Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BBS Actuarial Science, BBS Finance & BBS Financial Economics
END OF SEMESTER EXAMINATION

BSA 3108: Theory of Finance

DATE: 23rd July, 2018   TIME: 2 HOURS

INSTRUCTIONS:

(1) This Exam Contains five questions

(2) Attempt question one and any other two questions

(3) Question one is compulsory and carries 30 marks

(4) All the other questions carry 20 marks each

QUESTION 1   -   COMPULSORY

(a) Consider an investor with the following utility function (defined over wealth *w*):

$$u(w) = -exp(-\alpha w)$$

Identify the utility function and *explain* the risk preferences for the investor when α is interpreted as the coefficient of risk aversion. Support your explanation with suitable workings.   [5 marks]

(b) Under Mean Variance Optimization, expected return on a portfolio of risky assets is determined as the weighted average of the expected returns on the risky securities comprising the portfolio, with the weights being the relative proportions of the two assets in the portfolio. From this scheme, optimal portfolios can be obtained by minimizing risk at any given level of return.

     i. *State three main assumptions* of Mean Variance Optimization   [3 marks]

     ii. Three assets have the following attributes:

Asset i	Expected return (R_i)	Volatility (σ_i)
1	4%	6%
2	6%	12%
3	8%	18%

The correlation between assets 1 and 2 is 0.75; while the correlation between asset 3 and both of the other two assets is zero.

Calculate the expected return and the risk associated with the minimum variance portfolio assuming no short sale constraints. [8 marks]

(c) A securities market has only three risky securities, **A**, **B** and **C** with the following annual return attributes:

Attribute	Asset A	Asset B	Asset C
Market capitalization (Billions)	100	150	250
Expected return	4%	r_B	6%

The following assumptions hold:

- The Capital Asset Pricing Model (CAPM) holds in this market;

- The market price of risk is 10% per annum;

- The risk free rate is 3.3% per annum; and

- The expected annual return on the market portfolio is 5.3%

  i. *Calculate* the standard deviation of the annual returns on the market portfolio. *Justify* your response. [3 marks]

  ii. *Calculate* the expected return on asset B, r_B [2 marks]

  iii. *Calculate* the covariance of the annual returns on each asset with the annual return on the market portfolio. Briefly explain your results. [9 marks]

Total for Question 1: 30 marks

QUESTION 2   -   OPTIONAL

(a) Consider a lottery that pays either 50,000 or 10,000 with equal probability.

  i. *Calculate* the certainty equivalent for an investor with a utility function $\frac{1}{\gamma}w^\gamma$ when $\gamma = 0.25$. [3 marks]

  ii. *Comment* on the effect of changing the value of γ using the *Arrow-Pratt framework*. [5 marks]

(b) Suppose the expected return on an asset j is 10%, the expected return on the market portfolio is 12% and the risk free rate is 4%. You undertake to run the following regression:

$$R_j - R_f = \alpha_0 + \alpha_1\beta + \varepsilon$$

The estimated values for α_0 and α_1 are 1%(0.001) and 5%(0.001) respectively. Standard errors are in parentheses.

  i. Do you reject CAPM? *Discuss*. [6 marks]

ii. If you were to test this model again, how would you do things differently than was done here? [3 marks]

Total for Question 2: 20 marks

QUESTION 3 - OPTIONAL

(a) Critically analyse the following statement with regards to the Efficient Market Hypothesis (EMH)

"...given that markets are informationally efficient, no one ever profits from adjustments in asset prices ... "

[4 marks]

(b) Assume an individual with current wealth, W_0. The individual would like to spend an amount C, for current consumption and invest the rest for future consumption. Suppose the individual buys Q_s pure securities. Suppose further that state s can occur with probability π_s and the individual would like to maximize the utility of current consumption and future consumption.

i. *Formulate* a constrained optimization for the investor's decision. [2 marks]

ii. *Show* that the individual will purchase the securities in a given state until the marginal rate of substitution is equal to the price ratio of the states. [7 marks]

(c) List the advantages of using the following measures of investment risk in the context of a portfolio of bonds subject to credit risk.

i. Variance of return [2 marks]

ii. Value at Risk [2 marks]

iii. Tail Value at Risk [3 marks]

Total for Question 3: 20 marks

QUESTION 4 - OPTIONAL

(a) The CAPM has had lasting implications for Finance and Investments. Briefly discuss the implications of the *Capital Asset Pricing Model* for *Portfolio Selection* [8 marks]

(b) A securities market consists of the three securities A, B and C with the following information:

Security i	Price	Payoff in state 1	Payoff in state 2
A	270	486	216
B	216	0	432
C	p_c	648	324

Further assume that the two states have equal probability of occurrence.

i. Explain whether this market is complete [2 marks]

ii. Calculate the equilibrium price for asset C, p_c. Justify your response.[4 marks]

iii. Calculate the implicit risk free rate and state probabilities. [6 marks]

[6 marks]

Total for Question 4: 20 marks

QUESTION 5 - OPTIONAL

Finance has been touted as special among the social sciences since it is built on *"theoretical foundations"*, lends itself to solid *"empirical analysis"* and possesses a lot of *"practical applications"*. Seeing Finance from this point of view enables a full cycle of scientific research, from formal theory to tests and then to practical applications.

Discuss the foregoing claim in the context of the Arbitrage Pricing Theory (APT) as could be applied in Kenya under the *following subheadings*:

(a) The theoretical basis of the APT model [9 marks]

(b) Empirical considerations for APT in the Kenyan context [7 marks]

(c) Possible practical applications of the APT in Kenya [4 marks]

Total for Question 5: 20 marks