Portfolio Optimization: A comparison between the Mean-Variance Optimization Model, Mean Absolute Deviation Model, and Mean-Semi Absolute Deviation Model

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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Abstract

This study investigates the comparative performance of three portfolio optimization models—Mean-Variance Optimization (MVO), Mean Absolute Deviation (MAD), and Mean Semi-Absolute Deviation (MSAD)—in the context of the Kenyan stock market. The MAD and MSAD have been suggested as improvements to MVO since they use alternative risk measures. The study investigates which model generates a better optimized portfolio while emphasizing on the role of duration and time in determining the appropriate model to use in emerging markets characterized by significant economic volatility. Using monthly stock return data from 2011 to 2020, the study evaluates the models' performance across different investment horizons and distinct 12-month periods, and a comparison is done using the Sharpe and Sortino ratios. The findings were inconsistent across different durations and time periods. They revealed that while the MAD model outperformed during periods of high market uncertainty, the MSAD model proved effective for medium-term (2 years) investment horizons, and the MVO outperformed in short-term investment horizons (1 year). The results underscore the critical importance of duration and time over broader macroeconomic factors in portfolio optimization, providing practical insights for fund managers and investors. These findings contribute to the growing body of knowledge on alternative risk measures and their applicability in dynamic, real-world environments.

Keywords: Portfolio optimization, Mean-Variance Optimization (MVO), Mean Absolute Deviation (MAD), and Mean Semi-Absolute Deviation (MSAD).

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List of Abbreviations

CVaR	Conditional Value-at-Risk
MAD	Mean Absolute Deviation
MPT	Modern Portfolio Theory
MSAD	Mean Semi-Absolute Deviation
MVO	Mean-Variance Optimization
NASI	Nairobi All Share Index
NSE	Nairobi Securities Exchange
TA	Threshold Accepting
VaR	Value at Risk

CHAPTER 1: INTRODUCTION

1.1 Background to the Study

A portfolio is an appropriate collection of financial assets owned by one person or an entity. Portfolio optimization is determining the proportions of various financial assets to be held in a portfolio so that the portfolio performs better than any other according to some criterion (Ogutu, 2011). The goal of optimization varies depending on the desired result, which can be either the minimum volatility for a given level of return or the maximum return for a given level of volatility, while still maintaining portfolio efficiency (Rasmussen, 2003).

Kale (2009) stated that the Mean-Variance Optimization (MVO) model, developed by Markowitz (1952), continues to be the most widely used method for portfolio construction. It is based on the notion that investors are exceedingly risk averse and take on additional risk only when compensated by higher returns. The model maximizes the expected return of a portfolio for a given level of risk or minimizes the risk for a given level of expected return. It uses a quadratic programming problem to find the optimal weights to allocate to each investment alternative in the portfolio to achieve the desired risk-return trade-off. An investor can effectively lower risk by holding a combination of assets that are not entirely positively associated with one another, and it is a formalization and extension of diversification in investing.

According to Markowitz (2010), mean-variance analysis is not always applicable, for instance when return distributions are excessively dispersed. The mean-variance model, though widely used in the past, is predicated on the notions that investors are risk averse and that either the asset return distribution is normally distributed, or the investor's utility is a quadratic function of the rate of return. However, neither of these assumptions holds in practice (Chang,Yang, and Chang, 2009). Therefore, this inspired a vast amount of research aimed at improving the Markowitz Model both theoretically and computationally.

There have been several studies in portfolio optimization using some of the main alternative risk measures such as Value-at-Risk (VaR), Mean Absolute Deviation (MAD), semi-variance, Minimax, Conditional Value-at-Risk (CVaR), and Maximum Loss. Additionally, objective functions have been adopted to offer superior optimization results. This technique does not require the use of variance or mean of returns. Non-risk reward heuristic models such as algorithms and linear simulations have been run on data for portfolio selection with the most popular ones being the Genetic Algorithm (GA) and the Threshold Accepting (TA). These models do not dismiss volatility and mean as measures but are incorporated as part of specifications.

Konno and Yamazaki (1992) suggested the Mean-Absolute Deviation (MAD) model as an alternative to the MVO model since it does not assume that stock returns are normally distributed. The MAD model also minimizes the mean absolute deviation which is also a measure of risk. The mean absolute deviation corresponds to $MAD_p = E[|R_p - r_p|]$. It transforms the portfolio selection problem from a quadratic programming into a linear programming problem which is easier to compute and eliminates the need for a covariance matrix which is cumbersome to compute.

Downside risk measures how much an investor could lose if the price of an asset or security drops due to changing market conditions. The application of downside risk as a risk measure in portfolio optimization models is becoming increasingly popular among investors because it distinguishes between downside risk and upside potential.

Ibrahim and Kamil (2006) state that the mean absolute negative deviation from the mean is half of the mean absolute deviation from the mean, hence the corresponding mean-risk model is equivalent to the MAD model. A downside portfolio selection problem can be modelled as a linear programming problem by taking the mean absolute negative deviation as the downside risk measure. This is the Mean Semi Absolute Deviation (MSAD) portfolio optimization model – it considers only the returns that fall below a specified target or mean.

In an emerging market context, investment horizons and timing of portfolios are vital due to the heightened market volatility and economic disruptions. Fund managers can adjust their strategies more effectively by knowing how duration and time affect a portfolio’s resilience. This approach is significant to the Kenyan stock market, where notable fluctuations over the last ten years call for an emphasis on the performance of portfolio optimization models over a range of time periods and investment horizons.

The Mean-Variance Optimization model has been a method of choice for many financial modelers in the 21st Century (Estrada, 2006). Mean-variance analysis and factor models are the primary tools used by portfolio managers in the Kenyan market for portfolio selection (Masese, 2017). In addition to the MVO model, this study will use the MAD and MSAD models to determine the best model for asset allocation and portfolio optimization.

1.2 Problem Statement

According to Reilley and Brown (2012), diversification, number of securities to invest in, their allocation and the risk involved are important considerations in portfolio construction. According to Mwangangi (2006), 60% of the fund managers used the mean-variance optimization model when determining the proportions of asset allocation. However, the variability of the market affects the effectiveness of the variance as a risk measure, as it may not fully capture the complexities of market volatility. Moreover, the model makes bold assumptions that may not hold in practice. These limitations necessitate the use of alternative models that better account for market complexities.

The Kenyan stock market has experienced significant volatility over the last five years, characterized by periods of both sharp declines and recoveries. Although macroeconomic factors like interest rates and inflation have some influence on portfolio performance, this study focuses only on time and duration. These above variables provide a better understanding of how adaptable portfolio optimization models are across various investment horizons (duration) and time periods because they are directly related to their success.

Alternative models such as the Mean Absolute Deviation Optimization and Mean Semi-Absolute Deviation address the shortcomings of the MVO model. The MAD model uses a risk measure that exhibits robustness to extreme values and does not assume the normality of returns while the MSAD model focuses on downside risk; thus, it may be more relevant for asymmetric return distributions common in emerging markets like Kenya. However, no research has been conducted on the MAD and MSAD models in Kenya, representing a significant research gap.

This study evaluates the MAD and MSAD against MVO across varying durations and time periods to identify the model that provides the optimal portfolio selection method for fund managers and institutional investors.

1.3 Research Objective

1.3.1 Main Objective

The primary objective of this study is to compare the Mean-Variance Optimization (MVO), the Mean Semi Absolute Deviation (MSAD) model, and the Mean Absolute Deviation (MAD) model in asset allocation and optimal portfolio selection in the Kenyan context.

1.3.2 Specific Objectives

The specific for this study are:

1. To determine which model between the mean-variance, mean absolute deviation, and mean semi-absolute deviation generates a better-performing optimal portfolio.
2. To determine the performance of the models across different investment horizons (duration) and time periods.

1.4 Research Hypotheses

The research hypotheses for this study are:

1. The MAD and MSAD models build a better-optimized portfolio as compared to the MVO model.

2. The MSAD model consistently outperforms the MAD model and MVO model across different investment horizons (duration) and time periods.

1.5 Significance of the study

The Kenyan stock market is of interest given it is among the best-performing stock markets in Africa (Gachiri, 2014). The study compares the performance of various optimization models to determine which one best fits the Kenyan stock market, offering insights into the most suitable model for portfolio construction in this context. This study has important implications for various stakeholders in the Kenyan financial sector:

Investors

The findings of this study can provide both individual and institutional investors with alternative portfolio construction methods that have the capability of generating superior risk-adjusted returns by determining the optimal allocation of funds to each asset, during their different investment horizons.

Regulatory agencies

The results can inform future reviews of policy and regulatory guidelines for institutional investors in Kenya.

Fund managers and financial advisors

They will be able to advise clients, based on their investment horizons, on the best model to use for portfolio optimization in the Kenyan stock market context.

