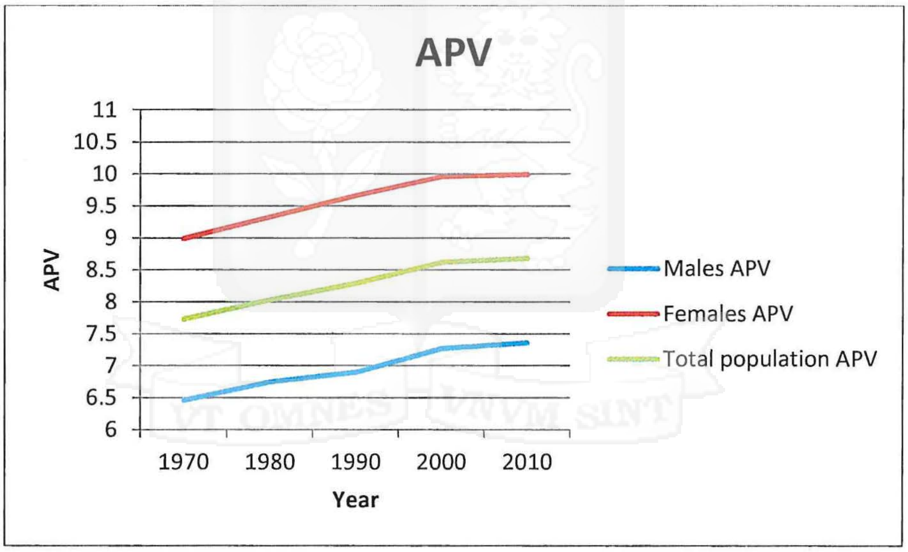


Figure 11: Males, Females, Total population APV for Taiwan population

Figure 11 shows an increase in the Actuarial Present Value with an increase in time.

From the evidence presented above, longevity risk is present and although longevity risk develops and reveals itself slowly over time, if left unaddressed it can affect financial stability by building up significant vulnerabilities in public and private balance sheets for life insurance companies. The ever reducing mortality rates, though good for life assurers in the long run, creates volatile cashflows in the short term. On a macroeconomic level, the effects of a longevity shock on the economy and markets are similar to the effects of aging they propagate through the size and composition of the labour force, public finances, corporate balance sheets, private saving and investment, and potential growth. While the effects of longevity risk perhaps act too slowly to cause sharp movements in asset prices, if unaddressed they add to balance sheet vulnerabilities, affecting fiscal sustainability and the solvency of private financial and corporate institutions. This in turn makes institutions and markets more prone to the negative effects of other shocks.

CHAPTER 5: DISCUSSIONS

5.1 Introduction

This chapter consists of four sections: The first section summarises the study, its design and implementation, conclusions and recommendations. The second section is a summary of the findings and the conclusion which are drawn from the key findings. The third section will involve recommendations for both policy and research that can be drawn from the findings of the research project. The last section will consist of more recommendations but with regard to asset liability modelling which the insurance companies could take into account.

5.2 Summary

This study involved forecasting of future mortality rates and how they impact on life insurance companies. Several studies have been examined in the past on mortality improvements in the world, however, new research is being done every year and the rates are volatile though improving hence prompting a research every now and then, most of the research being on the impact of longevity risk on pension schemes.

The study set out to achieve two main objectives:

a) Using the Lee-carter model to forecast mortality rates
b) Assessing the impact of mortality improvements in life insurance companies.

The research employed a study of Taiwan to represent developing countries like Kenya. I used Taiwan because the mortality rates are similar to Kenya according to a study done in 2017. The Kenyan mortality rate stands at 7 per 1000 population while Taiwan stands at 6.97 per 1000 population. Using the Lee-carter model to project the mortality rates, I was able to determine that there is evidence of longevity in the country for the past 100 years. I was also able to determine that there is a reduction in the mortality rates in the past years. Using the lee-carter function (lca) in Excel, I forecasted the mortality rates for the next 110 years and from the forecast there is a continued reduction in mortality. Using this knowledge, I then went ahead to determine the impact of the mortality reduction in the life insurance industry.

