Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN STATISTICAL SCIENCE
END OF SEMESTER EXAMINATION
STA 8301: MULTIVARIATE STATISTICAL ANALYSIS

Date: 23rd September 2021 Time: 3 Hours

Answer Question ONE and TWO other questions.
You must show *all* work where needed to receive *any credit*.

Question ONE (30 marks)

You are working as a statistician doing oversight for a food-safety and nutrition board, as part of a governmental investigation. The agency has selected 43 types of cereal produced by three companies (General Mills (G), Kellogg's (K), and Quaker (Q)). Eight numerical nutritional characteristics have been measured for each cereal. These 8 variables are *calories*, *protein*, *fat*, *sodium*, *fiber*, *carbohydrates*, *sugar*, and *potassium*. In addition, the data file gives the name of each cereal and the company that produced the cereal. Use the output from the analysis attached to answer the following questions:

(a) Are there notable associations/relationships between some of the variables? If so, describe them (8 marks).

(b) Can we find a few indices that describe the variation in the data set using a lesser dimension than the original set of variables? If so, what are those indices? Is there a convenient interpretation of any of the indices? (8 marks).

(c) What is the best model suggested by BIC and the number of groups? How would you characterize the groups? What proportion of the cereals were produced by Kellog's? (8 marks).

(d) Would you say the three companies are the same in terms of their average values for the eight variables? Why? (6 marks).

Question TWO (15 marks)

The Bumpus Bird dataset (Bumpus, 1898) consists of five body measurements on 49 female sparrows: X_1 = total length, X_2 = alar length, X_3 =length of beak and head, X_4 = length of humerus, and X_5 = length of keel and sternum (all in millimetres). Use the attached analysis results to answer the following questions:

(a) Write down the first 2 principal components (PCs) and interpret them (5 marks).

(b) What seems to be a reasonable number of PCs to use? Why? (5 marks).

(c) Identify any potential outliers (5 marks).

Question THREE (15 marks)

The matrix below shows the correlations between ratings on nine statements about pain made by 123 people suffering from extreme pain. Each statement was scored on a scale from 1 to 6, ranging from agreement to disagreement.

$$Pain.Corr = \begin{pmatrix} 1 & -0.04 & 0.61 & 0.45 & 0.03 & -0.29 & -0.3 & 0.45 & 0.3 \\ -0.04 & 1 & -0.07 & -0.12 & 0.49 & 0.43 & 0.3 & -0.31 & -0.17 \\ 0.61 & -0.07 & 1 & 0.59 & 0.03 & -0.13 & -0.24 & 0.59 & 0.32 \\ 0.45 & -0.12 & 0.59 & 1 & -0.08 & -0.21 & -0.19 & 0.63 & 0.37 \\ 0.03 & 0.49 & 0.03 & -0.08 & 1 & 0.47 & 0.41 & -0.14 & -0.24 \\ -0.29 & 0.43 & -0.13 & -0.21 & 0.47 & 1 & 0.63 & -0.13 & -0.15 \\ -0.3 & 0.3 & -0.24 & -0.19 & 0.41 & 0.63 & 1 & -0.26 & -0.29 \\ 0.45 & -0.31 & 0.59 & 0.63 & -0.14 & -0.13 & -0.26 & 1 & 0.4 \\ 0.3 & -0.17 & 0.32 & 0.37 & -0.24 & -0.15 & -0.29 & 0.4 & 1 \end{pmatrix}$$

The nine pain statements were as follows:

1. Whether or not I am in pain in the future depends on the skills of the doctors.
2. Whenever I am in pain, it is usually because of something I have done or not done.
3. Whether or not I am in pain depends on what the doctors do for me.
4. I cannot get any help for my pain unless I go to seek medical advice.
5. When I am in pain I know that it is because I have not been taking proper exercise or eating the right food.
6. People's pain results from their own carelessness.
7. I am directly responsible for my pain.
8. Relief from pain is chiefly controlled by the doctors.
9. People who are never in pain are just plain lucky.