Researchers

The study will spur interest in alternative portfolio optimization methods and add to the existing knowledge pool. This will provide grounds for additional research and future policy guidelines to improve the overall performance of the Kenyan market.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter provides the theoretical framework for analysing the existing theories related to asset allocation and portfolio optimization and a review of empirical studies undertaken by other researchers both locally and globally.

2.2 Theoretical Review

2.2.1 Mean-Variance Portfolio Theory

The Modern Portfolio Theory (MPT) is a finance theory developed by Harry Markowitz in the seminal 1952 paper, "Portfolio Selection." Its foundation is that investors can construct portfolios to maximize expected return for a given level of portfolio risk by strategically allocating asset proportions.

Markowitz (1952) through the analysis of the mean and variance, proposed a mathematical framework for selecting a portfolio based on the expected return and the variance of the return. Mean-variance analysis is the cornerstone of MPT, and it involves the construction of a portfolio that provides the minimum risk for a given expected return or, otherwise, the minimum risk for a given expected return. This dual goal is accomplished by considering not just the risks and returns of each asset individually, but also how those assets interact with one another through covariances within the portfolio.

Markowitz illustrated that in some instances, an investor's portfolio choice could be simplified to just managing two essential components of the portfolio: expected return and variance. The expected return represents the weighted average of the anticipated returns of each asset in the portfolio, based on historical data or analyst forecasts. Variance, on the other hand, captures the portfolio's risk, which reflects the overall volatility and the potential deviation from the expected return. The key concept of Markowitz's framework is diversification, which involves combining assets with varying correlations to reduce overall risk of a portfolio (Khan and Tanwani, 2024).

Therefore, this framework underscores that the pair-covariance of the return on all assets in the portfolio should be taken into consideration when determining the risk of a portfolio, which is determined by the variance of portfolio returns. As a result, Markowitz illustrated the efficient frontier, a curve representing the portfolios that maximize return for a particular risk. The efficient frontier is a pivotal concept in MVO, and it is typically a concave curve in the risk-return space, with portfolios below the frontier considered suboptimal as they do not exploit the available diversification benefits.

2.2.1.1 Assumptions and Drawbacks of Mean-Variance Optimization

While MVO model has been widely adopted in both academic research and practical portfolio management, it is not without its criticisms. Boasson, Boasson, and Zhou (2011) pointed out two limitations in Markowitz's mean-variance approach. First and foremost, it assumes that the distribution of investment returns is normally distributed. If the asset's underlying return data is not normally distributed, the variance is likely to produce inadequate findings. Fama (1968) demonstrates that asset returns do not follow a normal distribution. In the real world, asset returns frequently exhibit kurtosis (fat tails) and skewness, meaning that they are more likely to experience extreme returns than a normal distribution would predict. Consequently, asset returns tend to have an asymmetrical distribution e.g. a log-normal distribution.

Given the skewed distribution of the assets, the variance is an inefficient risk metric because it penalizes both positive and negative deviations from the mean returns equally, incorporating the beneficial upside variations of asset returns over the expected value as part of the risk. If the returns are asymmetrical, investors using standard deviation as a risk metric are more prone to making poor asset allocation decisions since the risk might be underestimated. Moreover, the construction of an efficient frontier using improper risk measures might lead to illogical results. Secondly, the mean-variance optimization model assumes that investors are rational and risk-averse ignoring the investor’s degree of risk aversion. This limitation arises because the variance can only determine the dispersion of returns distribution about a mean and cannot be tailored for each investor’s level of risk aversion (Boasson, Boasson, and Zhou, 2011).

Kritzman (2011) outlined three major drawbacks used by “old technology” against the mean-variance optimization model. The first one was the “Garbage in, Garbage out” mentality which emphasized the need for high-quality accurate data since individuals expected the model to offer great portfolios yet poor inputs were used (i.e. expected return, variances, and correlations) that would go against the theory leading to suboptimal outputs. Second, was that the optimizers were susceptible to input errors resulting in an understatement of risk overstatement of returns. The third one, which has already been discussed, is the foundation of the model on misleading assumptions particularly the normality of returns distribution.

Other assumptions are that all the investors simultaneously have access to information, there are no taxes and transaction costs, the risk of an asset is known ahead of time, and any investor can lend or borrow an infinite amount at a risk-free rate.

However, Markowitz (1959) reveals that variance as a risk metric has certain advantages over other downside risk measures affordability, convenience, and familiarity. The affordability stems from the fact that, in the past, efficient portfolios based on downside risk took two to four times as much computing time as those based on standard deviation. The difference in convenience arises from the fact that efficient portfolios require means, variances, and covariance as inputs, while those based on downside risk demand the utilization of the entire joint distribution of returns (Estrada, 2006).

2.2.1.2 Improvements to Mean-Variance Optimization

Better out-of-sample performance can result from adding constraints to the MVO problem as opposed to portfolios that are built without them. According to Kolm, Tütüncü, and Fabozzi (2014), exposure constraints made at the portfolio manager's discretion, trading constraints (discretionary limits on positions or trades), risk management constraints, transfer coefficients, and regulatory constraints are the most common types of constraints found in practice in portfolio optimization problems. Nonetheless, as they have the potential to distort the robustness and consistency of the asset allocation, these six caveats and recommendations must be applied cautiously. To make the estimations of return and risk more introspective than historical, judgment and constraints should be carefully included (Markowitz, 2010).

2.2.2 Alternatives to Mean-Variance Optimization

One of the alternatives to MVO, as suggested by Kolm et al. (2014), is the use of risk-based allocation strategies which remove expected returns from the portfolio construction process and focus solely on estimating and modeling risk parameters. The Global Minimum-Variance Model seeks to minimize portfolio volatility without considering expected returns resulting in portfolios with the least amount of risk for a particular set of assets. While the Optimal Risk Parity approach allocates funds based on the risk contribution of each asset, aiming for equal risk exposure across the portfolio.

Markowitz (2010) suggests that determining an investor's utility function and maximizing it is an alternative to MVO. However, due to the changing perception of investors, it is quite challenging to identify a suitable utility function to use.

Another strategy for creating more optimal portfolios and getting around the mean-variance analysis's drawbacks is to employ different risk or return measures in the analyses (Biglova, Ortobelli, Rachev, and Stoyanov, 2004; Jaaman, Lam, and Isa, 2013; Bonyo, 2015; Rollinger and Hoffman, 2013). The MAD, semi-variance, Minimax, VaR, CVaR, Maximum Loss, and lower partial moments are some of the alternative risk measures used together with the mean in portfolio optimization.

Heuristic models that function as generalized optimization models are becoming popular in portfolio optimization problems as they can account for many aspects of return distributions, (Gilli and Schumann, 2011). A study on portfolio optimization using Genetic Algorithms and a risk measure was conducted by Chang, Yang, and Chang (2009). The measures of risk utilized were semi-variance, variance, and variance with skewness measures and they were adopted as the objective functions. The study concluded that Genetic Algorithms, which is also a heuristic model, can efficiently produce near-optimal solutions without difficulty for the above-stated objective functions, hence, it is robust and efficient for solving optimization problems applying varying risk metrics. Masese (2017), compared the performance of the Threshold Accepting (TA) model and the MVO model and concluded that the TA model performs better for optimal portfolio construction.

2.2.3 Alternative Risk Measures

The Mean-Variance Optimization model, introduced in 1959, assumes that standard deviation is a sufficient measure of portfolio risk. However, this approach has limitations, as standard deviation only measures the dispersion of returns and may not fully capture the true risks in portfolio optimization. In real market data, relying solely on standard deviation or variance might be inadequate because it overlooks other aspects of risk beyond simple return variability.

Variance, commonly associated with normal distribution, is widely understood but can overlook extreme data points, potentially compromising study results (Gorard, 2015).

Multiple scholars have found that other measures of risk offer better asset allocation and portfolio selections than the mean-variance method (Gilli and Schumann, 2009; Tian, Cox, Lin, and Zuluaga, 2010). Consequently, other risk measures such as the mean absolute deviation which is based on the dispersion of the asset return distribution, and the VaR, MAD, Minimax, CVaR, Maximum Loss, lower partial moments, and semi-variance which are downside risk measures were introduced.

Hiroshi Konno and Hiroaki Yamazaki proposed the Mean-Absolute Deviation (MAD) model in 1992 as a substitute for Markowitz's Mean-Variance model. This framework measures risk as the mean absolute deviation. It is the expected absolute difference between an individual asset's returns its expected value. The computing benefits of MAD allowed the model to solve optimization problems involving more than 1,000 stocks much faster than the mean-variance model. Adoption of the MAD model in portfolio optimization is increasing because of its computational advantages over the Mean-Variance Model. When the data does not conform to a perfect normal distribution, mean absolute deviation is more effective than variance (Biglova et al., 2004; Jaaman et al., 2013).

