



Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN DATA SCIENCE AND ANALYTICS
SPECIAL EXAMINATION
DSA 8302 COMPUTATIONAL TECHNIQUES IN DATA SCIENCE

DATE: **May 6, 2024**

Time: **2 Hours**

Instructions

1. This examination consists of **FIVE** questions and an appendix to one of the questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question 1

- a) Distinguish between numerical and analytical methods in the solution of mathematical problems.
(3 Marks)
- b) Describe the bisection method and explain how it differs from the Newton-Raphson method.
(3 Marks)
- c) Given an initial guess $x=3$, find an approximate value of the root of the function $f(x) = x^2 - 2$ using 3 iterations. Provide a sample R code that you would use to solve this problem, stopping after 40 iterations.
(6 Marks)
- d) Using the inverse-transform approach to explain (mathematically) how you would generate random numbers from the exponential distribution, $f(y) = \theta \exp - (\theta y)$, and further provide an R code that will be used to generate random numbers from this distribution.
(8 Marks)

(TOTAL 20 Marks)

Question 2

Starting with the Newton-Raphson formula

$$x^{(n+1)} = x^{(n)} - \frac{f(x^{(n)})}{f'(x^{(n)})}$$

shown that the order of convergence of the Newton-Raphson method is

$$|\varepsilon^{(n+1)}| = k|\varepsilon^{(n)}|^2$$

as $n \rightarrow \infty$, with $k = \frac{1}{2} \left| \frac{f''(r)}{f'(r)} \right|$ provided $|f'(r)| \neq 0$.

(20 Marks)

Question 3

a) Consider the following data:

| | | | |
|---|---|---|---|
| X | 1 | 2 | 4 |
| Y | 1 | 3 | 1 |

Use quadratic spline interpolation to find the approximate value of y at x=3.

Hint:

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 2 & 0 & 1 & 0 \\ 0 & 4 & 0 & 2 & 0 & 1 \\ 0 & 16 & 0 & 4 & 0 & 1 \\ 4 & -4 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0.5 & -0.5 & -0.25 & 0.25 & 0.5 & -0.5 \\ -1.0 & 1.0 & 0.00 & 0.00 & 0.0 & -3.0 \\ -3.0 & 3.0 & 1.00 & -1.00 & -3.0 & 3.0 \\ 2.0 & -1.0 & 0.00 & 0.00 & 0.0 & 2.0 \\ 4.0 & -4.0 & 0.00 & 1.00 & 4.0 & -4.0 \end{pmatrix}$$

(10 Marks)

b) Consider the following data

$$(x_1, y_1) = (1,7), (x_2, y_2) = (1,23), (x_3, y_3) = (3,100)$$

Use polynomial interpolation to determine the value of the function at x=2.7.

Hint:

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 0.5 & -1 & 0.5 \\ -2.5 & 4 & -1.5 \\ 3.0 & -3 & 1.0 \end{pmatrix}$$

(10 Marks)

Question 4

Consider the general linear model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \sim MVN(\mathbf{0}, \sigma^2 \mathbf{I}),$$

where $\mathbf{Y} = (Y_1, \dots, Y_n)'$ is an $(n \times 1)$ vector of response values, \mathbf{X} is an $(n \times k)$ design matrix corresponding to the explanatory variables X_1, \dots, X_k , $\boldsymbol{\beta} = (\beta_0, \beta_1, \dots, \beta_k)'$ is the vector of parameters, and σ^2 is the variance.

Using a maximum likelihood approach, clearly showing the likelihood function, the log-likelihood function, and the score-vector, derive the maximum likelihood estimators of $\boldsymbol{\beta}$ and σ^2 .

(20 Marks)

Question 5

- a) Show that the probability density function of the Poisson distribution, $f(y) = \frac{e^{-\lambda} \lambda^y}{y!}$, $y = 0, 1, \dots$ belongs to the exponential dispersion family

$$f(y; \theta) = \exp \left[\frac{y\theta - b(\theta)}{\phi} \right] + c(y; \phi).$$

(3 Marks)

- b) Also show that the mean and the variance of the Poisson distribution are equal to $b'(\theta)$ and $b''(\theta)$, respectively.

(3 Marks)

- c) Consider the generalized linear model

$$g(\mu_i) = \mathbf{X}_i \boldsymbol{\beta}$$

where $\mathbf{Y} = (Y_1, \dots, Y_n)'$ is an $(n \times 1)$ vector of response values belonging to the Poisson distribution, \mathbf{X} is an $(n \times k)$ design matrix corresponding to the explanatory variables X_1, \dots, X_k , $\boldsymbol{\beta} = (\beta_0, \beta_1, \dots, \beta_k)'$ is the vector of parameters.

Derive an expression for the estimating equation and Hessian that would be used to estimate the vector of parameters $\boldsymbol{\beta}$.

(14 Marks)