Strathmore Institute of Mathematical Sciences (SIMS)
End of Semester Examination for the Degree of Bachelor of Business Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE: 29th July, 2022 **Time: 2 Hours**

Instructions

- **This examination consists of FIVE questions.**
- **Answer Question ONE (COMPULSORY) and any other TWO questions.**

1. (a) Assume that an investor's consumption preferences are given by a power utility function: $u(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$. Derive the relative risk aversion for this investor. **(4 Marks)**

   (b) Briefly, but intuitively explain the following concepts (in the context of asset pricing literature).

   (i) Mutual fund theorem **(3 Marks)**

   (ii) An arbitrage portfolio **(4 Marks)**

   (c) Briefly describe the following phenomenon as used in asset pricing

   (i) Momentum investing **(3 Marks)**

   (iii) Value premium puzzle **(4 Marks)**

   (d) In the seminal study of Fama and Macbeth (1973), atleast three main testable implications of the theoretical CAPM are outlined. Identify and briefly explain these testable implications. **(6 Marks)**

   (e) (i) Briefly explain the concept of Complete markets and why they are important in an economy. **(3 Marks)**

   (i) Briefly explain the main advantage of the Arrow-Debreu pricing over the capital asset pricing model **(3 Marks)**

2. (a) Let $\overline{R} = (\overline{R_1}, \overline{R_2} \ldots \overline{R_n})'$ be an $n \times 1$ vector of the expected returns of the n assets. Also let Σ be the $n \times n$ covariance matrix of the returns on the n assets. Σ is assumed to be of full rank, it is symmetric and positive definite. Let $\omega = (\omega_1, \omega_2 \ldots \omega_n)'$ be an $n \times 1$ vector of portfolio proportions, such that ω_i is the proportion of total portfolio wealth invested in the i^{th} asset. Finally, define e to be a vector of ones.

(i) Write down the optimization problem used to determine the global minimum variance portfolio. **(3 Marks)**

(ii) Write down the first order conditions of the optimization program obtained in part (i) and solve for the optimal weights **(5 Marks)**

(b) The prediction test of the CAPM plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$. Based on these estimates one may estimate the simple linear regression equation

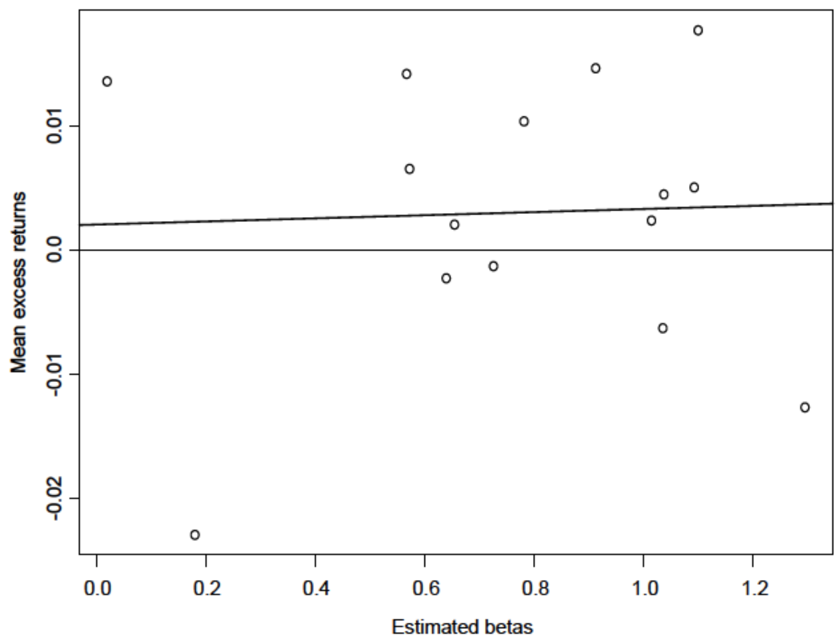
$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + error_i, i = 1, \ldots, N \quad (1)$$

if the CAPM were true, what should the estimated values for γ_0 and γ_1 be? **(3 Marks)**

(c) The following graph plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$ for 15 stocks based on monthly data over the five year period. Superimposed on the graph is an estimate of the security market line (SML) based on the simple linear regression

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + error_i, i = 1, \ldots, 15$$

