Strathmore University

Institute of Mathematical Sciences

Master of Science in Biomathematics

End of Semester 4 Examination 2018

BMA 8405: Ecological Modelling

Date: 17th August 2018 **Duration: 2½ Hours**

Answer ***Question*** **1** *any other* ***two*** *questions*

Question 1 (30 marks)

a. (i) Define a metapopulation and give an example of one. **(3 marks)**

(ii) State a possible mathematical model that would describe the population dynamics that would account for the reproduction, mortality and movements in the metapopulation **(7 marks)**

(iii) What, in the long run, would one be interested in about the population dynamics? **(2 marks)**

b. Consider the fishery harvesting model:

$$\frac{dN}{dt} = (14 - 3N)N - 8$$

(i) In the absence of harvesting obtain the intrinsic rate of growth and the carrying capacity of the fishery. **(3 marks)**

(ii) According to the model state the harvesting rate and graphically, or otherwise, indicate whether there exist equilibrium levels. Which of these are stable?

(8 marks)

c. Consider the density at any time t of the forest biomass $B(t)$, wood industries $W(t)$, synthetic industries $S(t)$ respectively, whereby we wish to conserve the forest by using wood alternative industries. The depletion of forest is mainly due to wood industries, population and pollution. We also assume that forest population grows logistically in the absence of wood based industries with intrinsic growth rate r and carrying capacity K. Let parameters c_1 and c_2 be the parameters to signify the competition effect of $B(t)$ on $W(t)$ and of $W(t)$ on $B(t)$, respectively. We also assume that wood based industries entirely depend on the forest biomass while the synthetic industries do not. We assume that a sufficient amount of synthetic is provided to synthetic industries at constant rate Q. Depletion rate of forest biomass is α and the growth rate of wood based industries in presence of forestry biomass is α_1. Finally, h_1 and h_2 are natural depletion rate of wood industries and synthetic, respectively. Formulate a mathematical model to depict the inter relationships between the forest and the two industries. **(7 marks)**

Question 2 (15 marks)

The dynamics of a population can be formulated by age-structured Leslie matrix

$$\begin{bmatrix} n_0 \\ n_1 \\ . \\ . \\ n_L \end{bmatrix}(t+1) = \begin{bmatrix} F_0 & F_1 & . & F_{L-1} & F_L \\ s_0 & 0 & . & 0 & 0 \\ 0 & s_1 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & s_{L-1} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ . \\ . \\ n_L \end{bmatrix}(t). \qquad (2.1)$$

i. Describe what each of the variables/parameters n_i, F_i, s_i, for $i = 1,2, \ldots , L$, stand for. **(7 marks)**

ii. Let (2.1) be rewritten as:

$$N(t+1) = LN(t) \qquad (2.2)$$

In a 4 – age group structured population, the initial sizes of the age groups are:

$$n_o(0) = 0, n_1(0) = 10, n_2(0) = 20, n_3(0) = 5$$

and

$$L = \begin{bmatrix} 0.5 & 2.4 & 1.0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}.$$

Determine the population distribution, $N(4)$, with respect to age, after 4 periods of demographical observations. **(8 marks)**

Question 3 [15 marks]

Consider an ecosystem in which the vegetation biomass V supports the Herbivores H, which are preyed on by the Carnivores C. The following model represents the population dynamics of the ecosystem:

$$\frac{dV}{dt} = rV\left(1-\frac{V}{K}\right) - \frac{\alpha HV}{a+V}$$
$$\frac{dH}{dt} = \frac{\beta HV}{a+V} - \gamma H - \delta HC$$
$$\frac{dC}{dt} = \varepsilon HC - \eta C$$

(i) Interpret each of the terms and parameters of the model. **(11 marks)**

(ii) The vegetation biomass is affected by rainfall, which may be assumed to be a function of the temperature $T^{\circ c}$. Show how you would modify the equation for vegetation biomass in order to incorporate the effect of temperature. **(4 marks)**

Question 4 (15 marks)

Consider the dynamics of a disease in a metapopulation in which there is only one community (region) where treatment for the disease can be done. When the infected individuals move to this community they may infect others in that community. Those who recover may choose to remain in the sub-region or return to the original community. The following SIR model describes such dynamics:

$$\frac{dS_1}{dt} = \lambda N_1 - \alpha_1 S_1 I_1 - \beta S_1 - \mu S_1 + v_1 R_1$$
$$\frac{dI_1}{dt} = \alpha_1 S_1 I_1 - \gamma I_1 - \mu I_1$$
$$\frac{dR_1}{dt} = \sigma R_2 - \mu R_1 - v_1 R_1$$
$$\frac{dS_2}{dt} = \lambda N_2 - \alpha_2 S_2 I_2 + \beta S_1 - \mu S_2 + v_2 R_2$$
$$\frac{dI_2}{dt} = \alpha_2 S_2 I_2 + \gamma I_1 - \mu I_2$$
$$\frac{dR_2}{dt} = -\sigma R_2 - \mu R_2 - v_2 R_2$$

i. Indicate the number of communities in the metapopulation. **(2 marks)**

ii. Interpret each of the variables and parameters of the model. **(8 marks)**

iii. Investigate whether there are possible equilibrium solutions of the model. **(5 marks)**