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Application of the Gumbel Copula for Economic Risk Aggregation: A Case of Kenyan Banks

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Reg. No: 072154

A Research Project submitted in partial fulfillment of the requirements for the Degree of Bachelor of Business Science in Actuarial Science at Strathmore University

December 2015

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Nairobi, Kenya

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VaR – Value at Risk

BIS – Bank of International Settlements

CDS – Credit Default Swaps

CDO – Collateralized Debt Obligations

KCB- Kenya Commercial Bank

NBK – National Bank of Kenya

DTB – Diamond Trust Bank

BBK – Barclays Bank of Kenya

CBK – Central Bank of Kenya

Kshs. – Kenyan Shillings

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Declaration

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Abstract

Commercial banks are some of the most heavily regulated institutions, not only in Kenya but also in other parts of the world. Increasingly, the Basel Accords have been revised to require banks to set aside enough funds to cover their risks. Commercial banks, however, face many risks chief among them market and credit risks. These risks tend to have different distributions which complicates the calculation of a firm-wide measure of risk to calculate economic capital. Many companies use value at risk as it is simple and easy to explain. However, value at risk is not wholly dependable due to the limitations associated with correlations and the fact that it does not fulfill the sub-additivity property. The copula function was therefore developed to deal with these challenges. This study applies the Gumbel copula in measuring dependence and economic capital using data from Kenyan banks.

Key words: Copulas Risk aggregation, Dependence, Value at risk, Gumbel copula

1 Introduction

1.1 Background

In the recent times the value and critical role of risk in enterprises has proved to be a major factor for evaluation in firms. This is because of its increasing importance in the determination of how a company's performance in terms of both profitability and operations is concerned. The downfall of many huge companies due to poor risk management e.g. Enron, Barings, Lehman Brothers and the Allied Irish Banks have reinforced the need for effective risk management.

The concept of risk and risk management however is not only attributable to modern times. According to Dunbar (2000), as early as 1800BC when the Code of Hammurabi was initiated in Babylon, there was indication of the use of the derivatives concepts i.e. options to provide financial cover incase crops failed.

The definition of risk however is what differs from time to time and from organization to organization. Risk is a central element in both insurance and finance for example. Individuals are assumed to be risk averse and thus would be willing to pay a premium over expected losses to reduce risk. How risk is considered in these two areas, however, is significantly different an actuary views risk as a component of an individual insured that cancels out at the level of the insurer due to the law of large numbers. The financial economist views risk as a combination of two factors, diversifiable risk that is irrelevant for pricing assets and systematic risk that enters into the asset pricing determination (Buhlmann, 1987). The management styles taken by these fields can therefore be substantially different.

Numerous theories and analyses have also been proposed on how to deal with risk e.g. scenarios (Laycock, Operational Risk Quantification: Scenarios, 2014), stress tests (Laycock, Operational Risk Quantification: Stress Tests, 2014), the maximum loss concept, subjective analysis (Banks & Dunn, 2003) , the Advanced Measurement Approach (Laycock, Operational Risk Management: The Advanced Measurement Approach, 2014) and the Value at Risk Measure. The Basel Accords have also been developed to regulate the capital requirements for financial institutions e.g. banks. The Basel Committee for the Banking Supervision of the Bank of International Settlements (BIS) has continually supervised these institutions to ensure that adequate capital is set aside to cover operational surprises (Tchernobai, 2006).

Lately firms have generally began measuring each risk separately by classifying them in the categories of operational, credit and market risk. These three measures are thereafter used to calculate the firm-wide measure (Nocco & Stulz, 2006) i.e. risk aggregation. This is because the recent financial crisis has proved that dependencies do exist between these risks; some of which have compounding effect. Credit risk and market risk for example can compound each other in some cases (Corrigan, Decker, Hoshino, Delft, & Verheugen, 2009). Risk aggregation, therefore, is the merging of different risk types into a single metric which provides a basis for assessing of economic capital. The risk aggregation models are useful because they are used by firms to support decisions about capital allocation and capital adequacy and solvency and also support risk management functions e.g. risk identification, monitoring and mitigation (Basel Committee on Banking supervision, 2010).

This approach, however, has three major problems that make it hard to effectively implement as proposed by Nocco and Stulz (2006) i.e. (1) the fact that these risks have different distributions, (2) the problem with estimation of correlations (indicates the strength and direction of a linear relationship between two random variables) due to non-subadditivity property of Value at Risk measure (VaR) (Artzner, Delbaen, Eber, & Heath, 1999) and the possibility of both model risk and a market liquidity problem (McNeil, Frey, & Embrechts, 2005) (3) the existence of non-quantifiable risks e.g. reputational and strategic risks; also classified as 'other risks' according to Tchernobai (2006).

The VaR measure; one of the most common measures used by companies; has an implication that total VaR is greater than firm wide VaR, the lesser value depending on the correlation between these risks. However since estimation of correlations is hard, this makes combining these marginal distributions not as straightforward as adding them up. In addition, the correlation measure is only a reasonable measure if the random variables are elliptically distributed and good if the random variables follow a multi-variate normal distribution. However, according to Nocco and Stulz (2006), the multivariate normal distribution is not appropriate, especially for credit and operational risks as the risks have fat tails. Correlation is also a scalar measure of dependence and therefore cannot tell us everything we would like to know about the dependence structure of risks. Thirdly, correlation is only defined where the variances of risks are finite and cannot be applied for heavy-tailed risks where the variances tend to infinity (Habiboellah, 2007).

These disadvantages therefore necessitate the need for an appropriate measure of dependence i.e. the copula.

