Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BACHELOR OF BUSINESS SCIENCE ACTUARIAL SCIENCE,
FINANCIAL ENGINEERING & FINANCIAL ECONOMICS
END OF SEMESTER EXAMINATION
BSM 3220: OPTIMIZATION METHODS IN FINANCE

Date: 5th December, 2024 **Time: 2 Hours**

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) State the strong duality theorem. [2 Marks]

(b) Write the following linear program in standard form. [4 Marks]

$$\begin{aligned} Min, \quad & x_1 + x_2 \\ subject \quad to: & \\ & x_1 + x_2 \geq 1 \\ & x_1 - x_2 \leq 0 \\ & x_1, x_2 \quad \text{unrestricted in sign.} \end{aligned}$$

(c) A company has the following short term financing problem.

Month	Jan	Feb	Mar	Apr	May	Jun
Net Cash Flow	-130	-90	150	-180	40	290

Net cash flow requirements are given in thousands of dollars. The company has the following sources of funds

- A line of credit of up to $100,000 at an interest rate of 1% per month,
- In any one of the first three months, it can issue 90-day commercial paper bearing a total interest of 2% for the 3-month period,
- Excess funds can be invested at an interest rate of 0.3% per month.

Formulate a linear program that maximizes the wealth of the company at the end of the month of June (do not solve). [7 Marks]

(d) State whether the given statement is TRUE or FALSE. [5 Marks]

(i) Unconstrained problems typically have interior solutions.

(ii) In a 0-1 integer programming problem involving a capital budgeting application (where $x_j = 1$, if a project is selected and $x_j = 0$, otherwise), the constraint $x_1 - x_2 \leq 0$ implies that if project 2 is selected, project 1 cannot be selected.

(iii) A local optimum of a linear program is a global optimum.

(iv) A linear program with an unbounded dual always has an optimal solution.

(v) A linear program with a bounded primal always has an optimal solution.

(e) A vet is treating a farm animal. He must provide minimum daily requirements of an antibiotic, a vitamin and a nutrient. He has two types of medicine available, tablets and liquid. The table summarises what the medicines contain and the requirements.

	Antibiotic	Vitamin	Nutrient
Tablet (units per tablet)	3	2	10
Liquid (units per dose)	2	4	50
Daily requirement (units)	18	16	100

The tablets cost \$0.38 each and liquid medicine costs \$1 per dose. The vet wants to find the cheapest way to treat the animal. By letting x and y represent the number of tablets which the vet prescribes per day, and the number of doses of liquid medicine, respectively, formulate and solve as a linear programming problem. [8 Marks]

(f) Given the following simplex tableau, perform the final pivot operation and read off the solutions. [4 Marks]

x	y	z	s_1	s_2	s_3	P	constant
$\frac{54}{11}$	$\frac{56}{11}$	0	1	0	$\frac{3}{11}$	0	$\frac{343}{11}$
$\frac{76}{11}$	$\frac{-32}{11}$	0	0	1	$\frac{3}{11}$	0	$\frac{200}{11}$
$\frac{7}{11}$	$\frac{4}{11}$	1	0	0	$\frac{1}{11}$	0	$\frac{30}{11}$
$\frac{25}{11}$	$\frac{-3}{11}$	0	0	0	$\frac{2}{11}$	1	$\frac{60}{11}$

QUESTION TWO (20 MARKS)

(a) Mr. Ollyin is buying new pillows for his hotel. He buys three types of pillows: soft, medium and firm.

He must buy at least 100 soft pillows and at least 200 medium pillows.
He must buy at least 400 pillows in total.
Soft pillows cost \$4 each. Medium pillows cost \$3 each. Firm pillows cost \$4 each.
He wishes to spend no more than \$1800 on new pillows.
At least 40% of the new pillows must be medium pillows.
Ollyin buys x soft pillows, y medium pillows and z firm pillows.

(i) In addition to $x \geq 0$, $y \geq 0$ and $z \geq 0$, find five inequalities in x, y and z that model the above constraints. [5 Marks]

(ii) Mr. Ollyin decides to buy twice as many soft pillows as firm pillows.