Use the results of the maximum likelihood (ML) factor analysis provided to:

(a) select the necessary number of common factors (7 marks).

(b) interpret the results of the factor solution after the varimax rotation (8 marks).

Question FOUR (15 marks)

The High School and Beyond (HSB) dataset contains demographic information and standardized test scores for 200 high school students. Suppose our goal is to compare the mean vectors (where the variables are the scores on *read*, *write*, *math*, *science*, and *socst*) among the different levels of *ses* (high, middle, and low socioeconomic classes). Using the analysis of the dataset attached, answer the following questions:

(a) Conduct the MANOVA F-test using Wilks' lambda to test for a difference in (*read*, *write*, *math*, *science*, *socst*) mean vectors across the three *ses* classes. Use a 0.05 significance level, and give the P-value of the test (5 marks).

(b) Check to see whether the multivariate normality assumption of your test is met. Do you believe your inference is valid? (5 marks).

(c) Examine the sample mean vectors for each group and informally comment on the differences among the groups in terms of the specific variables. Which approach would help determine pairwise significant differences in the sample mean vectors? (5 marks).

Question FIVE (15 marks)

The SIDS dataset was collected in an investigation of sudden infant death syndrome (SIDS). The two groups here consist of 16 SIDS victims and 49 controls. The *Factor68* variable arises from spectral analysis of 24 hour recordings of electrocardiograms and respiratory movements made on each child. All the infants have a gestational age of 37 weeks or more and were regarded as full term. Use the results of the analysis provided to answer the following questions:

(a) Use Fisher's linear discriminant analysis (LDA) to classify the children into the two groups, based on all four variables. Write down the discriminant function and obtain a plug-in estimate of the misclassification rate (5 marks).

(b) Perform the classification of the children using the logistic regression approach and obtain a plug-in estimate the misclassification rate. Write down the logistic regression model used (5 marks).

(c) Which of the two approaches is preferable, based on the estimated misclassification rates? Discuss an approach for estimating the misclassification rate, that may improve this comparison (5 marks)

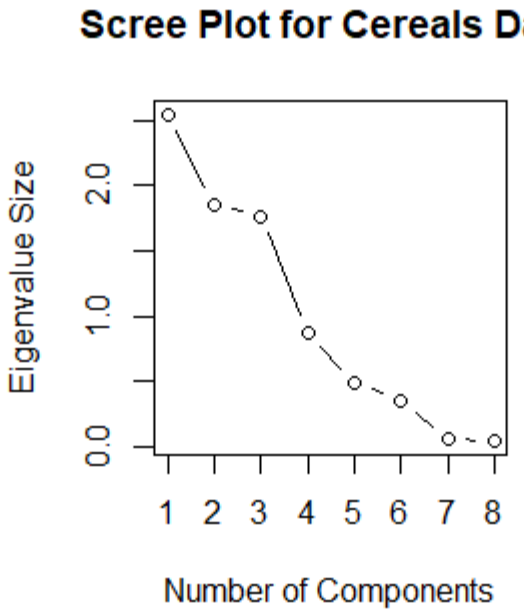
Results of Data Analysis (*R* Output) for Answering the Questions