The average excess return on the market index over this period was 0.0031. If the CAPM were true, what should the estimated values for γ_0 and γ_1 be? **(3 Marks)**



(d) The least squares estimates of the SML presented in part (c) are summarized below. Using these estimates, test the null hypothesis that the CAPM is true using a 5% significance level. **(6 Marks)**

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> summary(sml.fit)

Call: lm(formula = mu.hat ~ betas)
Residuals:
     Min       1Q  Median       3Q    Max
 -0.0252 -0.0047 0.00112 0.00936 0.0142


Coefficients:
               Value Std. Error t value Pr(>|t|)
(Intercept) 0.002 0.007       0.275   0.788
     Betas  0.001 0.009       0.143   0.888

Residual standard error: 0.0115 on 13 degrees of freedom
Multiple R-Squared: 0.00157
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3. Suppose a typical investor solves the following problem

$$\max_{\alpha} U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]$$
$$\text{subject to } c_t = e_t - \alpha p_t \text{ and } c_{t+1} = e_{t+1} + \alpha x_{t+1} \qquad (2)$$

where c_t and c_{t+1} denotes consumption at date t and $t+1$ respectively. The other parameters in the model are; e_t (investor's endowment at period t and $t+1$ respectively), p_t (asset price at time t), α (the number of units of the asset purchased by the investor), and x_{t+1} (payoff of the asset at period $t+1$), β represents the time preference parameter of the investor $(0 \leq \beta \leq 1)$. The general functional form $U(.)$ represents the investor's utility function. An often convenient utility function used in many applications is the power utility $u(c_t) = \frac{1}{1-\gamma}c^{1-\gamma}$

(i) Show that the price of an asset at any given time can be represented as follows: **(3 Marks)**

$$p_t = E_t[\frac{\beta u'(c_{t+1})}{u'(c_t)} x_{t+1}]$$

(ii) Provide an intuitive interpretation of the term $\frac{\beta u'(c_{t+1})}{u'(c_t)}$ and briefly discuss how this term influences the asset price p_t **(3 Marks)**

(iii) Show that a simple manipulation of the price equation in (i) can yield the following equation (ignoring time subscripts)

$$1 = E(mR)$$

where R represents the gross return of the asset, while $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$ **(2 Marks)**

(iv) Assuning the power utility function provided and the manipulations in parts (i)-(iii) above, one can show that the risk-free rate can be expressed as

$$R^f = \frac{1}{\beta}(\frac{c_{t+1}}{c_t})^{\gamma}$$

Intuitively discuss three factors that influence interest rates in an economy. **(6 Marks)**

(v) Show that the basic pricing equation ($1 = E(mR)$) can be rewritten as follows;

$$E(R) = R_f - \frac{Cov(u'(c_{t+1}), R)}{E(u'(c_{t+1}))}$$

Interpret this equation. **(6 Marks)**

4. (a) You are given the following information:

Security	**State 1**	**State 2**	**Security Prices**
j	\$12	\$20	$p_j = \$22$
k	24	10	$p_k = 20$

(i) Compute the prices of pure security state prices.**(6 Marks)**

(ii) What is the initial price of a third security i, for which the payoff in state 1 is \$6 and the payoff in state 2 is \$10? **(4 Marks)**

(b) There are three possible states in the future, states 1,2,and 3, and the consumption at each state is denoted by $c_1, c_2, \text{and} c_3$. There are one million homogeneous consumers with expected utility function

$$q_1 u(c_1) + q_2 u(c_2) + q^3 u(c_3)$$

, where $(q_1, q_2, q_3) = (0.5.0.3, 0.2)$, $u(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 0.5$, and with endowment $(y_1, y_2, y_3) = (100, 81, 64)$ Treating the state 1 good as numeraire, find the general equilibrium prices for the state 2 good and for state 3 good. **(10 Marks)**

5. In economics and finance, a puzzle refers to the inability of standard intertemporal economic models to rationalize the statistics that characterize financial markets. In this context, discuss the following puzzles (**Hint:** Illustrate the puzzle by data, graphs or mathematical forms. Further, highlight the various attempt(s) to solve the puzzle)

(i) Volatility puzzle (Grossman & Shiller, 1981) **(8 Marks)**

(ii) Equity premium puzzle (Mehra and Prescott, 1985) **(12 Marks)**

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