The term 'copula' was first used in a mathematical sense by Abe Sklar in 1959 (Sklar, 1959). The Sklar's theorem states that 'an n-dimensional copula (briefly, an n-copula) is an n-place real function C with Dom C = J", Ran $C = I$, which satisfies the following conditions: (i) $C(l,\dots,1,x,_{m},1,\dots,1) = xm$ for each $m = n$ and all xm in I, (ii) $C(xu\dots,x_n) = 0$ if $xm = 0$ for any m ^ n, (iii) C is n-increasing (Sklar, 1959). In simple words, 'any joint cumulative distribution function **F** can be written in terms of a copula and marginal cumulative distribution functions. If the marginals are continuous then the copula is unique for **F**' (Habiboellah, 2007). The copula is the information missing from the individual marginal to complete the joint distribution i.e. joint = copula + marginal. It is useful for measuring the dependence among different risks. The copula is preferable because (1) it helps in understanding dependence at a deeper level; (2) it expresses dependence on a quantile scale, useful for describing dependence in extreme outcomes and natural in a risk-management context (McNeil, Frey, & Embrechts, 2005). This study is therefore going to look at how to achieve this dependence measure using copulas especially for the banking sector.

1.2 Problem statement

The Kenyan banking, insurance and other financial sectors encounter risks as an inherent part of their operations. The Central Bank of Kenya has therefore formulated guidelines for banks and other financial institutions to follow so as to ensure stability and efficiency. These guidelines stipulate the stress testing requirements for financial institutions under the Prudential Guideline number 20 for banks. However, the exact methods to use when conducting stress test are not stipulated thus requiring financial institutions to their own methods. Consequently, for economic capital allocation, this generates the need for a tool that measures these risks and determines their interrelation for establishing the overall capital requirement to maintain stability.

The correlation approach was the more common method used. However since it is riddled with shortcomings researchers such as Nguyen and Molinari (2011) have proposed the use of copulas. These establish dependencies between the different risks affecting an institution thereby generating an overall loss distribution for the firm. The overall distribution can then be used as a basis to determine the overall reserving levels for the institution. This study, therefore applies these copulas, in particular the Gumbel copula, to establish dependence levels and to determine how to generate the overall loss distribution based on these dependencies for the Kenyan commercial banks.

1.3 Research objectives

To use copulas to calculate dependencies between market and credit risks.

To determine the overall loss distribution for use in determining the overall capital allocation for Kenyan commercial banks.

1.4 Research questions

How can copulas be used to calculate dependencies for the risks in Kenyan banks?

How can these dependencies then be used to calculate the overall loss distribution to be used for capital allocation in Kenyan commercial banks?

1.5 Significance of the study

Calculating the general risk tolerance level for the whole firm is a challenge as it involves combining all the different risks facing the firm that are quantifiable. These risks majorly operational, credit and market as classified by most firms all have different distributions that cannot be combined directly. According to Nocco & Stulz (2006), market risk behaves similar to the returns on a portfolio of securities which have a 'normal' or symmetric distribution; credit risk has an asymmetric distribution since a creditor will either pay or not while operational risk is also asymmetric but with a longer tail since there tends to be a large number of small losses but also a chance for large operational losses. Simply adding up the risks could therefore result in an over-estimate due to the non-subadditivity property while the value at risk measure involves use of correlations that have numerous limitations.

Problems associated with the correlation approach have made copulas a more dependent measure for dependence analysis. A copula can therefore be used, on the sell side; to price derivatives and on the buy side; to model portfolio risk in finance. This is because copula models (i) allow for greater flexibility in defining dependence than the linear risk aggregation models (Skoglund, 2010), (ii) allows the original shape of the sub-risk distributions to be retained and (iii) allow for the specification of more general dependence models than the normal dependence models.

In addition to banking, the copula can also be effectively used in insurance to adequately model dependent insurance portfolios so as to evaluate the overall risk exposure and capital allocations especially with the increasing alignment towards capital-based models. The fact that most insurance companies are moving towards securities-linked products also reinforces the importance of risk and effective risk aggregation methods are therefore required to accurately predict the risk levels.

2 Literature review

2.1 Risk and risk management

Risk is a central element for many financial institutions. It is generally defined as 'uncertainty regarding a specific outcome' (Tchernobai, 2006). In the financial sector these risks have mainly been classified into three major areas i.e. operational, credit and market risk with each of the sub-risks falling under these three categories. Valuation is also done with respect to each classification with most firms having a department set aside for the sole purpose of risk management and control. The Basel Accords, the Solvency Guidelines and for Kenya; the Prudential Guideline number 20 for Banks just reinforce how important risk management is. Coupled with the experiences during the 2008 financial crisis e.g. collapse of the Lehman Brothers, Merrill Lynch and Bear Sterns; collapse in credit rating of Icelandic banks; steady declines in equity and property bear markets and the decline in risk free rates, financial institutions ultimately saw the need for effective risk management.

Risk Management is defined as 'a process that identifies loss exposures faced by an organization and selects the most appropriate techniques for treating such exposures' (Rejda, 2008). It involves identifying the loss exposures, analyzing them, selecting the most viable techniques of dealing with them and adequate monitoring and implementation of the process. This is done through a process known as enterprise risk management, which enables management to effectively deal with uncertainty thus have a greater capacity to build value (Committee of Sponsoring Organizations of the Treadway Commission, 2004).

2.2 Risk measurement and the link to copulas

Risk can be measured using many different ways depending on one's objective and goal. Different risk metrics and models have been introduced over the years each with the aim of achieving a better more comprehensive evaluation of risk than the last. These metrics can either be linear or non-linear depending on the variables with the loss models being developed in line with the types of business being undertaken as proposed by Yang (2012). The linear types of risk metrics are mainly associated with correlations and regression measures while the non-linear measures are associated with portfolios that cannot be aggregated directly.