In risk management, the most debated risk metrics for estimating downside risk are Value at Risk (VaR) and Conditional Value at Risk (CVaR). VaR estimates the maximum loss of a portfolio at a given confidence level and time horizon. CVaR, also known as expected shortfall, is a percentile risk measure and is defined as the conditional expectation of losses when the actual loss exceeds the VaR threshold. Therefore, CVaR was developed as an improvement to the VaR as it accounts for risk beyond the VaR cut-off point and applies to non-symmetrical distributions.

The drawback with VaR and CVaR is that they need the investor to specify a short-term holding period i.e. 1 to 10 days, and a probability level of loss surpassing the VaR limit. A short-term horizon is not ideal for portfolio optimization for investors with a longer investment horizon. Moreover, the greater the time horizon, the lower the accuracy of the efficient VaR frontier (Campbell, Huisman, & Koedijk, 2001). Additionally, a lot of researchers contend that VaR is not an adequate risk measure since it does not fulfill the subadditivity requirement. Put another way, the combined VaR of several securities in a portfolio may not always be less than the total of the

VaRs of the individual securities if we combine them at a certain confidence level. It would not be ideal to have a combined VaR that is higher than the total of the VaRs.

Markwotiz (2010) gives preference to the semi-variance as a risk metric asopposed to the variance in producing optimized portfolios. Semi-variance is a statistical measure that only incorporates return variations that fall below a predetermined cutoff point, usually the mean or a target return. Unlike variance, which captures both positive and negative deviations from the mean, semi-variance provides a focused view of risk by isolating those deviations that represent actual losses or underperformance relative to the investor's expectations. However, Markowitz still insisted on using a mean-variance method, based on the geometric mean return due to the complexity of calculating the lower semi-variance.

Young (1998) proposed the Minimax (MM) model as an alternative risk measure. This model is based on game theory, which incorporates trade-offs between choices and rewards. The author's perspective rests on the idea that investors should always make choices that will shield them from the worst-case scenario by minimizing their maximum expected losses (Minimax criteria).

Only a few of the several alternative risk metrics that have been developed and used to conduct portfolio optimization are discussed here. This demonstrates that other researchers have been exploring alternatives to circumnavigate the drawbacks of the widely used mean-variance model for optimal asset allocation and portfolio selection.

2.2.4 Downside Risk

Downside risk is an important concept in modern portfolio theory and risk management. It emphasizes the possibility of losses in an investment or portfolio rather than focusing on the volatility of returns. Downside risk focuses exclusively on the negative side of the return distribution, in contrast to conventional risk metrics such as variance, valuate both positive and negative deviations from the mean equally. Due to this emphasis, downside risk can be particularly vital for investors risk-averse and financial institutions trying to reduce possible losses. The Mean-Variance approach has been the most widely used portfolio optimization model. The overall volatility of an asset is expressed using the standard deviation. However, this symmetric measure does not discriminate between volatility on the upside and downside. This concern has led to the development of downside risk measures that better align with investors' risk preferences.

Downside risk measurement only accounts for returns that go below a specific threshold. This threshold captures the risk perceptions across different investors. In contrast to the standard deviation, downside risk allows for various risk assessments based on the deviation from the threshold (Mwabaya, 2015).

Markowitz (2010) highlights that investors minimize downside risk for two major reasons. First, downside risk is the only type of risk that truly matters to them, as it directly relates to potential losses. Second, using a downside risk measure enables investors to make more informed decisions, especially when dealing with securities that do not follow a normal return distribution.

Markowitz (1959) proposed two downside risk measures: a semi-variance derived from the mean return and a semi-variance derived from a target return. Other downside risk measures include VaR, CVaR, Lower Partial Moments (LPM), MSAD, and Beta-CVaR. Downside risk measures are implemented in portfolio management to build portfolios that correspond with an investor's risk tolerance. Some critics argue that these measures can be overly conservative, potentially leading to suboptimal investment decisions. Additionally, downside risk metrics often require complex calculations and assumptions about the return distribution, which can introduce estimation errors.

2.2.4.1 Safety First Theory

Markowitz (2010) explores other substitutes in the occasion where the mean-variance approach is not viable. One of the predominant alternatives is establishing the investor's explicit utility function and maximizing its expected value. However, investors' preferences keep changing making it difficult to determine a suitable utility function to use.

To move away from utility functions, Roy (1952) was the first to evaluate other downside risk measures. The author defined the "safety first" theorem which assesses investment risk by calculating the probability that an investment's value will fall below a certain target or disaster level. It entails building a portfolio using the minimum acceptable return. Investors can mitigate the loss of the principal amount by establishing a minimum acceptable return. According to Roy, it is prudent for investors to minimize the likelihood of disaster. Investors would therefore favour an investment with the lowest possibility of falling below the disaster level.

2.2.5 Mean Absolute Deviation Model

The Mean Absolute Deviation model was suggested by Konno and Yamazaki (1991) as an alternative to the Mean-Variance Optimization (MVO) model. This measure of risk used by this model is the mean absolute deviation. The mean absolute deviation is the average of the absolute differences between individual asset returns and the expected return. The MAD is a linear programming model that overcomes the computational difficulties of the MVO which is a quadratic programming problem. The linear nature of the MAD model makes it computationally efficient, particularly for large-scale portfolios since it does not require calculation of the covariance matrix.

The MAD model does not rely on any distributional assumptions, unlike the MVO model, which assumes that stock returns are normally distributed, which is untrue. This makes the MAD model more flexible and applicable in real-world scenarios where returns are often skewed or exhibit kurtosis. Unlike standard deviation, which squares the differences from the mean and emphasizes extreme values, MAD uses the absolute value of these differences. This approach treats all deviations from the mean equally, making MAD less sensitive to outliers. As a result, it is a more robust measure, especially in markets where returns are highly volatile. This model often has fewer assets, thus decreasing the transaction cost incurred in portfolio revisions.

However, since it doesn't take into consideration variance or higher moments like skewness and kurtosis, the MAD model may lose some information about the return distribution, despite being robust and easier to use. The MAD model may not be in line with investors' wishes because it regards all deviations from the mean equally. The downside risk is usually of greater concern to investors, and the MAD model does not distinguish between deviations on the upside and downside. As a result, Kamil and Ibrahim (2006), suggest a downside measure of risk which is calculated by only taking into consideration the return below the mean as the risk and replaces the mean absolute deviation with the mean absolute negative deviation. This model is referred to as the Mean Semi Absolute Deviation (MSAD).

The MSAD model's focus on downside risk makes it more suitable for the Kenyan market, which likely has periods of high volatility and skewed return distributions. By only considering negative deviations from the mean, the MSAD model provides a more relevant measure of risk for risk-averse investors. This study explores portfolio optimization to identify the most effective risk measure for asset allocation and portfolio selection by comparing variance, mean absolute deviation, and mean absolute negative deviation.

2.2.6 The role of time and duration in portfolio optimization

Time and duration are important factors in portfolio optimization since they capture the dynamic nature of portfolio performance across economic cycles. Gilli and Schumann (2009) found that longer investment horizons frequently reduce short-term market volatility and produce more consistent risk-adjusted returns. Konno and Yamazaki (1992) further show that portfolio models' resilience and flexibility can be shown by how sensitive they are to different timeframes. In contrast to macroeconomic factors, which frequently have indirect or lagging effects, duration and time provide quick, useful information about how a portfolio is performing.

2.3 Empirical Review

The study conducted by Abdalla (2013) comparing the performance of the NSE-20 Share Index and an optimal portfolio constructed from eight of its constituent stocks provides valuable insights into portfolio optimization under Modern Portfolio Theory (MPT). The study evaluated the performance of the optimal portfolio by comparing its Sharpe ratio to that of the NSE-20 share index. It was found that the optimal portfolio performed better than the index. It was established that through the implementation of portfolio optimization techniques under the guidelines of MPT, a new market portfolio index that exceeds the NSE 20 Index could be created.

A comparison of the selection of portfolios by the single index model and the MVO model in the NSE-20 Share Index was conducted by Nyokangi (2016). The MVO model outperformed single index model over longer time horizons. The former model had higher portfolio returns than the MVO model but at a higher risk. This was evident when the portfolios under the two models were compared using the Sharpe ratio. Consequently, a risk-averse investor would favor the MVO model.

Masese (2017) compared the performance of the Threshold Accepting model with that of the Mean-Variance Optimization model in stock portfolio optimization. Using the Sharpe, Sortino, and Information ratios as the performance measures, the study established that the TA model performed better in portfolio optimization as compared to the MVO model. Additionally, it was noted that when comparing the models' performances across time, two of the four periods showed the best results for each model. Risk-taking investors would favour the TA model since it gives higher returns but at a higher risk. Nyokangi (2016) came to the same conclusion when comparing the MVO model and the single index model. However, Masese (2017) emphasized the relevance of additional studies on portfolio optimization particularly when other securities like Bills and Bonds are incorporated.

