STRATHMORE UNIVERSITY
SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
MASTER OF SCIENCE IN COMPUTER BASED INFORMATION SYSTEMS
END-OF-SEMESTER EXAMINATIONS [SEPT-DEC 2022 EC]
MCS 8112: Mathematics for Business Computing

DATE: 5th January 2023 Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question 1 (20 marks)

a) List the elements of the following sets. Assume the universe is $\mathbf{Z}^+$.
   (i) $\{x|x < 9\}$ (ii) $\{x|(x \; mod \; 2 = 1) \wedge (x < 10)\}$ **[3 marks]**

b) Let R be the relation $\{(a,a),(b,b),(c,c),(a,c),(a,d),(b,d),(c,a),(d,a)\}$ on the set$\{a,b,c,d\}$. Determine the (i) reflexive and (ii) symmetric closures of R. **[2 marks]**

c) Let P(x) denote the statement "x is a graduate student" and let Q(x) denote the statement "x has a full-time job." Write each statement in (i) and (ii) symbolically
   (i) All graduate students have full time jobs
   (ii) Someone who has a fulltime job is a graduate student **[2 marks]**

d) A connected planar graph has nine vertices having degrees 2,2,2,3,3,3,4,4, and 5.
   (i) How many edges are there?
   (ii) How many faces are there? **[4 marks]**

e) Use rules of inference to show that the following argument is valid.
   Hypotheses: If it is sunny I will go to the beach; If I go to the beach I will swim; I do not swim.
   Conclusion: It is not sunny, or I buy a new computer **[3 marks]**

f) Given a set B with$|B| = n$, use binomial theorem to show that $|P(B)| = 2^n$ **[3 marks]**

g) Prove that if a and b are positive even integers then ab is divisible by 4 **[3 marks]**

Question 2 (20 marks)

a) Verify the identity $\frac{n}{(n+1)!} + \frac{1}{(n+1)!} = \frac{1}{n!}$ **[3 marks]**

b) Find $\gcd(12345, 54321)$ **[6 marks]**

c) A company wants to pair up a product designer and a marketer to prepare for the launch of a new product. They have *m* product designers and *n* marketers to choose from. Using this scenario, on the number of ways in which they can pair up a product designer and a marketer explain why the identity

$mn = \binom{m+n}{2} - \binom{m}{2} - \binom{n}{2}$ is true. **[5 marks]**

(d) The password format for a particular website has five letters and three digits. How many different passwords are possible in each of the following situations?

(i) The letters follow the digits and the letters and digits can be repeated

(ii) The letters follow the digits and the letters and digits cannot be repeated

(iii) The three digits can be anywhere in the password, and the letters and digits can be repeated **[6 marks]**

Question 3 (20 marks)

a) State the order of the following linear homogeneous recurrence relation: $a_n = -a_{n-1} + 5a_{n-2} - 3a_{n-3}$ **[1 mark]**

b) Prove that $n^3 - n$ is divisible by 3 for any integer $n \geq 0$ **[6 marks]**

c) Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_0 = 1\ and a_1 = 2$ **[6 marks]**

d) A bit string from Bn is called "unfriendly" if no two adjacent bits are 1's. For example {0, 1} are bit string from B1 that are unfriendly and {00, 10, 10} are unfriendly bit strings from B2. Let a_n denote the number of unfriendly bit strings in Bn. Thus $a_1 = 2$ and $a_2 = 3$.

i) List the unfriendly bit strings from B3 and B4.

ii.) Determine a recursive formula for a_n **[7 marks]**

Question 4 (20 marks)

a) Determine whether the function $g: Z \to Z. given\ by\ g(x) = \left\lfloor \frac{x}{2} \right\rfloor$ is injective or surjective or both. **[4 marks]**

b) Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 500 bits of data? **[2 marks]**

c) Given $f(x) = 2x + 7$ and $g(x) = 1 - x^3$. Determine each of the following.

(i) $(f + g)(3)$ (ii) $f^{-1}(x)$. (iii) $f(g(x))$

[8 marks]

d) Consider the relation on $r\ on\ \mathbb{Z}$ defined by $r = \{(a, b): a \equiv b\ (mod\ 5)\}$. Show that r is an equivalence relation. **[6 marks]**

Question 5 (20 marks)

a) Define each of the following

(i) A graph
(ii) A complete graph
(iii) A planar graph
(iv) A connected graph **[5 marks]**

b) How many edges do the following graphs have?
(i)C_4 (ii) K_5 (ii) $K_{2,3}$ **[3 marks]**

c) Examine the adjacency matrices I and II and use them to answer the following questions.

$$\text{I} \quad \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{II} \quad \begin{matrix} P \\ Q \\ R \\ S \end{matrix}\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$$

(i) Determine whether the graphs with adjacency matrices I and II have an Euler circuit. How can you tell without drawing the graphs?
(ii) Draw and label a graph corresponding to each adjacency matrix I and II.
[7 marks]

d) A company has the following six marketing teams, Simba, Swara, Ndovu, Twiga, Chui, and Kifaru. Some of the marketers belong to more than one team as shown in the table below.

Team	**Marketers belonging to more than one team**
Simba	Chao, Sheena, Karis
Swara	Chao, Karis, Wayne
Ndovu	Sheena
Twiga	Karis, Vera, Wayne ,
Chui	Vera, Sheena
Kifaru	Sheena

Given that every team meets once a week, determine the fewest number of days needed to schedule all marketing team meetings in such a way that no two teams that share a team member meet on the same day.
[5 marks]