The Pearson's linear correlation is the main metric in the linear category and it measures only linear stochastic dependency of two random variables and can only take values between +1 and -

1 (Nguyen & Molinari, 2011). The other types of ratios are the Spearman's rank, the Kendall's tau, and the coefficient of tail dependence which have the advantage of being copula-based i.e. they can be expressed in terms of the underlying copula alone (Aas, 2004). These three measures, however, both have their own shortcomings as evidenced by Nguyen & Molinari (2011). The Pearson's linear correlation for example, does not always apply since it is not in all cases that the perfectly positive correlated random variables represent a correlation of positive one and vice versa for the perfectly negative correlated random variables. In addition, random variables with strong dependence may represent a correlation coefficient that is according to an amount close to zero. Also, extreme events with high losses can be severely underestimated by using this measure for dependencies (Szego, 2002). The Spearman's rank and Kendall's tau do not give full information about dependencies between random variables as it reduces the information to a single number. The coefficient of tail dependence also has the problem of not giving full information on the dependence structures.

Due to these shortcomings most firms are moving towards the non-linear measures. However, the non-linear measures still pose the problem of aggregating combined portfolios. This therefore introduces the problem of risk aggregation i.e. incorporating multiple types or sources of risk into a single metric as per the Basel Committee for the Banking Supervision. According to Yang (2012), there are four common approaches to calculate the combined loss distribution tail. They include (1) the use of direct simulation, (2) making use of some assumptions regarding loss distributions, (3) use of various approximations e.g. Delta-Gamma, Cornish-Fisher and Saddle Point and (4) the use of copulas which is the most popular and flexible.

2.3 Capital allocation and economic capital

Capital allocation is a crucial process in a firm's operations because it could determine just how successful a firm is in terms of generating returns and also in risk management. This is why research is frequently being conducted to improve on the capital allocation processes in a firm and the determining factor as to how much capital should be allocated and where it should be allocated. The Basel Capital Accords were also developed due to the need for effective capital allocation (Basel Committee on Banking Supervision, 2007).

Capital can also be subdivided into classes; economic capital being one of the most important since it is the 'amount set aside to cover losses that could occur from the firm's risk tolerance level' (Guegan & Jouad, 2012) or according to Dr. Yimin Yang, it is the 'amount of extreme losses that exceed expectations'[1] (Yang, 2012), thus associating it with both risk related returns and total risk calculations. The economic capital level should therefore be calculated using a single-loss model that is responsive to all risk factors and covers all portfolio products.

Through risk aggregation a firm can determine the overall capital allocation for the firm to cover the different risks that the firm will face, especially the quantitative risks. For insurers, these dependencies have long been recognized as primary factors driving the aggregate loss process (Tang & Valdez, 2006). In the past, however, the focus has mainly been on the use of linear correlations which, as shown before have numerous shortfalls.

2.4 Introduction to copulas

The word 'Copula' is a Latin noun meaning a 'link, tie, bond' (Cassell's Latin Dictionary). Copulas, as a dependence measure, have been in existence for quite some time since the first paper explicitly relating copulas to the study of dependence among random variables was published in 1981 (see (Schweizer & Wolff, On nonparametric measures of dependence for random variables, 1981)). First introduced by Sklar (1956), the copula has proved to be a very useful tool in financial applications. In defining a copula, Meucci (2011) summarized that it is the 'information missing from the individual marginal distributions to complete the joint distribution' or the dependence function. It has also been defined as 'functions that join or " couple multivariate distribution functions to their one dimensional marginal distribution functions, or alternatively as 'multivariate distribution functions whose one-dimensional margins are uniform on the interval (0, 1)' (Nelsen, 2006).

Other researchers define a copula as a tool that is 'used to separate the pure randomness of one variable (for example, a financial asset) from the interdependencies between it and other variables. By doing so, one can model each variable separately and, in addition, have a measure of the relations between those variables' (Rachev, Stein, & Sun, 2009). Copulas have therefore been seen as the factor that combines marginal distributions to get a joint distribution i.e. the 'glue' between the marginals.

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[1]Dr Yang terms economic capital as something 'extreme'; normally associated with a preset loss tolerance level (a so-called confidence level) and is determined by the tail of the loss distribution; while Guegan and Jouad view it as an preset amount depending on the tolerance level already available. These two definitions, in essence, are the same since we use the tolerance level of the company to calculate the economic capital in both cases.

Through the Sklar's theorem, one could be able to fit copulas to empirical data by the use of maximum likelihood since it provided the probability density function of the copula from the joint and marginal distributions. Copulas' application in finance, however, was introduced much later by Embrechts et al (1999). Subsequently this was also put in practice by Li (2000) when he used the Gaussian Copula to price derivatives i.e. Collateralized Debt Obligations. This way highly complex risks were modeled much faster and with greater flexibility since he was able to model default correlation without even looking at historical default data (Li, 2000). He used market data for credit default swaps to check default risk because when the price of a credit default swap goes up, it indicates that default risk has risen. This meant that instead of using actual historical default data which is hard to find, data on the market prices of credit default swaps (CDS) could be used instead assuming that the CDS markets were pricing default risk correctly.

Other researchers also used copulas to model portfolio risk (see (Meucci, Gan, Lazanas, & Phelps, 2007)) thus reinforcing the importance of copulas. They have also been used for risk aggregation (see (Nguyen & Molinari, 2011) and (Koll, Kurth, & Vogt, 2011)), aggregation of economic capital (see Yang (2012)) or to measure the dependence between alternative risk transfer products (Blum, Dias, & Embrechts, 2002) and also, with time, using copulas to define the correlation structure between market variables (Hull, 2006).

Copulas are also being used in many other ways according to Nelsen (2006) e.g. to express Chapman- Kolmogorov equations for the transition probabilities in real stochastic processes as a product of copulas (Darsow, Nguyen, & Olsen, 1992) thereby enabling researchers dealing with Markov Processes to 'capture the Markov property of such processes in a framework as simple and perspicuous as the conventional framework for analyzing Markov chains (Schweizer, 1991)'

2.5 Classifications of copulas

There are two major classifications of copulas; the elliptical copulas and the Archimedean copulas (see e.g. (Nguyen & Molinari, 2011) and Skoglund (2010)).