Boasson, Boasson, and Zhou (2011) conducted a study on portfolio optimization using semi-variance as the measure of risk. The results indicated that portfolios constructed using semi-variance simultaneously minimized downside risk and achieved similar or better-expected returns as compared to those that used the variance since the portfolios had limited upside gain and higher downside risk. Moreover, semi-variance may offer investors an asset allocation method that minimizes holdings in both assets with a higher variance and assets with higher semi-variance along with higher correlation among themselves.

Mwabaya (2015), extended the Mean Semi-variance model to the Kenyan stock market and compared its performance to that of the MVO model and further investigated whether the geometric mean-variance can approximate the semi-variance approach. The results showed that there was not much difference between the semi-variance and mean-variance approaches since the NSE monthly equity returns seemed to exhibit a symmetrical distribution. According to Hull (2008), if returns follow a normal distribution semi-variance as a risk metric it is proportional to the variance. Furthermore, portfolio performance and F-test confirmed that the geometric mean-variance method can approximate the semi-variance method. However, further studies should be conducted to confirm if it gives similar results when returns are not normally distributed.

Birungi (2021) conducted a thorough analysis of the risk metrics in portfolio optimization for the Uganda Securities Exchange. This study uses Value-at-Risk(VaR), variance, Conditional Value-at-Risk(CVaR), and covariance as risk measures to be used in the following models: the Mean-Variance (MV), Mean-Conditional Value-at-Risk (CVaR), and Mean Absolute Deviation (MAD) models which are traditional portfolio optimization models and the Robust Portfolios and Covariance Estimation Models(Shrinked Mean-Variance (SMV and Alternative Covariance Estimator (ACE) Models)). It was found that the optimal portfolio with the highest return was produced by the conventional models of portfolio optimization. The suggested risk models provide less risky portfolios. The study concludes that optimization techniques and assigning weights to portfolio assets are more important than any particular risk metric.

Li, Wu, and Ojiako (2012) analysed and compared the performances of various portfolio selection models including the MAD model, the VaR, the MVO model, and Young's Minimax model using four-year data of shares listed on the Hong Kong Hang Seng Index (HIS). The analysis of the results showed that the portfolios of the MVO and MAD models seemed to be more diversified than the rest. However, the authors suggested the use of additional portfolio optimisation techniques such as the CVaR model, and the use of data extending a longer period as opportunities for further research.

Bonyo (2015) tested the effectiveness of CVaR against variance for the years 2007 to 2014 and inferred that CVaR outperformed variance since it simultaneously minimized downside risk and achieved similar or better returns. The study concluded that CVaR is a better risk measurement in asset allocation and portfolio optimization than variance in the Kenyan stock market. Kasenbacher et al. (2017) compared the Mean-Variance and Mean Absolute Deviation models. The results showed that the Mean Absolute Deviation model performed better than the Mean-Variance Optimization model when applied to the S&P 500 data.

Silva et al. (2017) investigated the efficacy of alternative portfolio optimization models that combine the conditional value at risk (CVaR) and the mean absolute deviation (MAD) when the returns of financial assets are highly volatile. It is discovered that while the traditional models yield higher returns for portfolios, alternative optimization models that combine the CVaR and MAD were able to generate portfolios with less risk.

Zhang and Zhou (2013) conducted an empirical study analysing the efficiency of the Mean Semi Absolute Deviation model in portfolio selection. The model's efficiency was tested using 20 stocks in the Chinese securities market without an option of short selling. The semi-variance absolute values were used to compare the risk of the MVO and MSAD models. It found that the semi-variance absolute values of the MVO model were larger, hence, the portfolio risk used by the MSAD model is greater than that of the MVO model. Therefore, the MSAD model can effectively guide the decision in asset allocation.

The commonly used MVO model has weaknesses that have motivated other scholars to develop alternative optimization models. The studies mentioned above have investigated whether alternative risk measures and portfolio optimization models can outperform the MVO model in different markets using different sets of data. The objectives of this study align with earlier studies since it analyses and compares different optimization models in attempt to determine which performs better in optimal portfolio construction. Moreover, this study investigates the consistency of the performance of the three different models across different varying time periods, which is like the study conducted by Nyokangi (2016).

As an improvement over MVO, more research can be conducted by considering additional risk metrics such as CVaR, Mean Absolute Deviation, partial drawdown, and semi-variance in a risk-return analysis, as recommended by Jaaman et al. (2013) and A. H. Chen, Fabozzi, and Huang (2012). Mwabaya (2015) researched portfolio optimization using semi-variance as the measure of risk, however, no study uses mean absolute deviation and the mean absolute negative deviation as risk measures in the Kenyan stock market. This study will contribute to the existing knowledge on asset allocation and portfolio optimization models within the Kenyan context, particularly considering the market's dynamic conditions across different time periods.

This study focuses on evaluating the use of the MVO, MAD, and MSAD models for optimal portfolio construction using the shares listed in the NSE and comparing the portfolios selected by the different models.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

Research methodology, according to Kothari (2014), is a sequential theoretical assessment of the techniques applied in a particular field of study. This study's methodology is described in this chapter. It gives a detailed explanation of the sampling strategy, sample size, population under study, kind of data gathered, and data processing methods.

3.2 Research Design

This study utilized both quantitative and exploratory research designs. The quantitative aspect involved collecting and analysing data on stocks to construct optimal portfolios. Additionally, the study was exploratory as it compared three different models to determine which was best for asset allocation and portfolio selection. The exploratory design was also used in previous studies (Boasson et al., 2011; Masese, 2017; Mwabaya, 2015) which focused on a similar topic.

The study also focused on understanding how duration and time influenced portfolio performance. The exclusion of macroeconomic variables is deliberate, as the study prioritized direct measures of time-sensitive portfolio adaptability without the added complexity of interpreting lagging macroeconomic factors.

3.3 Population and Sampling

This study analyzed portfolio returns using different asset allocations obtained from each of the portfolio optimization models from January 2011 to December 2020. All securities listed on the NSE all share index (NASI) as of December 2020 formed the population of this study. They were sixty-six (66) in number.

This duration, January 2011 to December 2020, was chosen since the data was readily available. Moreover, this period faced changing economic and market environments that contributed to varying market volatility. Therefore, the period can offer valuable insights into the resilience and effectiveness of various portfolio optimization models, making the findings highly relevant for portfolio managers, investors, and fund managers in Kenya.

This research used a sample size of 8 stocks that were active as of December 2020, had fully disclosed their prices, and had superior risk-adjusted returns. The study does not consider any entity that was newly enlisted, delisted, or suspended during the period.

The data used was from the daily price lists of the NASI. Throughout the research period, the daily returns were computed and converted into matching monthly stock returns. Using this return, the expected value (returns), standard deviations, and covariances were computed. Daily returns are more erratic; thus, monthly returns are utilized.

The average interest rate on the 91-day Treasury Bills issued by the Central Bank of Kenya is used as an equivalent of the risk-free rate.

3.4 Data Collection Methods

The study utilized secondary data covering the period from January 2011 to December 2020. The daily price lists of securities listed on the NSE were obtained from the NSE website, while the Treasury Bill interest rates were sourced from the CBK and the National Treasury website. The secondary data was appropriate for this study since the sources were credible and they arise from day-to-day trading, therefore practical in implementation.

3.5 Data Analysis

The study was quantitative, thus descriptive and inferential statistics were used. The data was collected, cleaned, and transformed to ensure it was accurate and consistent. Thereafter, an analysis was done using the R programming language which minimized the risk subject to the constraints explained in the subsections below. Tables were generated to present and visualize the statistics.

The goal is to determine the asset weights at which the different risk measures will be minimized while maintaining the necessary rate of return, across the different MAD, MVO, and MSAD models.

The out-of-sample technique was used to assess the performance of the three models. The data was split into the training and testing data sets. The training data set included stock data spanning a period of 7 years starting January 2011 to December 2017. This data was used in the optimization process to generate the optimal weight of the stocks in the portfolio. The testing data included stock data spanning a period of 3 years starting January 2018 to December 2020. On the other hand, this data set was utilized to obtain the expected return and risk of the portfolios using the weights obtained from the training data. To find out if the investment horizon mattered for the best asset allocation, the performance of the 1-year, 2-year, and 3-year portfolios was also compared. Varying 12-month time periods from January 2018 to December 2020 are also used to check the consistency in the performance of the MVO, MAD, and MSAD models in optimal portfolio construction across different economic environments.

