Strathmore University

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION

EMT 2101 Engineering Mathematics I

Instructions — **Date**: 17th October, 2023

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) The entropy change $\triangle S$, for an ideal gas is given by:

$$\triangle S = \int_{T_1}^{T_2} C_v \frac{dT}{T} - R \int_{V_1}^{V_2} \frac{dV}{V}$$

where T is the thermodynamic temperature, V is the volume and $R = 8.314$. Determine the entropy change when a gas expands from 1 litre to 3 litres for a temperature rise from $100K$ to $400K$ given that: [4 Marks]

$$C_v = 45 + 6 \times 10^{-3} T + 8 \times 10^{-6} T^2$$

(b) When determining the surface tension of a liquid, the radius of curvature, ρ, of part of the surface is given by:

$$\rho = \frac{\sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3}}{\frac{d^2y}{dx^2}}$$

Find the radius of curvature of the part of the surface having the parametric equations $x = 3t^2$, $y = 6t$ at the point $t = 2$. [5 Marks]

(c) Given

$$f(x) = \operatorname{artanh} x, \qquad x \in \mathbb{R}, \quad -1 < x < 1.$$

(i) Show clearly that [4 Marks]

$$f(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \qquad x \in \mathbb{R}, \quad -1 < x < 1.$$

(ii) Without the use of any calculating aid, solve the equation [2 Marks]

$$\operatorname{artanh} x = \ln 3$$

(d) A metal disc has a radius of 5.0 cm and is of thickness 2.0 cm. A semicircular groove of diameter 2.0 cm is machined centrally around the rim to form a pulley. Determine, using **Pappus' theorem**, the volume and mass of metal removed and the volume and mass of the pulley if the density of the metal is $8000kgm^{-3}$. [4 Marks]

(e) Given that $g(x) = [5(3x-1)]^4$, decompose g into its component functions and find its inverse. Is the inverse a function? [3 Marks]

(f) A sinusoidal voltage has a maximum value of $120V$ and a frequency of $50Hz$. At time $t = 0$, the voltage is (a) zero and (b) $50V$.
Express the instantaneous voltage v in the form $v = A\sin(\omega t \pm \alpha)$ [4 Marks]

(g) (i) Prove the validity of the below hyperbolic identity by using the definitions of $\cosh x$ in terms of the exponentials. [2 Marks]

$$2\cosh 3x \cosh x \equiv \cosh 4x + \cosh 2x$$

(ii) Hence solve the equation

$$\cosh 4x \cosh 2x - 6\cosh x = 0$$

giving the answer as an expression involving exact natural logarithms. [2 Marks]

QUESTION TWO (15 MARKS)

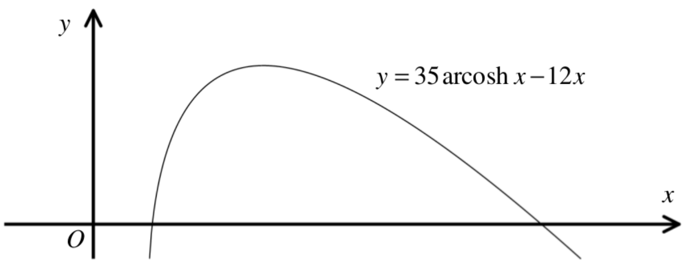
(a) The equation of a tangent drawn to a curve at point (x_1, y_1) is given by: [4 Marks]

$$y - y_1 = \frac{dy_1}{dx_1}(x - x_1)$$

Determine the equation of the tangent drawn to the parabola $x = 2t^2$, $y = 4t$ at the point t.

(b) The figure below shows the graph of the curve with the equation

$$y = 35\operatorname{arcosh} x - 12x, \qquad x \in \mathbb{R}, \quad x \geq 1.$$



The curve has a single stationary point with coordinates $\left(\frac{a}{b}, c\ln 6 - d\right)$ where a, b, c and d are positive integers.
Determine the values of a, b, c and d. [6 Marks]

(c) In electrostatics,

$$E = \int_0^\pi \left\{ \frac{a^2 \sigma \sin\theta}{2\epsilon\sqrt{(a^2 - x^2 - 2ax\cos\theta)}} d\theta \right\}$$

where a, σ and ϵ are constants. x is greater than a and x is independent of θ. Show that

$$E = \frac{a^2\sigma}{\epsilon x}$$

[5 Marks]

QUESTION THREE (15 MARKS)

(a) The current in an a.c circuit at any time t seconds is given by:

$$i = 5\sin(100\pi t - 0.432) \quad amperes$$

Determine:

(i) the amplitude, frequency, periodic time and phase angle (in degrees), [2 Marks]

(ii) the value of current at t = 8 ms, [1 Marks]

(iii) the time when the current is first a maximum, [2 Marks]

(iv) the time when the current first reaches 3A. [2 Marks]

(b) Solve the equation $2\cosh 2x + 10\sinh 2x = 5$ giving your answer in terms of a natural logarithm. [3 Marks]

(c) Determine the coordinates of the centroid of the area lying between the curve $y = 5x - x^2$ and the x-axis. [5 Marks]

QUESTION FOUR (15 MARKS)

(a) Determine the area enclosed by the two curves $y = x^2$ and $y^2 = 8x$. If this area is rotated $360^{\cdot}$ about the x-axis determine the volume of the solid of revolution produced. [5 Marks]

(b) Consider the following hyperbolic equation, given in terms of a constant k.

$$2\cosh^2 x = 3\sinh x + k$$

(i) Find the range of values of k for which the above equation has no real solutions.
. [3 Marks]

(ii) Given further that $k = 1$, find in exact logarithmic form, the solutions of the above equation. [2 Marks]

(c) Determine by integration the region and area bounded by the three straight lines $y = 4 - x$, $y = 3x$ and $3y = x$. [5 Marks]

QUESTION FIVE (15 MARKS)

(a) Given that $x > 0$ and $y > 0$, solve the simultaneous equations [5 Marks]

$$\cosh(4x - 3y) = 1$$

$$y = \frac{1}{x}e^{\operatorname{arsinh}\frac{4}{3}}$$

(b) The curve $\mathcal{C}$ has equation

$$y = \cosh(2\operatorname{arsinh} x), \qquad x \in \mathbb{R}$$

(i) Find an expression for $\frac{dy}{dx}$. [2 Marks]

(ii) Show clearly that [2 Marks]

$$\frac{d^2y}{dx^2} = \frac{4}{1+x^2}\cosh(2\operatorname{arsinh} x) - \frac{2x}{(1+x^2)^{3/2}}\sinh(2\operatorname{arsinh} x)$$

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(iii) Hence, show further that [2 Marks]

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ky = 0,$$

for some value of the constant k.

(c) The average value of a complex voltage waveform is given by:

$$V_{AV} = \frac{1}{\pi}\int_0^{\pi}(10\sin\omega t + 3\sin 3\omega t + 2\sin 5\omega t)d(wt)$$

Evaluate V_{AV} correct to 2 decimal places. [4 Marks]

END OF PAPER