2.5.1 Elliptical copulas

The elliptical or implicit copulas (Aas, 2004), which include the Gaussian (Li, 2000) as well as the Student-t copulas are more popular with scenarios involving the Monte Carlo simulation process ((Koll, Kurth, & Vogt, 2011) as evidenced by Trivedi & Zimmer (2005). These copulas are termed so due to the fact that they do not have a previously implied form but are derived or implied by already known or existing multivariate distributions. Their implementation has followed the following steps traditionally according to Meucci (2011); (i) Monte-Carlo scenarios are drawn from elliptical or related distributions, (ii) the scenarios are channeled through the respective (quasi-)analytical marginal cumulative distribution functions subsequently getting the grade scenarios, (iii) the grade scenarios are then then fed into flexible parametric quantiles thus obtaining the desired joint scenarios.

Here the Gaussian copula is that of the normal distribution while the Student-t copula is that of the student-t distribution. The elliptical copulas generally have the following format;

$$C_E(u_1, \dots, u_n : \alpha) = H_{\alpha^{(n)}}(H^{-1}(u_1), \dots, H^{-1}(u_n)) \quad (1)$$

where the multi-index α denotes the parametrical dependence upon a covariance matrix $\sum$ and, additionally for the Student-t copula, the degree of freedom v (Koll, Kurth, & Vogt, 2011). $H^{(n)}$ represents the n-dimensional Gaussian or Student-t distribution, H standing for their univariate counterparts.

For the Gaussian copula, the dependency in the tails of a distribution that is multivariate with a Gaussian copula goes to zero (Embrechts, McNeil, & Straumann, 2002) which generally implies, according to Nguyen & Molinari (2011) that the single random variables of the joint distribution function are almost independent in case of high realizations. This therefore makes modeling of insurance risks using the Gaussian copula inappropriate.

The Gaussian copula has also been blamed for causing the economic crisis due to its popularity at that time in valuing Collateralized Debt Obligations (CDO's). This was because the copula function allowed no space for unpredictability since the correlation function was assumed to be constant despite the fact that in the real world this was not the case. Its focus on CDS markets which had only been existence for a short period of time could also have been another of its problems because the CDS market had been existence when the house prices had soared creating the illusion that default risk was low. In addition, the Gaussian copula lacked tail dependence thus the probability of extreme events was not adequately modeled. However, researchers e.g. Yang (2011) have criticized this deduction citing human ignorance of the assumptions and risks associated with market models.

Student-t copulas however do not feature independence in the tails of the distribution (Tang & Valdez, 2006). This copula allows for joint fat tails and an increased probability of joint extreme events in comparison to the Gaussian copula (Aas, 2004). According to Hull (2006), when modelling the Student-t copula, the variables of interest are mapped onto new variables that are assumed to have a student-t distribution, the mapping being 'fractile-to-fractile' as for the Gaussian copula.

2.5.2 Archimedean copulas

The Archimedean copulas which include the Frank, Clayton and Gumbel copula are non-elliptical copulas. Yang (2012) considers this a very rich class with broad applications. They are also easy to construct and have many classes of copulas within it; in addition to the fact that they have many positive properties (Nelsen, 2006). Since elliptical copulas do not have 'fat tails' which is a common phenomenon with large losses, Archimedean copulas come in handy at this level. The Clayton copula, for example, has a 'thin tail', the Frank has 'no tail' while the Gumbel has a 'fat tail'; making each of these copula suitable for different distributions.

The Gumbel copula is similar to the Student-t copula in that they both are tail-dependent. The difference comes in because the Gumbel is only tail dependent in the upper tail (see Nguyen & Molinari (2011)). This makes these copulas useful when modelling extreme events; either those with high losses and high dependence or those with lower or common losses and appear independent (Zwiesler, 2005).

The Frank copula is completely tail-dependent [(Junker & May, 2005) and (Venter, 2002)]; with the dependence structure being similar to that of the normal copula even though the dependence in the tail is even lower. The Clayton copula, which was first introduced by Clayton in 1978 is a solution to the fact that the student-t copula does not allow for asymmetries (see Aas (2004)). It follows the following equation as retrieved from Skoglund (2010);

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1} \tag{2}$$

And as θ approaches 0 we obtain the independence copula, whereas as θ approaches ∞ we obtain the perfect dependence copula; where θ is the copula parameter restricted on the interval $(0, \infty)$. If θ=0 then the marginal distributions become independent; when $\theta \rightarrow \infty$ the Clayton copula approximates the Fréchet-Hoeffding upper bound (Clayton, 1978). Due to the restriction on the dependence parameter, the Fréchet-Hoeffding lower bound cannot be reached by the Clayton copula. This suggests that the Clayton copula cannot account for negative dependence (Clayton, 1978).

According to Meucci (2011), for larger markets, one has to narrow down to formulas that are mainly available in the elliptical class i.e. analytical and quasi-analytical formulas. However, one can draw scenarios directly from parametric copulas, which are limited to the Archimedean copula class and a few extensions. However the parameters of the Archimedean family are not easy to interpret and it is also hard to simulate when the N is large. This therefore serves to prove that both are important and have their strongpoints and weaknesses.

Many researchers have analyzed copulas from their own perspectives and come up with different formulas and designs of calculation. One thing that is evident, however, is that when analyzing the different types of risks it is imperative that the distribution of each risk be determined first so as to determine which type of copula is most appropriate for modeling such a risk. Estimation of parameters can then follow from this to get the appropriate aggregated risk measure.