By evaluating performance across 1-year, 2-year, and 3-year horizons, and varying 12-month periods from 2018 to 2020, the study demonstrated how portfolio optimization models responded to real-world market fluctuations. The decision to exclude macroeconomic variables simplified the analysis and aligned with the study's goal of evaluating time-sensitive portfolio strategies.

Thereafter, the Sharpe and Sortino ratios were used to assess the performance of the portfolios, to determine which model outperformed the rest. The daily returns of the individual shares using the following equation:

$$r_{j,t+1} = ln\left(\frac{P_{j,t+1}}{P_{j,t}}\right) \quad for\ j = 1,\ldots,N \ \ and \ \ t = 0,\ldots,251$$

The expected return of the portfolios is the weighted average of the expected returns of securities within the portfolio. The arithmetic mean method is used:

$$E[r_p] = \sum_{j=1}^{N} w_j E[r_j]$$

These terms have the same meaning across the different models:

w_j represents the weights of asset j – it is the proportion of the portfolio value invested in asset j

$E[r_j]$ is the expected return for security j

r_j denotes the random return representing return per period of asset j

σ_{jk} is the covariance between assets j and k

N is the number of assets in each portfolio.

ρ denotes the parameter representing the minimal rate of return required by an investor

M_0 represents the total amount of capital being invested.

r_f is the risk-free rate and it was used as the minimal rate of return required by an investor.

T represents the total number of periods.

Since monthly stock data returns were used in the optimization process, the geometric mean of the monthly portfolio returns was calculated over the respective periods under analysis. The monthly geometric mean of portfolio returns was used to analyse the performance of the portfolios and to compute the Sharpe and Sortino ratios. The geometric mean is preferred over the arithmetic mean as it accounts for compounding and coveys what average rate of return per period would have been for the entire investment duration. It is expressed as:

$$R_{(0,T)} = \left(\prod_{t=1}^{T}(1 + R_t\right)^{\frac{1}{T}} - 1$$

3.5.1 Mean-Variance Portfolio Optimization Model

As proposed by Markowitz (1952), the mean-variance optimization model seeks to minimize variance at a given level of expected returns. The expected return, variance, and covariance of each asset return in the sample are calculated and employed in the optimization process. Using the Markowitz view, we will optimize finding the mean-variance or risk-return efficient frontier. This means that every portfolio on this frontier yields the highest expected return for the given level of risk.

The expected return, $E[r_p]$, of the portfolio is calculated as follows:

$$E[r_p] = \sum_{j=1}^{N} w_j E[r_j]$$

The standard deviation of the portfolio, σ_p , which denotes the risk measure according to the MVO model is computed as follows:

$$\sigma_p = \sqrt{\sum_{j=1}^{N}\sum_{k=1}^{N} w_j w_k \sigma_{jk}}$$

This model is a quadratic programming model. We formulate the mathematical model as follows:

$$Minimize \left\{ \sigma_p^2 = \sum_{j=1}^{N}\sum_{k=1}^{N} w_j w_k \sigma_{jk} \right\}$$

Subject to;

1) $\sum_{j=1}^{N} w_j = 1$, this constraint ensures that all the money is invested in the risky portfolio.
2) $w_j \geq 0 \quad \forall \; j = 1, \dots, N$, no short selling is allowed.
3) $\sum_{j=1}^{N} w_j E[r_j] \geq \rho M_0$, ensures that the minimum portfolio return will achieve investors' minimum requirements,
4) $0 \leq w_j \leq u_j$, *for j = 1,…,N,* limits of the asset proportions.

3.5.2 Mean Absolute Deviation Model

Konno and Yamazaki (1991) proposed the mean absolute deviation as a risk measure to overcome the weaknesses of Markowitz's model. This model minimizes the mean absolute deviation instead of the variance. MAD results in a linear programming model which is computationally more tractable than the MVO model. However, Bower and Wentz (2005) stated that the mean absolute deviation is theoretically the same as the variance when the returns follow a normal distribution. Literature also shows that if returns are normally distributed, the MAD, MVO, and Minimax models yield the same results.

The mathematic expression for the mean absolute deviation model is:

$$E\left|\sum_{j=1}^{N} r_j w_j - E\left(\sum_{j=1}^{N} r_j w_j\right)\right| \quad for\; j = 1, \dots, \mathrm{N}$$

The MAD model is mathematically formulated as follows:

$$Minimize\ g(r) = E\left[\left|\sum_{j=1}^{N} r_j w_j - E\left(\sum_{j=1}^{N} r_j w_j\right)\right|\right]$$

The mathematic expression for the mean absolute deviation model is:

1) $\sum_{j=1}^{N} w_j r_j \geq \rho M_0$, the portfolio's expected return exceeds some minimum return required by investors ρM_0.
2) $\sum_{j=1}^{N} w_j = 1$, this ensures that all the capital is invested in the risky portfolio.
3) $0 \leq w_j \leq u_j$, *for j = 1,…,N,* shows the limits of the asset proportions.

According to Konno and Yamazaki, the expected value of the random variable can be approximated by the average over time. That is:

$$r_j = E[r_j] \cong \frac{1}{T} \sum_{t=1}^{T} r_{jt}$$

r_{jt}= realization of a random variable r_j at time t, for t = 1,…,T

Here, r_j reflects the average return that asset j is expected to generate over time.

We then substitute the previous equation into the equation below:

$$Minimize\ g(r) = E\left[\left|\sum_{j=1}^{N} r_j w_j - E\left(\sum_{j=1}^{N} r_j w_j\right)\right|\right]$$

The resulting solution is as follows:

$$E\left(\sum_{j=1}^{N} r_j w_j\right) \approx \sum_{j=1}^{N} w_j E[r_j] = \sum_{j=1}^{N} w_j \left(\frac{1}{T} \sum_{t=1}^{T} r_{jt}\right)$$

The expected portfolio returns for a given period $E\left[\sum_{j=1}^{N} r_j w_j\right]$ can be written as:

$$E\left(\sum_{j=1}^{N} r_j w_j\right) = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt} w_j$$

Rewrite the objective function using the substitution:

$$g(r) = E\left[\left|\sum_{j=1}^{N} r_j w_j - \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt} w_j\right|\right]$$

This is the final equation:

$$g(r) = E\left[\left|\sum_{j=1}^{N} r_j w_j - E\left(\sum_{j=1}^{N} r_j w_j\right)\right|\right] = \frac{1}{T}\sum_{t=1}^{T}\left[\left|\left(\sum_{j=1}^{N}(r_{jt} - r_j) w_j\right)\right|\right]$$

Since the absolute value in this equation is nonlinear, the variable z_{jt}, is used to linearize it. Let $z_{jt} = y_{jt} - E[y_j]$ (which is the deviation of the asset return from its expected return in period t). Substituting this into the above equation gives:

$$g(r) = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} |z_{jt}| w_j$$

$|z_{jt}| w_j$ is the deviation of the portfolio's return from the expected return in period t. If we let $b_t = \sum_{j=1}^{N} z_{jt} w_j \quad for\ t = 1, \ldots, T$ which represents the total deviations, we have:

$$g(r) = \frac{1}{T}\sum_{t=1}^{T} b_t$$

The objective function representing the average deviations of the portfolio’s returns from the expected return T periods is as follows:

$$Minimize\ g(r) = \frac{1}{T}\sum_{t=1}^{T} b_t$$

Subject to:

1) $b_t \pm \sum_{j=1}^{N} z_{jt}w_j \geq 0, \quad for$ t = 1,$\cdots$ T, b_t is a linear form that uses the appropriate portfolio allocation; to describe returns on asset j at time t. This constraint considers the variation of values that fall and rise above the portfolio's expected value,

2) $\sum_{j=1}^{N} w_j r_j \geq \rho M_0,$ this constraint implies the portfolio's expected return exceeds some minimum return (ρM_0),

3) $0 \leq w_j \leq u_j$, *for j = 1,…,N,* shows the limits of the asset proportions,

4) $\sum_{j=1}^{N} w_j = 1$, this ensures that all the capital is invested in the risky portfolio.

This becomes our linear optimization problem which equally penalizes both negative and positive deviations. Research reveals that investors appreciate larger positive deviations and steer clear of smaller negative deviations in portfolio return.

3.5.3 Mean Semi-Absolute Deviation

Kamil and Ibrahim (2007) proposed the Mean Semi-Absolute Deviation (MSAD) portfolio optimization model, which utilizes the mean absolute negative deviation as a downside risk measure. This approach is derived from the traditional mean absolute deviation but only pays attention to the negative deviations from the mean return, making it a more effective tool for managing downside risk in portfolios.

