STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

BSM 3114- Differential Equations with Financial Applications

BBS (Financial Engineering)

Date: Wednesday, 19th August 2020 Time: $2\frac{1}{2}$ Hours

INSTRUCTION: Answer question ONE and any other TWO questions.

QUESTION ONE (30 Marks)

a) Classify the following differential equations as either linear/non-linear or homogeneous/non-homogeneous: [2 Marks]

i. $y'' + y'(Sin[t]) + ty + \frac{2}{y}e^t = 0$

ii. $y''Cos[t] + p(t)y' + q(t)y - r(t) = 0$

b) Given that $t(0) = 1$, find the particular solution by the method of variable separation:

$t' = \frac{t\ Sin[x]}{t+1}$ [4 Marks]

c) Solve the differential equation by direct integration: $\frac{d}{dx}[y\ f(x)] = g(x)$ hence evaluate $\frac{d}{dx}[y\ Cos(x)] = Cos(x)$ [5 Marks]

d) Solve by the method of integrating factor the differential equation

$(x^2 - 1)y' + 2xy = x$ [5 Marks]

e) Give the differential equation represented by the system $x' = -x + 6y$

$y' = x - 2y$ [3 Marks]

f) By making the substitution $v = x^2 + y^2$, find the general solution to

$yy' + x = (x^2 + y^2)^{0.5}$ [5 Marks]

g) Express in matrix form the system $x_1' = x_1 + 3x_2 + e^t$

$x_2' = 3x_1 + x_2 + 2e^{3t}$ [2 Marks]

h) Express the equation below as a system; $Pu'' + Qu' + Ru = S(t)$ by letting:

i. $x_1 = u$ and $x_2 = u'$

ii. $x_1 = u'$ and $x_2 = u$

iii. What can you say about your answers in i and ii above? [4 Marks]

QUESTION TWO [20 Marks]

a) Given that $x(0) = 1$ and $y(0) = 2$, solve the system $x' = 5x + 3y$, $y' = -6x - 4y$ [6 Marks]

b) Using the substitution $v = \frac{y}{x}$, solve $y' = \frac{y-4x}{x-y}$ [5 Marks]

c) By investigating the three possible cases $(\lambda < 0, \ \lambda = 0, \ \lambda > 0)$, find the non-zero functions y which are the solutions to $y'' + \lambda y = 0$ with $y(0) = 0$ and $y(t) = 0, t > 0$. [9 Marks]

QUESTION THREE [20 Marks]

a) Given the non-linear system $\frac{du}{dt} = u(v-1), \frac{dv}{dt} = 4 - v^2 - u^2$, determine; [10 Marks]

i. Its equilibrium points.

ii. The Jacobian matrices corresponding to each equilibrium point in i above.

iii. The eigenvalues corresponding to each Jacobian matrix.

iv. Whether the points are stable or unstable.

b) Two commodities in a company are defined by the non-linear system;

$$\frac{da}{dS} = -b - ab + b^2$$

$$\frac{db}{dS} = -7a + ab + 2a^2$$ where S represents time.

i. Find the relationship between the two commodities. [3 Marks]

ii. Plot the phase plane diagram and analyse it. [7 Marks]

QUESTION FOUR [20 Marks]

a) Classify the following as either semi-linear or quasi-linear; [3 Marks]

i. $u_t + u_x + u^2 = 0$

ii. $u_t + b(u)u_x = 0$

iii. $Sin(t) + uu_x + u_y - 2 = 0$

b) Find a solution to the equation $u_t + au_x = 0$ using the method of characteristics. [7 Marks]

c) Using an appropriate substitution, show that the equation $y' = H(rx + sy)$, with $r, s \in \mathbb{R}$ is actually separable hence solve $y' - (2x - 3y + 1)^2 = 0$ with $y(0) = 1$ [10 Marks]

QUESTION FIVE [20 Marks]

a) Using the method of separation of variables, solve the equation $u_t = 4xu_x$ given that $u(x,0) = 3Sin(\pi x) - 4Sin(10\pi x),\ 0 \leq x \leq 5$ [7 Marks]

b) Show that if $a \in \mathbb{R}$, then $u(x,t) = Sin(at)Cos(x)$ is a solution to $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$. [5 Marks]

c) Using the method of Laplace transform, solve $\frac{\partial W}{\partial x} + x\frac{\partial W}{\partial t} = 0$ subject to $W(x,0) = 0$ and $W(0,t) = t$ [8 Marks]