3 Research methodology

3.1 Introduction

The research methodology explains the processes undertaken in answering the research questions and fulfilling the research objectives. This involves methods ranging from choosing the research sample, the research design, data collection methods and data analysis and synthesis methods

Research sample

The sample comprises data from the ten Kenyan banks listed in the Nairobi Securities Exchange. These are;

Table 1: Sample of Commercial banks

Bank	Assets (million kshs.)[2]	Market share
Kenya Commercial Bank (KCB)	304,112	13.1%
Equity Bank	215,829	9.3%
Co-operative bank	199,663	8.6%
Standard Chartered Bank	195,493	8.4%
Barclays Bank of Kenya (BBK)	185,102	7.9%
CFC Stanbic Bank	135,378	5.7%
NIC Bank	101,722	4.4%
Diamond Trust Bank (DTB)	94,512	4.1%
I&M Bank	91,520	3.9%
National Bank of Kenya (NBK)	67,155	2.9%

The data includes financial statements ranging as far back as 10 years i.e. from 2005 to 2014, in order to get as many data points as possible and observe trends.

[2] Retrieved from http://fortuneofafrica.com/kenya/assets-and-market-share-of-commercial-banks/

3.2 Research design

The study type is exploratory. An exploratory study is one in which a researcher has an idea or has observed something and seeks to understand more about it. An exploratory research project is an attempt to lay the groundwork that will lead to future studies, or to determine if what is being observed might be explained by a currently existing theory[3].' The approach will therefore be exploratory as we seek to understand more about application of copulas and expand their use to the Kenyan banking sector.

3.3 Data collection methods

The data used is collected from financial statements of the specific banks that can be retrieved from the company websites together with the Think Business Banking Survey East Africa for the 10 years as explained above. To increase the data points, 2000 simulations are created using Monte Carlo simulation. The main information required is the different risk levels for the banks.

3.4 Data analysis

The aim of the analysis techniques is to calculate an overall risk measure that integrates credit and market by considering the dependence between these risks. Operational risk will not be included for now as it poses challenges in modelling. The basic idea therefore follows the following steps as proposed by Li (2012); First, the marginal loss distributions for credit and market are determined from the data; a risk correlation matrix and copula function are then used to describe the risk correlation; Thereafter, Monte Carlo simulation is applied to generate the respective risk losses and finally the copula function is applied to the simulated data to get the Total Value at Risk (VaR) and consequently the Total loss distribution (Li J. , 2012).

3.4.1 The copula model

Assume the total assets in a bank are denominated *a*, the total risk of all the losses will be:

$$Y = aX_1 + aX_2 + aX_3 \tag{3}$$

Where Y is the total risk and X_1, X_2 and X_3 credit, market and operational loss rate respectively. Assuming X_1, X_2 and X_3 have the marginal cumulative distribution F_1, F_2 and F_3 respectively.

The random vector for the loss rate $X^T = (X_1, X_2, X_3)$ has a dependence structure characterized by its joint distribution using the copula function. According to Tang & Valdez (2006); a copula

[3] Read more: http://study.com/academy/lesson/purposes-of-research-exploratory-descriptive-explanatory.html

function can be used to get the joint distribution as it links the cumulative marginal distributions together i.e.

$$F(X_1, X_2, X_3) = C\ [F(X_1), F(X_2), F(X_3)] \quad (4)$$

The joint distribution is divided into two parts; one specifying the marginal distribution and another the dependency structure. This study uses credit and market risk as the main variables.

The data from financial statements is used to estimate the risks i.e. credit risk and market risk. Operational risk poses difficulties in modelling since it is highly behavioral; meaning it is hard to predict for example; who will commit fraud, for what amount and when such an incident will occur. This leaves credit and market risk. The variables under each main variable, according to Li (2012), are 1) income on investment 2) profit and loss from fair value changes 3) exchange gains and losses and 4) net interest income to model market risk and loan provisions to model credit risk.

Due to the fact that banks have different asset sizes, the earnings are converted to a return based measurement for direct comparison by dividing risk earnings by total assets i.e.

$$r_{i,t} = \frac{y_{i,t}}{Asset_{i,t}} \quad (5)$$

Assuming that y are the risk earnings at a given time t for an asset i, giving r which is the return on asset measure which determines the risk level for a specific bank.

After this we introduce the copula measure. Here we use the Gumbel copula, an Archimedean copula which is advantageous in that the copula function C is explicitly given. The Gumbel copula also has the advantage of compatibility with fat-tail distributions since it has a fat tail and sensitivity to loss correlations. In the Gumbel copula heavier losses are associated with higher correlations which is desirable because in the real world when the situation is getting worse, things tend to go bad altogether (Yang, 2012). This copula, however, has the limitation that the copula parameter must always be greater than 1.

Given u and v as the main univariate variables, the copula function C can be written as:

$$C_{(u,v)} = exp(-(-\ln u)^{\theta} + (-\ln v)^{\theta}) \quad (6)$$

Where Θ > 1 is the parameter which controls the behavior of the copula including correlation between the losses.

First get the Kendall's tau (τ), which can be calculated as:

$$\check{\tau} = \frac{2}{N(N-1)} \sum_{i<j} sign((x_i - x_j)(y_i - y_j)) \quad (7)$$

Here x and y are the risk measures and n is the number of time periods.

If u and v are independent uniform distributions and $W = C(u, v)$, we define a function $K(w) = Prob\ \{W < w\}$. For the Gumbel copula,

$$K(w) \ = Prob\{W < w\} = w - \frac{1}{\theta} wlnw \quad (8)$$

The function $K(w)$ can be estimated by:

$$\widehat{K(w)} = \frac{The\ number\ of\ z_i\ such\ that\ z_i \leq w}{N} \quad (9)$$

Where $z_i = \frac{The\ number\ of\ pairs\ (x_j,y_j) such\ that\ x_j < x_i\ and\ y_j < y_i}{N-1}$ (10)

We can now estimate the parameter Θ in two ways. This study uses the first method for estimation.