The mean absolute deviation is replaced with the mean absolute negative deviation, forming our measure. The MSAD objective function can be written as:

$$h(r) = \frac{1}{T}\sum_{t=1}^{T} \max\,(R - R_t, 0)$$

$$h(r) = \max\left(\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt}w_j - \frac{1}{T}\sum_{j=1}^{N} r_{jt}w_j\right), 0\right)$$

where:

$$R_t = \frac{1}{T}\sum_{j=1}^{N} r_{jt}w_j$$, R_t is the actual return of the portfolio at time t

$$R = \frac{1}{T}\sum_{t=1}^{T} R_t = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt}w_j$$, R is the the average portfolio return over all periods

$\max(R - R_t, 0)$, measures the extent to which the actual return falls below the mean return

The objective function h(r) represents the average of these absolute negative deviations across all periods across all periods.

To linearize the objective function, we introduce the auxiliary variable b_t to represent the absolute negative deviations from the mean.

$$b_t \geq R - R_t$$

$$b_t \geq 0$$

By use of substitution, we arrive at the equation below:

$$b_t \geq \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt}w_j - \frac{1}{T}\sum_{j=1}^{N} r_{jt}w_j$$

Substitute the auxiliary variables into the objective function:

$$h(r) = \frac{1}{T}\sum_{t=1}^{T} b_t$$

b_t effectively absorbs the $\max(\mathrm{R} - \mathrm{R_t}, 0)$

The linearized objective function that represents the average of the absolute negative deviations from the expected return for T periods is as follows:

$$Minimize\ h(r) = \frac{1}{T}\sum_{t=1}^{T} b_t$$

Subject to

1) $b_t \geq 0 \ for \ t = 1,..,T$, this ensures that is b_t non-negative

2) $b_t \geq \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} r_{jt}w_j - \sum_{j=1}^{N} r_{jt}w_j \ for \ t = 1,..,T$, this constraint ensures that the auxiliary variable b_t is at least as large as the negative deviation from the mean. This captures the downside risk,

3) $\sum_{j=1}^{N} w_j r_j \geq \rho M_0$, this constraint means the portfolio's expected return exceeds some minimum return (ρM_0),

4) $\sum_{j=1}^{N} w_j = 1$, this ensures that all the capital is invested in the risky portfolio.

5) $0 \leq w_j \leq u_j$, *for j = 1,…,N,* shows the limits of the asset proportions.

This becomes our linear optimization problem which specifically penalizes only the negative deviations below the mean return. The MSAD model focuses on minimizing these downside deviations, aligning with the literature that suggests investors are particularly concerned with and prefer to avoid negative deviations from the mean return while being less sensitive to positive deviations.

3.5.4 Measuring Portfolio Performance

Bolton et al (2011) state that risk and return are the primary metrics used to compare portfolio performance and are integrated to create ratios or indexes that are used to identify the most optimal portfolio. Gerken (2015) suggested comparing portfolios using multiple performance measures.

The three models were used to generate portfolios, and performance ratios were used to compare the performance of each portfolio. Performance ratios are financial metrics employed to assess the risk-adjusted return of an asset or investment portfolio. These ratios help investors evaluate how effectively a portfolio manager or investment strategy is generating returns relative to the amount of risk taken. The portfolio's mean ($E[r_p]$) and variance (σ_p^2) are used. To establish the optimal portfolio construction model, the portfolios generated were ranked using performance metrics.

Pekár et al. (2016) found that when two optimization models are compared, the model that performs better has portfolios with greater performance ratios. To determine which model performed better in the Kenyan market, Nyokangi (2016) evaluated the Single index and MVO models for stock optimization by maximizing the Sharpe ratio. Abdalla (2013) also used the Sharpe statistics to compare the performance of the NSE 20 index to a recently developed market portfolio index. Along with the Sharpe ratio, Ogutu (2014) also used the Jensen and Treynor ratios to determine which of the Single index, MVO, and (1/N) naïve portfolio models were the best portfolio optimization models. Masese (2017) compared the effectiveness of the TA and MVO models using the information, Sortino, and Sharp ratios.

This study employs the following three ratios:

Sharpe Ratio

This ratio was developed by Sharpe (1966) as a method of assessing the performance of mutual funds. It measures the excess return earned over the risk-free rate of interest per unit of an investment. It is computed as:

$$\frac{E[r_p] - r_f}{\sigma_p}$$

r_f is the average risk-free rate over the given required periods.

A portfolio with a higher Sharpe ratio is recommended since it offers better risk-adjusted returns.

Sortino Ratio

Frank Sortino introduced the Sortino ratio in 1981 as a refinement of the Sharpe ratio. It substitutes the standard deviation with the downward deviation as a measure of risk. According to Rollinger and Hoffman (2013), it thereby only considers the riskiness of returns that go below the needed rate of return.

This is particularly relevant for the Mean Semi-Absolute Deviation Model, which focuses on minimizing downside risk. It is obtained as follows:

$$\frac{E[r_p] - r_f}{\sigma_L(r_p)}$$

$$\text{Where } \sigma_L(r_p) = \frac{1}{n_L}\sum^{n}\left(Min(r_p - r_f), 0\right)$$

$\sigma_L(r_p)$ is the downside deviation of the portfolio returns, which measures the risk of negative returns.

Better risk-adjusted performance is shown by a higher Sortino Ratio, especially when considering downside risk.

CHAPTER 4: DATA ANALYSIS, FINDINGS AND DISCUSSION

4.1 Introduction

This chapter presents the results of the study. The analysis employed the use of three models in the determination of optimal asset weights over the period 2011 to 2017 as well as the performance and resilience of the models over the period 2018 to 2020. This informed the decision to reject or fail to reject the research hypotheses.

4.2 Portfolio Selection

The portfolio in this study was constructed using daily price data from the NASI on the NSE. It comprised of eight stocks, selected by first identifying the two companies with the highest market capitalization across all sectors on the NSE that had been trading throughout the 2011 to 2020 period. These companies were chosen for their market liquidity, strong presence, and stability, making them well-suited for modelling portfolio performance with a focus on risk minimization. Large-cap stocks are typically more resilient during economic downturns and tend to perform consistently across varying market conditions. Risk-adjusted returns, measured by the Sharpe ratio, were calculated for each stock, with only those showing positive risk-adjusted returns being included in the portfolio. To ensure diversification, the portfolio contained stocks from six of the eleven sectors listed on the NSE.

4.3 Variable Selection and Transformation

The prices used are the daily adjusted closing prices of every stock in the sample. The daily log returns are obtained from these prices and transformed into monthly compounded returns to facilitate portfolio analysis. Log returns are used in this study because they allow for easier aggregation over time when compared to simple returns.

The 91-day Treasury Bill rate corresponding to each month was obtained from the CBK and the National Treasury website. The rates are first converted into effective interest rates and then a geometric average of the rates is calculated for the respective training and the testing periods. The 91-day Treasury Bill averages are used as the risk-free rates for the corresponding periods.

4.4 Optimal Portfolio Selection

The Mean-Variance Optimization, Mean Absolute Deviation, and Mean Semi Absolute Deviation models were used to determine the optimal asset weights for the portfolio of selected stocks. The optimize.portfolio function under the Portfolio Analytics package in R was used to select the stocks and allocate weights for the optimal portfolio.

4.4.1 Mean-Variance Optimization Model

This study follows the process described in section 3.5.1 to find the optimal asset weights under this model. The results are shown in Table 4.1 below.

Stock	Weight	Portfolio Returns	Portfolio Risk
SCOM	0.3439	1.3448%	3.7084%
ORCH	0.0126		
JUB	0.0068		
BAT	0.4035		
KNRE	0.0212		
SASN	0.2120		

Table 4.1: Stock composition, return, and risk of optimal portfolio selected by the MVO model using data from 2011-2017

From the table above, we can see that BAT, SCOM, and SASN have the largest allocations. BAT is in the Manufacturing sector, SCOM is in the Telecommunications sector, SASN is in the Agricultural sector, KNRE and JUB are in the Insurance sector and ORCH is in the Investments sector.

BAT was selected because of its low risk while SCOM and SASN were chosen because of their high returns relative to the risk.

4.4.2 Mean Absolute Deviation Model

This study follows the process described in section 3.5.2 to find the optimal asset weights under this model. The results are shown in Table 4.2 below.

Stock	Weight	Portfolio Returns	Portfolio Risk
SCOM	0.2720	1.3147%	4.1750%
ORCH	0.0020		
JUB	0.2300		
CRWN	0.0280		
BAT	0.3160		

SASN	0.1360		
CTUM	0.0160		

Table 4.2: Stock composition, return, and risk of optimal portfolio selected by the MAD model using data from 2011-2017

From the table above, we can see that BAT (Manufacturing), SCOM (Telecommunications), and JUB (Insurance) have the largest allocations. CTUM is in the Investments sector and CRWN is in the Construction sector.

BAT was selected because of its low risk while SCOM and JUB were chosen because of their high returns relative to the risk.