5.3 Conclusions

From the results of the model, we can identify a common trend of mortality changes with age using the standard Lee-Carter model of the Taiwan data. Moreover, we estimated the parameters using the Singular Value Decomposition approach and forecasted the values of k_t using the ARIMA method. Lastly, we forecasted the life expectancies at birth. The results were clear that there is a noted decrease in mortality rates with age and time. The actuarial present value that is supposed to be used in the calculation of annuities has also increased due to the increase of the life expectancy with time. We can therefore conclude that if life insurers and pensioners do not take the changes in the mortality rates into consideration by still using the life tables year in year out, they will end up paying more annuities and more volatile annuity payments. The life insurance companies should therefore reserve for longevity risk in order to avoid going into liquidation.

I used Taiwan data to fit the Lee-Carter Model since Kenyan data was not readily available on the Human Mortality Database and not up to date in World Health Organization. According to World Population 2300 published by the United Nations, life expectancy at birth is expected to increase in the future for both developed and developing nations with the only difference being the rate of increment. Future research focusing on Kenya especially will therefore be recommended.

5.4 Recommendations

Based on the conclusion, we have clearly observed that longevity risk exists for annuity providers such as life assurers and pension providers. The life insurance companies could therefore be exposed to higher than expected payout ratios. With regards to this, we recommend that longevity risk management ideas should be implemented. Actuaries and insurance companies tend to use the latest available mortality tables and usually update then after several years, around every 10 years and a lot can change in 10 years in the mortality rates. Therefore I would recommend the companies to take into account any changes that could have occurred in between those years.

5.5 Asset Liability Management

If life insurers and pension plans retain longevity risk as part of the business, asset liability modelling will therefore be important to ensure that the assets that they hold are sufficient to meet the expected liability requirements. The companies may for instance come up with solvency buffers where the required solvency buffer will be calculated by measuring the effect of predetermined adverse changes in longevity to reduce the probability of underfunding.

The life insurers and pension providers should also come up with up to date data for the Kenyan population in order to use more relevant data and come up with accurate mortality rate projections specific to Kenya.

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Appendix 1

The steps followed to download the Rstudio packages needed were:

library(demography)

library(forecast)

library(lifecontigencies)

To import all available annual data by single years of age (1*1) years for Taiwan:

TaiwanDemo<-(country="TWN", username= "username", password= "password", label= "Taiwan")

View(TaiwanDemo)

To plot the log death rates against age for age 0 to 100

Par(mfrow=c(1,3))

Plot(TaiwanDemo, series= "male", datatype = "rate", main = "Male rates")

Plot(TaiwanDemo, series= "female", datatype = "rate", main = "Female rates")

Plot(TaiwanDemo, series= "total", datatype = "rate", main = "Total rates")

To check the mortality rates trend run the following:

>Par(mfrow=c(1,3))

>Plot(TaiwanDemo,series="male",datatype="rate", plot.type="time", main="Male rates",xlab="Years")

>Plot(TaiwanDemo,series="female",datatype="rate", plot.type="time", main = "Female rates",xlab="Years")

>Plot(TaiwanDemo,series="total",datatype="rate", plot.type="time", main="Total rates",xlab="Years")

To fit the model run the following lines

>taiwanLcaM<-lca(taiwanDemo,series="male",max.age=100)

>taiwanLcaF<-lca(taiwanDemo,series="female",max.age=100)

>taiwanLcaT<-lca(taiwanDemo,series="total",max.age=100)

To plot a_x these are the steps I followed:

>par(mfrow=c(1,3))

>plot(taiwanLcaT$ax, main="ax", xlab="Age",ylab="ax",type="l")

>lines(x=taiwanLcaF$age, y=taiwanLcaF$ax, main="ax", col="red")

>lines(x=taiwanLcaM$age, y=taiwanLcaM$ax, main="ax", col="blue")

>legend("topleft" , c("Male","Female","Total"),

+ cex=0.8,col=c("blue","red","black"),lty=1)

To plot b_x these are the steps I followed:

>plot(taiwanLcaT$bx,main="bx", xlab = "Age", ylab = "bx",type = "l")lines(x = taiwanLcaF$age, y=taiwanLcaF$bx, main="bx", col="red")

>lines(x=taiwanLcaM$age, y=taiwanLcaM$bx, main="bx", col="blue")