Based on Kendall's tau,

$$\check{\theta} = \frac{1}{1-\check{\tau}} \quad (11)$$

Based on $K(w)$ estimate,

By minimizing the $K(w)$ function with its estimator using $\int_0^1 (K(w) - \widetilde{K(w)})^2 dw$ to obtain;

$$\check{\theta} = -\frac{4}{27 * \alpha^*} \quad (12)$$

3.4.2 Copula Simulation Process

Simulate copula pairs $\{(u_i, v_i)\}$ using Θ and the following steps (Yang, 2012):

Step 1: Draw two independent uniform variates u' and v'.

Step 2: Calculate $S_* = -\ ln(\theta - 1) + ln(-ln\grave{u}) - \frac{1}{\theta-1} ln(\grave{u}\grave{v})$ (13)

Step 3: Solve $z + \ln z = S_*$ for z using the iteration formula

$$z_n = \frac{1 - ln z_{n-1} + S_*}{1 + \frac{1}{z_{n-1}}} \tag{14}$$

Step 4: Calculate $v = exp\{-[((\theta - 1) * z)^\theta - (-ln\grave{u})^\theta]\}$ (15)

Step 5: $u_* = \grave{u}$ and $v_* = \grave{v}$; generate a desired copula pair $(u_* v_*)$

Step 6: Repeat steps one through five for more pairs

3.4.3 Economic Capital Aggregation

For each copula pairs $p_* = (u_*, v_*)$, find the loss L_U from L_X associated with the percentile u_*, and the loss L_V from L_Y associated with v_*.

We then calculate $L_{P^*} = L_U + L_V$ (16)

$\{L_{P^*}\}$ for all pairs of (u_*, v_*) is the aggregated loss distribution

The copula code is directly implemented in R (See Appendix 1)

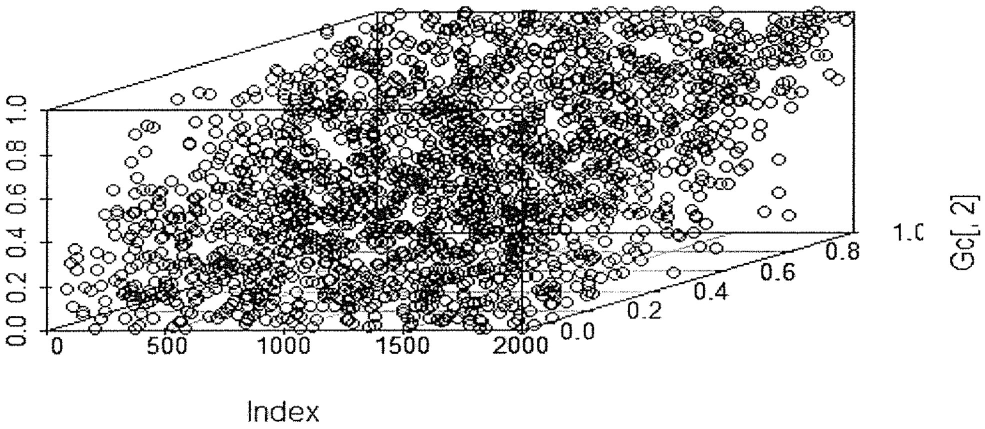
For this study, the R code for the gumbel copula is used instead of the formula. First the scatterplots in 3 dimensions were derived to show the gumbel distribution tail. The uniform variates are also derived from the data based on a simulation using Sklar's theorem. After this the copula dependence parameter is calculated using a maximum likelihood estimate. Thereafter the distributions i.e. the cumulative distribution functions, probability distribution functions, a simulation plot and contour graphs are derived. The code used for the maximum likelihood parameter is given in the appendices (see Appendix 2)

This dependence parameter is then used to get the value at risk at different percentiles.

4 Research findings

The plots below are given for the overall banking sector in Kenya. These plots serve to show how the gumbel copula is distributed and the tails of this distribution.

4.1 Figure 1: 3 Dimensional scatterplot for the overall banking sector

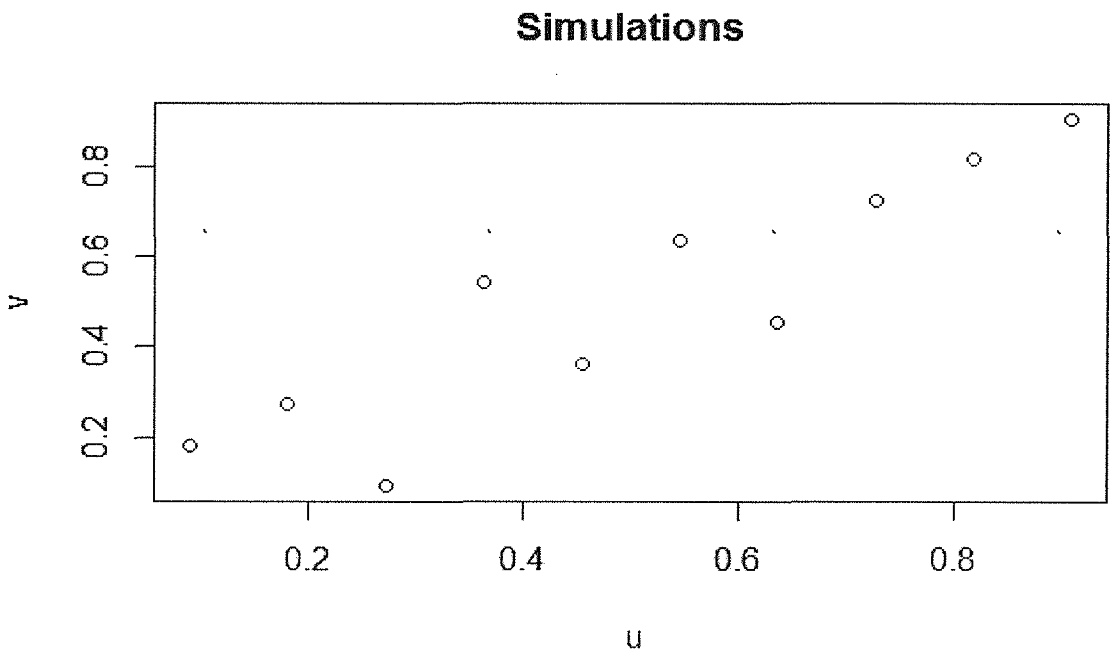


The scatterplot shows a gumbel copula simulation. The simulation code is given in the appendices. It is noticeable that the higher tail is more defined that the lower tail i.e. the property of the gumbel copula heavily skewed towards the right tail instead of the left tail.