4.4.3 Mean Semi-Absolute Deviation Model

This study follows the process described in section 3.5.3 to find the optimal asset weights under this model. The results are shown in Table 4.3 below.

Stock	Weight	Portfolio Return	Portfolio Risk
SCOM	0.436	1.4022%	3.8659%
JUB	0.008		
BAT	0.326		
SASN	0.224		
CTUM	0.014		

Table 4.3: Stock composition, return, and risk of optimal portfolio selected by the MSAD model using data from 2011-2017

From the table above, we can see that SCOM (Telecommunications), BAT (Manufacturing), and SASN (Agricultural) have the largest allocations.

BAT was selected because of its low risk while SCOM and SASN were chosen because of their high returns relative to the risk.

4.5 Portfolio Performance

4.5.1 Portfolio performance in the training period

In the training period, the three portfolio optimization models selected stocks from different sectors implying that optimal stock portfolios were well diversified.

The MVO model selected 6 stocks, the MAD model selected 7 stocks, and the MSAD model selected 5 stocks, which consist of the top market players in each sector. This result aligns with the conclusion reached by Dou, Gallagher, Schneider, and Walter (2014), which highlights that diversification across sectors enhances portfolio performance.

The results of the training period are summarized below:

	Portfolio Return	Portfolio Risk	Sharpe Ratio	Sortino Ratio
MVO	1.3448%	3.7084%	-2.0880	-0.9031
MAD	1.3147%	4.1750%	-1.9767	-0.8938
MSAD	1.4022%	3.8659%	-2.1115	-0.9048

Table 4.4: Portfolio risk, return, and performance ratios of the portfolios

The MSAD had the highest return, followed by the MVO model while the MAD model had the lowest return. Regarding risk, the MAD model had the highest, followed by the MSAD model, while the MVO model had the lowest risk.

The Sharpe and Sortino ratios are negative, indicating that the portfolio's returns are less than the return on the 91-day Treasury Bill, suggesting that the investors would have been better off investing in a 91-day Treasury Bill.

Below is the performance ranking of the three portfolio optimization models:

RANKING

	Sharpe Ratio	Sortino Ratio
MVO	2	2
MAD	1	1
MSAD	3	3

Table 4.5: Ranking of the MVO, MAD, and MSAD models by the Sharpe and Sortino ratio 2011-2017

This ranking is determined by the values of the three performance ratios discussed in section 3.5.4, with higher performance ratios leading to better rankings. A portfolio with a higher ranking is regarded as having superior performance. The MAD model outperforms the rest of the models in the training period while the MSAD model has the worst performance.

4.5.2 Performance of the models across different investment horizons

To investigate whether the investment horizon matters when choosing which portfolio optimization method to use, the performance of the 1-year, 2-year, and 3-year portfolios is compared. This is because some investors might want to hold their portfolio for a short period of one year or less, while other investors such as institutional investors might want to hold their portfolios for a longer period i.e 3 years.

The optimal weights obtained for each model will be used to create the 1-year, 2-year, and 3-year portfolios. The Sharpe and Sortino ratios are thereafter calculated to identify which portfolio optimization method outperforms the rest in the different investment horizons.

The results of the 1-year, 2-year, and 3-year portfolios are as follows:

	Portfolio Return	Portfolio Risk	Sharpe Ratio	Sortino Ratio
		1-year		
MVO	-1.8641%	4.8691%	-1.9501	-0.8977
MAD	-1.7177%	4.0696%	-2.3673	-0.9271
MSAD	-1.7903%	4.4436%	-2.1186	-0.9113
		2-year		
MVO	1.2358%	3.9213%	-2.1962	-0.9134
MAD	-1.1563%	3.3259%	-2.6701	-0.9389
MSAD	-0.8297%	3.8070%	-2.1858	-0.9126
		3-year		
MVO	-0.9833%	4.2898%	-1.9112	-0.8887
MAD	-1.0886%	4.1100%	-2.0950	-0.9048
MSAD	-0.6360%	4.0860%	-1.9935	-0.8910

Table 4.6: Portfolio risk, return, and performance ratios of the MVO, MAD, and MSAD models over a 1-year, 2-year, and 3-year investment horizon

It is observed that the MVO model has the highest risk among all models across all investment horizons, while the MAD model has the lowest risk in the 1-year and 2-year investment horizons and the MSAD model has the lowest risk in the 3-year investment horizon.

Despite yielding negative returns across all investment horizons, the MAD model achieves relatively better returns in the 1-year investment horizon, while the MSAD model achieves relatively better returns in the 2-year and 3-year investment horizons.

Below is the performance ranking of the three portfolio optimization models across the different investment horizons:

RANKING

	Sharpe Ratio	Sortino Ratio
	1-year	
MVO	1	1
MAD	3	3
MSAD	2	2
	2-year	
MVO	2	2
MAD	3	3
MSAD	1	1
	3-year	
MVO	1	1
MAD	3	3
MSAD	2	2

Table 4.7: Ranking of the MVO, MAD, and MSAD models by the Sharpe and Sortino ratios over a 1-year, 2-year, and 3-year investment horizon

According to the results, the MVO model delivers the best performance for the 1-year and 3-year investment horizon while the MSAD model performs best for the 2-year horizon.

4.5.3 Performance of the models in varying 12-month periods

To assess the consistency, reliability, and robustness of the MVO, MAD, and MSAD models, their performance is compared across different 12-month periods in 2018, 2019, and 2020.

Testing across multiple time periods evaluates whether the model can adapt to different economic cycles, assessing its effectiveness in dynamic, real-world environments. This approach provides insights into the practical application of the model for portfolio optimization, independent of external economic influences.

The results of the portfolios across different time periods are as follows:

	Portfolio Return	Portfolio Risk	Sharpe Ratio	Sortino Ratio
		2018		
MVO	-1.8641%	4.8691%	-1.9502	-0.8977
MAD	-1.7177%	4.0696%	-2.3673	-0.9271
MSAD	-1.7903%	4.4436%	-2.1187	-0.9113
		2019		
MVO	-0.5540%	2.8413%	-2.8603	-0.9483
MAD	-0.5277%	2.4973%	-3.3170	-0.9608
MSAD	-0.1840%	3.0442%	-2.4226	-0.9300
		2020		
MVO	-0.8347%	5.1163%	-1.4517	-0.8348
MAD	-1.2704%	5.5333%	-1.4345	-0.8318
MSAD	-0.5240%	4.7631%	-1.4755	-0.8389

Table 4.8: Portfolio risk, return, and performance ratios of the MVO, MAD, and MSAD in 2018,2019 and 2020

The Sharpe and Sortino ratios are negative, indicating that the portfolio underperforms relative to the risk-free 91-day Treasury Bill.

The results show that the MVO model exhibits the highest risk in 2018, the MSAD model has the highest risk in 2019, and the MAD model carries the highest risk in 2020.

Although all portfolios across the different periods yield negative returns, the MAD model achieves relatively better returns in 2018, while the MSAD model performs better in 2019 and 2020. All portfolios exhibited higher risk in 2020 because of high market volatility and economic uncertainty.

Below is the performance ranking of the three portfolio optimization models across the different periods:

RANKING

	Sharpe Ratio	Sortino Ratio
2018		
MVO	1	1
MAD	3	3
MSAD	2	2
2019		
MVO	2	2
MAD	3	3
MSAD	1	1
2020		
MVO	2	2
MAD	1	1
MSAD	3	3

Table 4.9: Ranking of the MVO, MAD, and MSAD models by the Sharpe and Sortino ratios in 2018, 2019, and 2020

The model with a higher ranking is considered to have superior performance. According to the results, the MVO model delivers the best performance in 2018, the MSAD model performs best in 2019, and the MAD model performs best in 2020.

4.6 Summary

4.6.1 1-year, 2-year, and 3-year investment horizon analysis

Testing across different investment horizons was conducted to determine whether the choice of portfolio optimization model depends on the investment horizon. The effective risk-free rates corresponding to each investment horizon were higher than the portfolios' monthly returns, resulting in negative Sharpe and Sortino ratios across all models.

The results show that the MVO model exhibited the highest monthly risk across all investment horizons, but it also delivered the best performance in the 1-year and 3-year investment horizons according to the performance ratios. The MAD model delivered the highest monthly returns across all investment horizons but delivered the worst performance according to the Sharpe and Sortino ratios. The MSAD model performed best in the 2-year horizon and moderately in the 1-year and 3-year investment horizon.

4.6.2 Varying 12-month periods (2018, 2019 and 2020)

Testing the models in these distinct economic cycles highlights their adaptability and effectiveness in dynamic, real-world environments. The analysis reveals that all models produced negative Sharpe and Sortino ratios, indicating underperformance relative to the risk-free rate offered on 91-day Treasury Bill.