Question ONE:

```
> cor.mat <- cor(cereal.new2)
> round(cor.mat, 3)
```

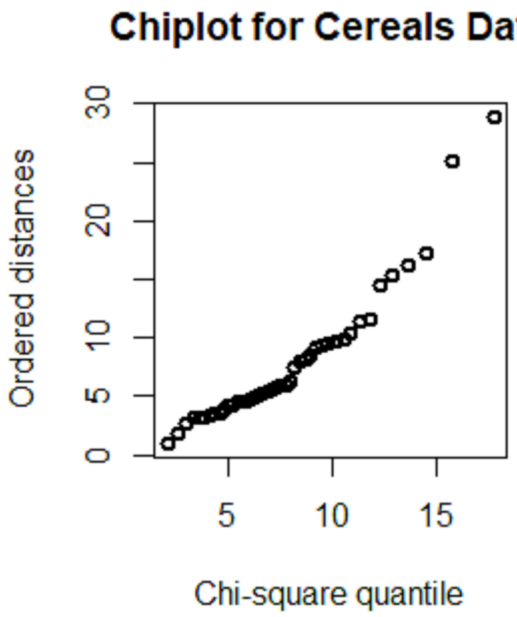
	Calories	Protein	Fat	Sodium	Fiber	Carbohydrates	Sugar	Potassium
Calories	1.000	0.033	0.388	0.337	-0.019	0.256	0.580	0.141
Protein	0.033	1.000	0.206	0.094	0.513	-0.076	-0.400	0.500
Fat	0.388	0.206	1.000	0.010	0.164	-0.323	0.187	0.312
Sodium	0.337	0.094	0.010	1.000	0.043	0.567	-0.049	0.114
Fiber	-0.019	0.513	0.164	0.043	1.000	-0.241	-0.034	0.929
Carbohydrates	0.256	-0.076	-0.323	0.567	-0.241	1.000	-0.315	-0.223
Sugar	0.580	-0.400	0.187	-0.049	-0.034	-0.315	1.000	0.081
Potassium	0.141	0.500	0.312	0.114	0.929	-0.223	0.081	1.000

```
> summary(cereal.pca, loadings=TRUE)
Importance of components:
                          Comp.1    Comp.2    Comp.3    Comp.4
Standard deviation     1.5961107 1.3618964 1.3299737 0.9318172
Proportion of Variance 0.3184462 0.2318452 0.2211037 0.1085354
Cumulative Proportion  0.3184462 0.5502914 0.7713951 0.8799306
                          Comp.5     Comp.6      Comp.7
Standard deviation     0.7051956 0.59785320 0.245974321
Proportion of Variance 0.0621626 0.04467856 0.007562921
Cumulative Proportion  0.9420932 0.98677173 0.994334648
                           Comp.8
Standard deviation     0.212891551
Proportion of Variance 0.005665352
Cumulative Proportion  1.000000000

Loadings:
              Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
Calories       0.114  0.657  0.135  0.100  0.454  0.136  0.469  0.289
Protein        0.421 -0.228  0.253  0.372  0.551 -0.420 -0.242 -0.171
Fat            0.316  0.302 -0.166  0.687 -0.432  0.251 -0.125 -0.209
Sodium                0.273  0.592        -0.502 -0.556
Fiber          0.558 -0.135        -0.376         0.194  0.453 -0.525
Carbohydrates -0.238  0.113  0.632         0.132  0.535 -0.370 -0.297
Sugar                 0.566 -0.362 -0.375  0.124 -0.254 -0.435 -0.371
Potassium      0.583               -0.296 -0.116  0.211 -0.409  0.585
```




```
> summary(best, parameters=TRUE)
----------------------------------------------------
Gaussian finite mixture model fitted by EM algorithm
----------------------------------------------------

Mclust VEV (ellipsoidal, equal shape) model with 2 components:

 log-likelihood  n df       BIC       ICL
      -886.6437 43 82 -2081.706 -2081.802

Clustering table:
 1  2
29 14

Mixing probabilities:
        1         2
0.6733348 0.3266652

Means:
                    [,1]       [,2]
Calories      107.9439974 107.830668
Protein         1.9638542   3.498337
Fat             0.8261418   1.287171
Sodium        190.0554997 160.697055
```