4.2 Summary of the cumulative distributions, contours and Simulations

The simulation code for the overall banking sector is given in Appendix 3

Figure 2: Simulations for the overall banking sector



The simulations show a positive dependence between the variables.

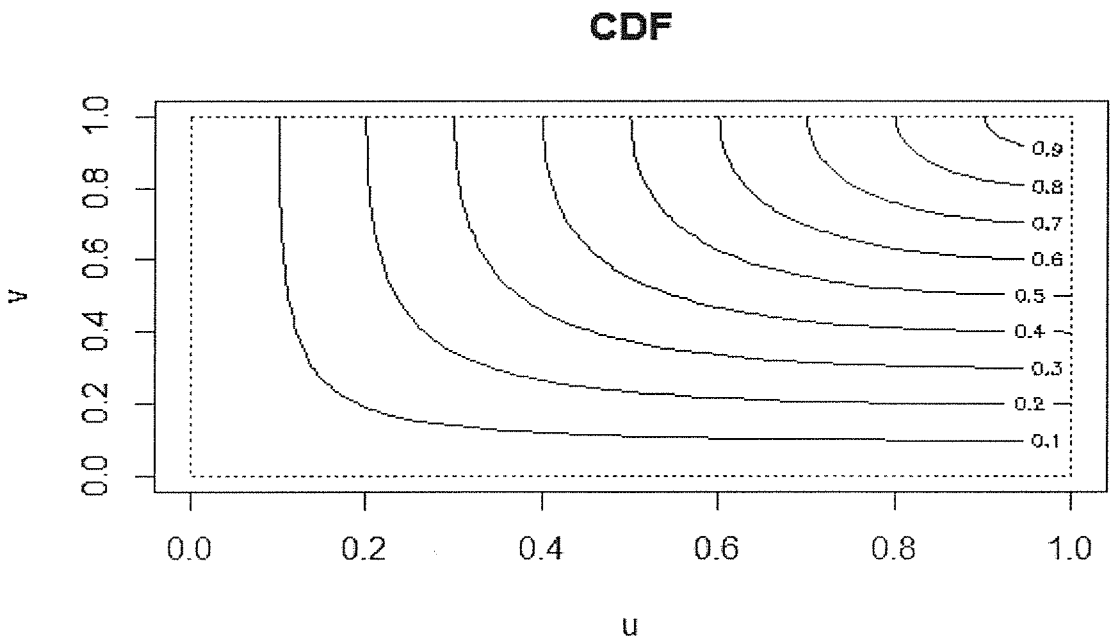
Table 2: Kendall's tau and Parameter values for the 10 banks

Company	Kendall's tau	Theta
Barclays Bank	-0.3778	0.7258
CFC Stanbic Bank	0.6	2.5
Co-operative Bank	-0.6444	0.6081
Equity Bank	0.9111	11.25
KCB	-0.5111	0.6618
Standard Chartered Bank	0.0667	1.0714
DTB	0.7333	3.75
I&M Bank	0.6444	2.8125
NBK	-0.4667	0.6818
NIC Bank	0.9556	22.5

From the table it is evident that Barclays Bank, Co-operative Bank, KCB and National Bank of Kenya all have negative taus. This means that we cannot get their dependence ratios based on the gumbel copula. We therefore use the six banks left for the analysis.

The Gumbel copula cannot be used for data that is negatively correlated, i.e. data with the Kendall's tau being less than zero. This means that the parameter theta must be greater than 1 for each company for the gumbel copula to work. The table below gives the Kendall's correlation and theta variables for each risk. The banks with a negative tau will therefore not be included in the analysis as the Gumbel copula cannot be used on them.

Figure 3: Contour plots for the cumulative density function for the overall banking sector



The contour plots above for the CDF represent the three dimensional relationship in a two dimensional manner. Notice the widening of the plots over the higher values of v. The tail dependence for a gumbel distribution is more heavily inclined upwards i.e. at the extremes.

4.3 Dependence function and Value at Risk

Table 3: Dependence Functions and Value at Risk for the chosen Banks

Company	Dependence function	Value at Risk	
		1%	7.5%
CFC Stanbic Bank	3.027	89	150
Equity bank	14.55	130	310
Standard Chartered	0.9245	509	778
DTB	8.344	288	360
I&M Bank	3.267	150	174
NIC Bank	22.63	548	611
Overall banking sector	9.616	5434	6573

The overall banking sector has a higher value justifiably because it is a conglomerate of all the banks. The banks with high dependence functions e.g. Equity also have a lower value at risk if their sizes are taken into account. In addition, those with low dependence functions e.g. Standard Chartered also seem to have considerably higher Value at risk. It is notable, however, that some of the variables seem to be moving in line with the dependence function instead of being lower e.g. NIC bank. This could mean that there are other factors other than the dependence structure that could influence the Value at Risk e.g. volatility, exposure, risk management measures etc.

Those companies with a higher dependence structure, however, should benefit from reduced Value at Risk due to the diversification effect.