The results show that the MVO model exhibited the highest risk in 2018, the MSAD model in 2019, and the MAD model in 2020. Despite yielding negative returns in all periods, the MAD model achieved relatively better returns in 2018, while the MSAD model outperformed in 2019 and 2020.

Further highlighting the different capabilities of the models in the different time periods are the performance rankings based on Sortino and Sharpe ratios. The MVO model had the best performance and the highest ranking in 2018. The MSAD model produced better results in 2019, but the MAD model took the lead in 2020.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This study utilized three risk-reward models, each employing a distinct risk measure to construct optimal portfolios of stocks listed in the NASI for the period 2011 to 2017. The optimal portfolio weights obtained in the training period were applied to calculate the expected returns and risks of the portfolios constructed in the testing period from 2018 to 2020. Thereafter, the models' performance across different investment horizons and varying 12-month periods were compared using different performance metrics – Sharpe and Sortino ratios. This comparison led to identifying the model that delivered the best performance.

The MAD model outperformed the MVO and MSAD models in the training period. This is in tandem with what Gilli and Schumann (2009) found, that other risk measures offer better asset allocation and portfolio selections than the mean-variance method. However, the MSAD model performs the poorest, and therefore, we reject the first research hypothesis stated in section 1.4.

When comparing performance across different investment horizons, the MVO model outperforms the rest in the 1-year and 3-year investment horizon, while the MSAD model outperforms the rest in the 2-year investment horizon. The MAD model ranked last in all investment horizons. It was observed that the MAD and MSAD model had lower risk than the MVO model with the MAD model having the lowest risk in the 1-year and 2-year investment horizons, while MSAD had the lowest risk in the 3-year investment horizon. The results show that there is inconsistency in the performance of the models across the different investment horizons and none of the models outperforms the others in all investment horizons. We conclude that the MVO model performs best for a shorter-term investment horizon such as 1 year, while both the MVO and MAD models perform best for medium-term investment horizons.

When comparing the performance across varying 12-month periods, the MVO model performed best in 2018. The MSAD model performed best in 2019, which is in line with what Ibrahim and Kamil (2006) found, that the model with downside risk (MSAD) performed better than the MAD and MVO models. The MAD model performed best in 2020 which is consistent with what Kasenbacher et al. (2017) found, that the MAD model outperforms the MVO model and provides larger risk-adjusted returns. This shows that the MAD model is suitable for periods with increased market volatility and high economic uncertainty since the COVID-19 pandemic, which occurred in 2020, caused a sharp decline in the Kenyan stock market.

In conclusion, there was an inconsistent performance of all models in the different investment horizons and in the varying 12-month periods. Therefore, we reject the second hypothesis stated in section 1.4.

This study demonstrates that duration and time are important factors for evaluating the performance of portfolio optimization models. By focusing on these factors, the research provides practical insights for fund managers navigating the dynamic conditions of Kenya's stock market. Unlike macroeconomic factors, which often have lagging effects, duration and time have a direct influence on portfolio performance, making them crucial for decision-making.

5.2 Limitations of the study

First and foremost, the study focused exclusively on stocks as securities for portfolio construction, with only 8 out of 66 stocks selected for analysis. This approach may have excluded other potentially valuable stocks and alternative investment options, such as Treasury Bills, Treasury bonds, REITs, and corporate bonds, which could have enriched the portfolio analysis.

Moreover, a three-year testing period was utilized to evaluate the performance of the models across different investment horizons and varying 12-month periods. However, this duration was insufficient to effectively compare the models' performance over medium-term investment horizons (1 to 5 years) and long-term investment horizons (more than 5 years).

5.3 Suggestions for further studies

This study used a testing period of three years to assess the performance of the models across different investment horizons and varying 12-month periods. This duration was not sufficient for a comprehensive comparison of the models' performance over medium and long-term investment horizons. Further studies could benefit from extending the testing period to provide for a more thorough analysis.

There is a need for further research on portfolio optimization especially when considering other securities like Treasury Bonds and Bills, and REITs which were not used in this study.

Further studies can also be done by considering other risk measures like partial drawdown and Minimax as an improvement over the MVO model and the use of Robust Portfolios and Covariance Estimation Models such as the Shrinked Mean-Variance (SMV) and the Alternative Covariance Estimator (ACE) Models in the Kenyan stock market.

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Appendices

Appendix A

A. Portfolio Formation R codes

A.1 Mean-Variance Optimization

```r
# Portfolio Optimization MVO Model
# Constraints
pspec_mvo <- portfolio.spec(assets = colnames(train_data2))

pspec_mvo <- add.constraint(portfolio = pspec_mvo,  type = "full_investment")

pspec_mvo <- add.constraint(portfolio = pspec_mvo, type = "long_only")

pspec_mvo <- add.constraint(portfolio=pspec_mvo, type="box", min = 0.0, max = 
0.5)
upper_bound <- 0.5

# variance objective
pspec_mvo <- add.objective(portfolio=pspec_mvo, type="risk",name="StdDev")

# Optimize the portfolio
optimized_portfolio_mvo <- optimize.portfolio(R = train_data2, portfolio = ps
pec_mvo, optimize_method = "ROI", search_size = 5000, trace = TRUE)
```

A.2 Mean Absolute Deviation Model

```r
# Portfolio Optimization MAD Model
# Custom MAD risk function
mad_risk_function <- function(R, weights) {
  # Ensure x is a matrix and weights are a vector
  if (is.data.frame(R)) {
    R <- as.matrix(R)
  }

  # Calculate portfolio returns
  portfolio_returns <- Return.portfolio(R, weights)

  # Calculate Mean Absolute Deviation (MAD)
  expected_return <- mean(portfolio_returns)
  mad <- mean(abs(portfolio_returns - expected_return))

  return(mad)
}

# Portfolio Optimization MAD model
```

```
# Setting up the constraints
pspec_mad <- portfolio.spec(assets = colnames(train_data2))

# Constraints for the portfolio
pspec_mad <- add.constraint(portfolio = pspec_mad,  type = "full_investment")
pspec_mad <- add.constraint(portfolio = pspec_mad, type = "long_only")
pspec_mad <- add.constraint(portfolio=pspec_mad, type="box", min = 0.0, max =
0.5)


# Add custom MAD risk objective
pspec_mad <- add.objective(portfolio = pspec_mad, type = "risk", name = "mad_
risk_function")



# Optimize the portfolio using the random search method
set.seed(718)
optimized_portfolio_mad <- optimize.portfolio(R = as.matrix(train_data2), por
tfolio = pspec_mad, optimize_method = "random", search_size = 5000, trace = T
RUE)
```

A.3 Mean Semi Absolute Deviation Model

```
# Portfolio Optimization MSAD Model
# Define the Mean Semi-Absolute Deviation (MSAD) Function for Optimization

# Custom MSAD risk function
msad_objective <- function(R, weights) {
  # Ensure x is a matrix and weights are a vector
  if (is.data.frame(R)) {
    R <- as.matrix(R)
  }

  # Calculate portfolio returns
  portfolio_returns <- as.numeric(Return.portfolio(R, weights))

  # Calculate mean portfolio return
  expected_return <- mean.geometric(portfolio_returns)

  # Calculate Mean Semi-Absolute Deviation (MSAD)
  msad <- mean(pmax(expected_return - portfolio_returns, 0))

  return(msad)
}
```

```r
# Setting up the constraints for MSAD optimization
pspec_msad <- portfolio.spec(assets = colnames(train_data2))

# Add constraints
pspec_msad <- add.constraint(portfolio = pspec_msad, type = "weight_sum", min
_sum = 1.00, max_sum = 1.00)
pspec_msad <- add.constraint(portfolio = pspec_msad, type = "long_only")
pspec_msad <- add.constraint(portfolio = pspec_msad, type = "box", min = 0.0,
max = 0.5)


# Add MSAD risk objective with mean return as target
pspec_msad <- add.objective(portfolio = pspec_msad, type = "risk", name = "ms
ad_objective")

# Optimize the portfolio
set.seed(7111)
optimized_portfolio_msad <- optimize.portfolio(R = train_data2, portfolio = p
spec_msad, optimize_method = "random", search_size = 5000, trace = TRUE)
```

Appendix B

B. Codes of stocks used

Code	Company	Company Sector
BAT	British American Tobacco	Manufacturing and Allied
CRWN	Crown Paints Kenya Ltd	Construction and Allied
CTUM	Centum Investments Co Ltd	Investment
JUB	Jubilee Holdings Ltd	Insurance
KNRE	Kenya Re-Insurance Corporation	Insurance
ORCH	Kenya Orchards Ltd	Investment
SASN	Sasini Limited	Agriculture
SCOM	Safaricom	SCOM

Table B.1: Stocks considered in the optimization process