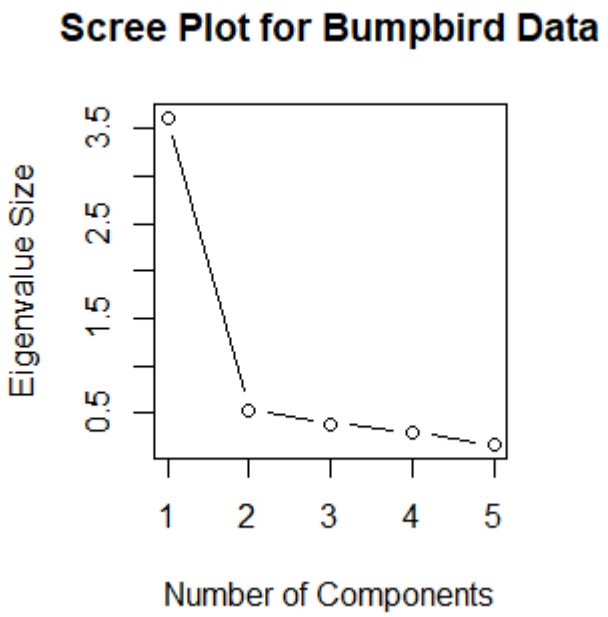
```
Fiber           1.0491944   3.084180
Carbohydrates  14.9181546  12.890572
Sugar           8.1402648   6.500624
Potassium      56.7758155 141.396969

> table(cereal$Company, best$classification)
     1  2
  G 14  3
  K 13  7
  Q  2  4

> summary(cereal.manova, test="Wilks")
          Df  Wilks approx F num Df den Df    Pr(>F)
Company    2 0.2684   3.8372     16     66 5.368e-05 ***
Residuals 40
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

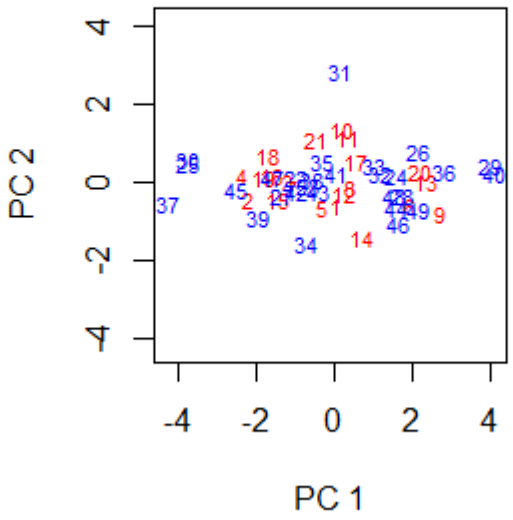


Question TWO:

```
> summary(bump.pc, loadings=TRUE)
Importance of components:
                          Comp.1    Comp.2     Comp.3    Comp.4
Standard deviation     1.9015726 0.7290433 0.62163056 0.5491498
```

```
Proportion of Variance 0.7231957 0.1063008 0.07728491 0.0603131
Cumulative Proportion  0.7231957 0.8294965 0.90678139 0.9670945
                           Comp.5
Standard deviation     0.4056199
Proportion of Variance 0.0329055
Cumulative Proportion  1.0000000

Loadings:
   Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
X1  0.452         0.690  0.420  0.374
X2  0.462 -0.300  0.341 -0.548 -0.530
X3  0.451 -0.325 -0.454  0.606 -0.343
X4  0.471 -0.185 -0.411 -0.388  0.652
X5  0.398  0.876 -0.178        -0.192
```





Question THREE:

```
>factanal(covmat=pain.corr,factors=2, rotation="none", n.obs=123)

Call:
factanal(factors = 2, covmat = pain.corr, n.obs = 123, rotation = "none")

Uniquenesses:
[1] 0.539 0.695 0.318 0.448 0.579 0.356 0.464 0.436 0.766

Loadings:
  Factor1 Factor2
1  0.648   0.202
2 -0.358   0.421
3  0.724   0.396
4  0.691   0.273
5 -0.304   0.573
6 -0.511   0.619
7 -0.556   0.477
8  0.710   0.244
9  0.483