5 Conclusion and points for further study

Comparing the 1% Value at Risk with the average capital using traditional means;

Table 4: Comparison with traditional methods

Bank	1% Value at Risk	Average capital - traditional
CFC Stanbic Bank	89	7,858.7
Equity Bank	130	15,378.2
Standard Chartered Bank	509	12,837.1
DTB	288	6,098.3
I&M Bank	150	6,554.6
NIC Bank	548	6,456.8

The figures under the traditional approach are much higher than those for the copula VaR. This could be because of the fact that operational risk was excluded in the analysis; in addition to the diversification increase. The diversification effect therefore decreases the overall capital requirements. What portion exactly has been reduced could be a point for further study. In addition, all the factors leading to low consistency levels could also be investigated further and allocated accordingly.

The gumbel copula is a useful tool for integrating risks especially for extreme events and for positively correlated variables. For variables sensitive to tail losses and more sensitive to large than small losses, the gumbel copula fits perfectly as the copula to use. In addition to its pre-specified parameter value, it is easier to calibrate and use. When it comes to negative correlations, however, the applicability of the gumbel copula diminishes. It is therefore important that for future study, different copulas will be analyzed and applied in each scenario.

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6 Appendices

6.1 Appendix 1: Copula code for the Gumbel Copula

#The code is tailor made from Mwamba (2015) using a dimension of 2 (dim=2) i.e. a data set made up with 2 columns (time series). To #adopt the programme to the data you must change the dim, change the value in the loop=i=1,2; #change the parameters in mu.gpd, sigma.gpd and x.gpd.

```
Library(copula)

rStable=function(Alpha=2,Beta=0,g=1/2^0.5,d=0,n=1)

{

#uniform variate Q

  a = -pi / 2

  b = pi / 2

  Q =(b - a) * runif(n) + a

#Exponential variate W

  w = -log(1 - runif(n))

#Stable(alpha,beta,1,0)

  A1 = 1 / Alpha

  q0 = atan(Beta * tan(pi * Alpha / 2)) * A1

  z = sin(Alpha * (q0 + Q)) / (cos(Alpha * q0) * cos(Q)) ^ A1 * ((cos(Alpha * q0 + (Alpha - 1)
* Q)) / w) ^ (A1 - 1)

   #Stable(alpha,beta,g,d)

S = g * z + d
```

```
S
}
#----------------------------------------
#----------------------------------------
GumbelCopula= function(n=1,theta,dim=6)
{
U=matrix(runif(n*dim),n,dim)
Y=matrix(rStable(1/theta,1,(cos(pi/(2*theta)))^theta,0,n),n,1)
s=matrix(0,n,dim)
   for (i in 1:dim)
   {
   s[,i]=-log(U[,i])/Y
   }
X=exp(-(s^(1/theta)))
X
}
#----------------------------------------
# Example
# theta=1.931663, dim=6, n=2000
#----------------------------------------
Gc=GumbelCopula(2000,1.931663,6)
```

```
kendall.tau=matrix(0,6,6)

mu=vector("numeric",6)

sigma=vector("numeric",6)

maxi=vector("numeric",6)

mini=vector("numeric",6)

for (i in 1:6)

{

mu[i]=mean(Gc[,i])

sigma[i]=sd(Gc[,i])

maxi[i]=max(Gc[,i])

mini[i]=min(Gc[,i])

   for (j in 1:6)

   {

   if  (i==j)  kendall.tau[i,j]=1  else  kendall.tau[i,j]=as.numeric(cor.test(Gc[,i],Gc[,j],method
="kendall")$estimate)

   }

}

#---------------------------------------

# tau

#---------------------------------------

theta=1.931663
```

```
tau= 1-(1/theta)

tau

print.noquote("mean:")

print.noquote(mu)

print.noquote("estdev: ")

print.noquote(sigma)

print.noquote("max: ")

print.noquote(maxi)

print.noquote("min: ")

print.noquote(mini)

print.noquote("kendall tau matrix: ")

print.noquote(kendall.tau)

print.noquote("tau: ")

print.noquote(tau)

library("scatterplot3d")

scatterplot3d(Gc[,1],Gc[,2],Gc[,3])

#----------------------------------------


# Generates dim=6 samples

# drawn from Gumbel Copula

# and Generalized Pareto Distribution Marginals
```

```
#---------------------------------------

mu.gpd=c(0.01331,0.00839,0.009,0.00834,0.01377,0.0138)

sigma.gpd=c(0.013982,0.012628,0.01352,0.014634,0.012029,0.017504)

x.gpd=c(0.13757,0.38541,0.32658,0.49331,-0.0701,0.29163)

X=mu.gpd+sigma.gpd*((Cc^-x.gpd)-1)/x.gpd

library(fExtremes)

mydata<-as.timeSeries(X)

############"The Portfolio VaR "############################

VaR(mydata,alpha=c(0.01,0.025,0.05,0.075),type="sample",tail="lower")

###########"The Conditional VaR For Each Asset in the Portfolio"###############

CVaR(mydata,alpha=c(0.01,0.025,0.05,0.075),type="sample",tail="lower")
```

6.2 Appendix 2: Code for Maximum Likelihood Estimation

```
library(copula)

#NB: for each bank; input the two risk variables and call it "Data"

## "Data" has only two columns

library(timeSeries(data)

data=as.timeSeries(data)

u = pobs(data)##

gumbel.cop=gumbelCopula(2, dim = 2)

fitCop = fitCopula(gumbel.cop,u, method="ml")##

fitCop (Mwamba, 2015)
```

6.3 Appendix 3: Simulation code

```
par(mfrow=c(2,2))

  persp(gumbel.cop, pCopula, main="CDF",

  xlab="u", ylab="v", zlab="C(u,v) ) ")

  contour(gumbel.cop, pCopula, main="CDF",

  xlab="u", ylab="v")

  plot(u, main="simdata",

  xlab=" u", ylab="v” (Zivot, 2012)
```