>legend("topright" , c("Male","Female","Total"), cex = 0.8,col = c("blue","red","black"), lty = 1)

To plot k_t these are the steps I followed:

>plot(taiwanLcaT$kt, main="kt", xlab="Year",ylab="kt",type="l")

>lines(x=taiwanLcaF$year, y=taiwanLcaF$kt, main="kt", col="red")

>lines(x=taiwanLcaM$year, y=taiwanLcaM$kt, main="kt", col="blue")

>legend("topright" , c("Male","Female","Total"), cex=0.8,col=c("blue","red","black"),lty=1)

Appendix 2

To forecast the future values of k_t

```
> fM<-forecast(taiwanLcaM,h=110)
> fF<-forecast(taiwanLcaF,h=110)
> fT<-forecast(taiwanLcaT,h=110)

> par(mfrow=c(1,3))
> plot(fM$kt.f,main="Male")
> plot(fF$kt.f,main="Female",)
> plot(fT$kt.f,main="Total")
```

Pattern of past and future rates

```
> ratesM<-cbind(TaiwanDemo$rate$male[1:100,],fM$rate$male[1:100,])
> ratesF<-cbind(TaiwanDemo$rate$female[1:100,],fF$rate$female[1:100,])
> ratesT<-cbind(TaiwanDemo$rate$total[1:100,],fT$rate$total[1:100,])

>par(mfrow=c(1,1))
>plot(seq(min(TaiwanDemo$year),max(TaiwanDemo$year)+110),ratesF[65,], col
= "red",xlab="Years",ylab="Death Rates",type = "l")
>lines (seq(min(TaiwanDemo$year),max(TaiwanDemo$year)+110),ratesM[65,], col
= "blue",xlab = "Years",ylab = "Death Rates")
>lines(seq(min(TaiwanDemo$year),max(TaiwanDemo$year)+110),ratesT[65,],
>col="black",xlab = "Years",ylab = "Death Rates")
> legend("topright" , c("Male","Female","Total"), cex = 0.8,col =
c("blue","red","black"),lty = 1);
```

Appendix 3

Actuarial Projections

```
createActuarialTable<-function(yearOfBirth,rate){
+ mxcoh<-rate[1:nrow(rate),(yearOfBirth-min(TaiwanDemo$year)+1):ncol(rate) ]
+ cohort.mx <- diag(mxcoh)
+ cohort.px=exp(-cohort.mx)
#get projected Px
+ fittedPx=cohort.px #add px to table
+ px4Completion=seq(from=cohort.px[length(fittedPx)], to=0, length=20)
+ totalPx=c(fittedPx,px4Completion[2:length(px4Completion)])
#create life table
+ irate=1.04/1.02-1
+ cohortLt=probs2lifetable(probs=totalPx, radix=100000,type="px",
+ name=paste("Cohort",yearOfBirth))
+ cohortAct=new("actuarialtable",x=cohortLt@x, lx=cohortLt@lx,
+ interest=irate, name=cohortLt@name)
+ return(cohortAct)}
Getting annuity APV
getAnnuityAPV<-function(yearOfBirth,rate) {
+ actuarialTable<-createActuarialTable(yearOfBirth,rate)
+ out=axn(actuarialTable,x=65,m=12)
+ return(out)}
Male cohorts
rate<-ratesM
for(i in seq(1920,2000,by=10)) {
+ cat("For cohort ",i, "of males the e0 is",
+ round(exn(createActuarialTable(i,rate)),2),
+ " and the APV is :",round(getAnnuityAPV(i,rate),2),"\n")}
Female cohorts
rate<-ratesF
for(i in seq(1920,2000,by=10)) {cat("For cohort ",i, "of females the e0 at birth is",
```

```
+ round(exn(createActuarialTable(i,rate)),2),
+ " and the APV is :",round(getAnnuityAPV(i,rate),2),"\n")}
```

Total population cohort

```
rate<-ratesT
for(i in seq(1920,2000,by=10)) {cat("For cohort ",i, "of total population the e0 is",
+ round(exn(createActuarialTable(i,rate)),2),
+ " and the APV is :",round(getAnnuityAPV(i,rate),2),"\n")}
```