                Factor1 Factor2
SS loadings       2.950   1.447
Proportion Var    0.328   0.161
Cumulative Var    0.328   0.489
```

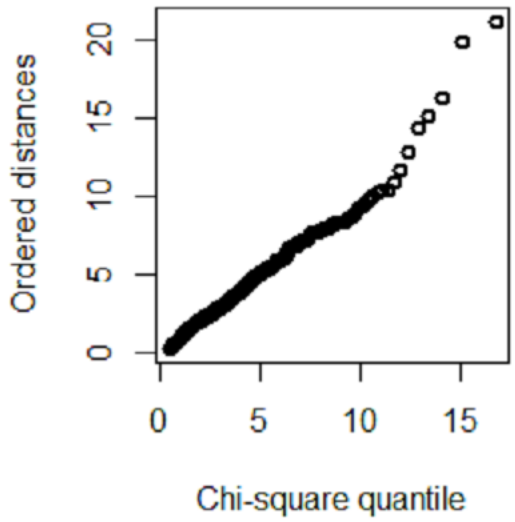
```
Test of the hypothesis that 2 factors are sufficient.
The chi square statistic is 60.41 on 19 degrees of freedom.
The p-value is 3.33e-06

> factanal(covmat=pain.corr,factors=3, rotation="none", n.obs=123)

Call:
factanal(factors = 3, covmat = pain.corr, n.obs = 123, rotation = "none")

Uniquenesses:
[1] 0.404 0.518 0.336 0.455 0.499 0.171 0.496 0.239 0.754

Loadings:
  Factor1 Factor2 Factor3
1  0.607   0.297   0.374
2 -0.458   0.288   0.435
3  0.610   0.506   0.189
4  0.621   0.400
5 -0.408   0.441   0.375
6 -0.677   0.591  -0.145
7 -0.626   0.331
8  0.674   0.481  -0.275
9  0.446   0.169  -0.135

               Factor1 Factor2 Factor3
SS loadings      3.004   1.501   0.624
Proportion Var   0.334   0.167   0.069
Cumulative Var   0.334   0.501   0.570

Test of the hypothesis that 3 factors are sufficient.
The chi square statistic is 19.2 on 12 degrees of freedom.
The p-value is 0.0838

> factanal(covmat=pain.corr,factors=4, rotation="none", n.obs=123)

Call:
factanal(factors = 4, covmat = pain.corr, n.obs = 123, rotation = "none")

Uniquenesses:
[1] 0.433 0.600 0.297 0.482 0.169 0.005 0.536 0.005 0.731

Loadings:
  Factor1 Factor2 Factor3 Factor4
1  0.496   0.125   0.391   0.391
```

```
2 -0.494           0.375
3  0.485   0.353   0.333   0.483
4  0.562   0.321   0.177   0.260
5 -0.412   0.259   0.738  -0.223
6 -0.748   0.660
7 -0.594   0.284          -0.157
8  0.752   0.655
9  0.371   0.188  -0.100   0.293

               Factor1 Factor2 Factor3 Factor4
SS loadings      2.826   1.301   0.996   0.620
Proportion Var   0.314   0.145   0.111   0.069
Cumulative Var   0.314   0.459   0.569   0.638

Test of the hypothesis that 4 factors are sufficient.
The chi square statistic is 9.72 on 6 degrees of freedom.
The p-value is 0.137

> fact(r=pain.corr,method="norm",rotation="varimax",maxfactors=3)
$`eigen.values`
[1] 3.442 1.930 0.942 0.754 0.515 0.444 0.422 0.326 0.225

$method
[1] "multivariate normal - varimax rotation"

$loadings
      Factor1 Factor2 Factor3
 [1,]   0.649  -0.372   0.190
 [2,]  -0.126   0.194   0.655
 [3,]   0.794  -0.144   0.116
 [4,]   0.725  -0.106  -0.092
 [5,]   0.015   0.292   0.645
 [6,]  -0.083   0.825   0.377
 [7,]  -0.225   0.590   0.325
 [8,]   0.815   0.062  -0.304
 [9,]   0.437  -0.076  -0.221