(A) Show that three of your answers in part (a) become [3 Marks]

$$\begin{aligned} 3x + 2y &\geq 800 \\ 2x + y &\leq 600 \\ y &\geq x \end{aligned}$$

(B) On the provided graph paper, draw a suitable diagram to represent Ollyin's situation, indicating the feasible region.
. [4 Marks]

(C) Use your diagram to find the maximum total number of pillows that Ollyin can buy. [2 Marks]

(D) Find the number of each type of pillow that Ollyin can buy that corresponds to your answer to part (C). [2 Marks]

(b) Put the linear programming problem into an initial simplex tableau: [4 Marks]

$$\begin{aligned} \text{Maximize} \quad & P = 15x - 23y + 35z \\ \text{Subject to:} \quad & 5x + 8y + 7z \leq 154, \\ & 5x + 8y + 7z \leq 154, \\ & 12x + 10y + 13z \leq 210, \\ & 11x + 14y + 26z \leq 611, \\ & 0 \leq x \leq 200, \\ & y \geq 0, \\ & z \geq 0. \end{aligned}$$

QUESTION THREE (20 MARKS)

The owner of a machine shop is planning to expand by purchasing some new machines-presses and lathes. The owner has estimated that each press purchased will increase profit by \$100 per day and each lathe will increase profit by \$150 daily. The number of machines the owner can purchase is limited by the cost of the machines and the available floor space in the shop. The machine purchase prices and space requirements are as follows.

Machine	Required Floor Space (ft^2)	Purchase Price
Press	15	\$ 8,000
Lathe	30	4,000

The owner has a budget of \$40,000 for purchasing machines and 200 square feet of available floor space. The owner wants to know how many of each type of machine to purchase to maximize the daily increase in profit.

(i) Formulate an integer linear programming model for this problem. [5 Marks]

(ii) Determine the integer solution to this problem using the branch and bound method. Display the B& B tree [15 Marks]

QUESTION FOUR (20 MARKS)

(a) Given the following simplex tableau, select the next pivot element, perform pivoting and read off the solutions. [6 Marks]

x	y	z	s_1	s_2	s_3	P	constant
4	8	9	1	0	0	0	260
-4	-6	9	0	1	0	0	80
2	1	4	0	0	1	0	390
-18	-13	-3	0	0	0	1	0

(b) Welte Mutual Funds, Inc., just obtained $100,000 by converting industrial bonds to cash and is now looking for other investment opportunities for these funds. Based on Welte's current investments, the firm's top financial analyst recommends that all new investments be made in the oil industry, steel industry, or in government bonds. Specifically, the analyst identified five investment opportunities and projected their annual rates of return. The investments and rates of return are as follows:

Investment	**Projected Rate of Return** (%)
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest Steel	6.4
Huber Steel	7.5
Government bonds	4.5

Management of Welte imposed the following investment guidelines.

- Neither industry (oil or gas) should receive more than $50,000.
- Government bonds should be at least 25% of the steel industry investment.
- The investment in Pacific Oil, the high-return but high-risk investment, cannot be more than 60% of the total oil industry investment.

The management is keen on portfolio recommendations that would maximize returns for the available $100,000. Formulate this problem as linear program problem (do not solve).

[14 Marks]

QUESTION FIVE (20 MARKS)

(b) Consider the following optimization problem:

$$\begin{aligned}
&\text{Choose} \quad c_u \quad \text{and} \quad c_d \quad \text{to} \\
&\text{maximize} \quad \frac{2}{3}\log(c_u - 3) + \frac{1}{3}\log(d_u - 3), \quad \text{subject to} \\
&\frac{4}{5}\left(\frac{1}{2}c_u + \frac{1}{2}c_d\right) = 6, \\
&c_d \geq 6 \quad \text{and} \quad c_u \geq 6.
\end{aligned}$$

This is a single-period choice of investment for consumption in a binomial model with translated log utility, initial wealth of 6, actual probabilities $2/3$ and $1/3$, risk-neutral probabilities $1/2$ and $1/2$, and risk-free rate of 25% (and therefore discount factor $4/5$).

(i) What are the objective function, choice variables, and constraints? [5 Marks]

(ii) What are the Kuhn-Tucker conditions? [8 Marks]

(iii) Solve the optimization problem. [7 Marks]

END OF PAPER