Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

MASTER OF SCIENCE IN STATISTICAL SCIENCES

END OF SEMESTER EXAMINATION

STA 8205: COMPUTATIONAL STATISTICS

Date: 5th May 2023 **TIME: 3 Hours**

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION 1 (30 Marks)

a. Briefly define the following terms (6 marks)
   a. Random Numbers.
   b. Simulation.
   c. Jacknief.

b. Given the following numbers, generate 3 random numbers (2 marks)

$$X_0 = 27, a = 7, c = 43, m = 100$$

c. 1000 random numbers were obtained using Linear congruential generators are shown in the table below.

Cell	Observed
1	100
2	96
3	98
4	85
5	105
6	93
7	97
8	125
9	107
10	94

Using the Chi-square test at 0.1 significant level, perform the appropriate test for these random numbers (7 marks)

d. Using $\mathrm{Poi}(2)$ distribution, simulate two random values given the following random numbers: 0.721 and 0.128 from $\mathrm{U}(0,1)$. (5 marks)

e. Suppose you want to find the integral of the following function:

$$\int_{-2}^{5} -x^3 + 6x^2 - x + 17dx$$

Describing the Monte Carlo integration, and using any 3 values for x, obtain the value of the integral. (7 marks)

f. Name any three random number generators (3 marks)

Question Two (15 Marks)

a. Outline the acceptance rejection method of sampling, hence or otherwise describe how you would draw $X \sim N(\mu, \sigma^2)$ (9 marks).
b. Assume that you have the following data set: $X_1, \ldots, X_n$. Let M_n be the median. Discussing what is bootstrapping, outline how you would obtain the bootstrap estimate of the variance, MSE and confidence interval of M_n (6 marks).

Question Three (15 Marks)

a. Suppose you have the following N =5 numbers: 0.44, 0.81, 0.14, 0.05, 0.93. Compare the CDF, $F(X)$ of the uniform distribution, with that of the empirical CDF $S_0(x)$, of the N sample observations using the Kolmogorov-Smirnov test (10 marks)
b. Briefly explain the expectation maximization algorithm (5 marks)

Question Four (15 Marks)

a. Variance reduction a is done in such a way that one can reach the same precision with lower numbers of draws or equivalently, a higher precision with the same number of draws.

Using the following integral as an example,

$$I = \int_0^1 e^x dx$$

and the following 3 draws from $\mathrm{U}(0,1)$: 0.44, 0.81, 0.14, discuss two variance reduction methods (15 marks)

Question Five (15 Marks)

a. Describe the Markov Chain Monte Carlo sampling technique (10 marks).
b. Simulate three random values from $Exp(0.1)$ distribution using the random values: 0.113, 0.608 and 0.003: Hint: The pdf of exponential distribution is: $f(x) = \lambda e^{-\lambda x}, x > 0$ (5 marks).

Kolmogorov-Smirnov Critical Values

Degrees of Freedom (N)	$D_{0.10}$	$D_{0.05}$	$D_{0.01}$
1	0.950	0.975	0.995
2	0.776	0.842	0.929
3	0.642	0.708	0.828
4	0.564	0.624	0.733
5	0.510	0.565	0.669
6	0.470	0.521	0.618
7	0.438	0.486	0.577
8	0.411	0.457	0.543
9	0.388	0.432	0.514
10	0.368	0.410	0.490
11	0.352	0.391	0.468
12	0.338	0.375	0.450
13	0.325	0.361	0.433
14	0.314	0.349	0.418
15	0.304	0.338	0.404
16	0.295	0.328	0.392
17	0.286	0.318	0.381
18	0.278	0.309	0.371
19	0.272	0.301	0.363
20	0.264	0.294	0.356
25	0.24	0.27	0.32
30	0.22	0.24	0.29
35	0.21	0.23	0.27
Over 35	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19