$communalities
[1] 0.596 0.482 0.664 0.545 0.501 0.829 0.504 0.761 0.246

$importance
                   Factor1 Factor2 Factor3
variance explained   2.507   1.331   1.291
percent explained    0.279   0.148   0.143
```

```
$residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -0.082  -0.014   0.001  -0.001   0.018   0.066
```

Question FOUR:

```
> hsb.manova <- manova(cbind(read, write, math, science, socst) ~ ses)
> hsb.manova
Call:
   manova(cbind(read, write, math, science, socst) ~ ses)

Terms:
                     ses Residuals
resp 1           1832.358 19087.062
resp 2            858.715 17020.160
resp 3           1307.091 16158.704
resp 4           1561.578 17945.922
resp 5            2528.18  20408.01
Deg. of Freedom         2       197

Residual standard errors: 9.843203 9.294985 9.056703 9.544425 10.17811
Estimated effects may be unbalanced

> summary(hsb.manova, test="Wilks")
          Df   Wilks approx F num Df den Df    Pr(>F)
ses        2 0.85105   3.2418     10    386 0.0004946 ***
Residuals 197
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

> means.by.grps <- cbind(tapply(read,ses,mean), tapply(write,ses,mean),
                         tapply(math,ses,mean), tapply(science,ses,mean),
                         tapply(socst, ses, mean))
> means.by.grps
          [,1]     [,2]     [,3]     [,4]     [,5]
high   56.50000 55.91379 56.17241 55.44828 57.13793
low    48.27660 50.61702 49.17021 47.70213 47.31915
middle 51.57895 51.92632 52.21053 51.70526 52.03158
```



Question FIVE:

```
> dis.sid.new <- lda(Group ~ BW + Factor68 + HR + Gesage, data=sids.new,
                     prior=c(0.5,0.5))
> dis.sid.new
Call:
lda(Group ~ BW + Factor68 + HR + Gesage, data = sids.new, prior = c(0.5,0.5))

Prior probabilities of groups:
  0   1
0.5 0.5

Group means:
        BW  Factor68       HR Gesage
0 3437.857 0.3108163 129.2408  40.00
1 2964.688 0.4018125 132.9500  39.25

Coefficients of linear discriminants:
                  LD1
BW       -0.001148097
Factor68  9.966188556
HR        0.001144709
Gesage   -0.137896906

> group<-predict(dis.sid.new, sids.new, method='plug-in')$class
> table(group,Group)
```

```
     Group
group  0  1
    0 41  3
    1  8 13

> glm.fit.sids <- glm(Group ~ BW + Factor68 + HR + Gesage, data = sids.new,
                      family=binomial)
> summary(glm.fit.sids)
Call:
glm(formula = Group ~ BW + Factor68 + HR + Gesage, family = binomial,
    data = sids.new)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.5976  -0.6425  -0.3204  -0.1153   2.6181

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)  2.3376266 13.7711841   0.170  0.86521
BW          -0.0017276  0.0007537  -2.292  0.02190 *
Factor68    16.2727269  5.9595617   2.731  0.00632 **
HR          -0.0054844  0.0261659  -0.210  0.83398
Gesage      -0.0742118  0.3589607  -0.207  0.83621
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 72.549  on 64  degrees of freedom
Residual deviance: 49.713  on 60  degrees of freedom
AIC: 59.713

Number of Fisher Scoring iterations: 5

> group.probs<-predict(glm.fit.sids, sids.new, type='response')
> pred.group <- rep(0,times=nrow(sids.new))
> pred.group[group.probs > 0.5] <- 1
> table(pred.group,Group)
          Group
pred.group  0  1
         0 46  9
         1  3  7
```