STRATHMORE UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

END OF SEM EXAMINATION

MASTER OF SCIENCE IN BIOMATHEMATICS

BMA 8203-STOCHASTIC MODELLING

Date: 30th August,2022 Time: 3 Hours

INSTRUCTIONS

Answer QUESTION ONE and ANY OTHER TWO questions

Question One (20 Marks)

(a) Give the definitions of a Gaussian white noise, standard white noise and the Wiener process. (3 marks)

(b) Let $W(t)$ be a standard Brownian motion. Define

$$X(t) = \exp\{W(t)\}, \text{ for all } t \in [0, \infty).$$

(i) Find $E[X(t)]$, for all $t \in [0, \infty)$. (3 marks)

(ii) Find $Var(X(t))$, for all $t \in [0, \infty)$. (3 marks)

(iii) Let $0 \leq s \leq t$. Find $Cov(X(s), X(t))$. (5 marks)

(c) Consider the SDE:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t),$$

where $-\infty < \mu < \infty$, $\sigma > 0$. Using Coefficient Matching Method, find the solution of $X(t)$). (6 marks)

Question Two (20 Marks)

Consider the following Markov chain one-step transition matrix, where some elements are replaced by $*$:

$$P = \begin{pmatrix} 1/2 & 1/2 & ? & ? \\ 1/3 & 1/4 & ? & 1/4 \\ 1/4 & ? & 1/4 & 1/4 \\ ? & ? & 1/3 & 2/3 \end{pmatrix}$$

(a) Explain why the missing elements of P can be determined from the available information, and find them. (4 marks)

(b) Draw the transition diagram and use it to explain why this Markov chain possesses an equilibrium distribution. (4 marks)

(c) Without performing any calculations, describe as best you can, the eigen-values of the matrix P. (4 marks)

(d) Find the equilibrium distribution. (8 marks)

Question Three (20 Marks)

(a) Find the mean, autocorrelation and autocovariaance of the random process

$$X(t) = A\cos\omega(t),$$

where ω and A is $U(0,1)$. (9 marks)

(b) X is a Poisson distributed random variable with parameter λ. Calculate $\mathbb{E}\left[3^X\right]$ (4 marks)

(c) Let $\{B(t), t > 0\}$ be a standard Brownian motion. Consider a stochastic process $\{V(t), t > 0\}$, with drift $\beta > 0$, diffusion σ^2 and initiial condition $V(0) = \nu$, is given by

$$V(t) = \nu e^{-\beta t} + \frac{\sigma}{\sqrt{2\beta}} B\left(e^{-\beta t} - 1\right).$$

Find $\mathbb{E}\left[V(t)\right]$ and $Var\left(V(t)\right)$ (7 marks)

Question Four (20 Marks)

(a) Suppose the following table gives the initial distribution of people in the three blood G=groups

Group	State	Proportion
M	1	39%
N	2	127%
MN	3	34%

With transition matrix

$$\begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0.06 & 0.80 & 0.14 \\ 0.12 & 0.25 & 0.63 \end{bmatrix}$$

Write a program in R that finds the blood groups composition after 10 years. (13 marks)

(b) Let B_t be a Brownian motion. Show that the process $X_t = -B(T-t)-B(t), \quad t < T < \infty$ is Brownian motion on $[0, T]$ (7 marks)

Question Five (20 Marks)

(a) Given a Legendre polynomial $P_n(x)$, with explicit representations

$$P_n(x) = \sum_{k=0}^{n} \binom{n}{k} \binom{n+k}{k} \left(\frac{x-1}{2}\right)^k.$$

Write an R code to plot line graphs of the given polynomial with $xlim = c(-1, 1)$ and $ylim = c(-1, 1)$ for $n = 0, 1, 2, \cdots, 6$. (12 marks)

(b) Let $Y_1, Y_2, \cdots$ be a sequence of independent random variables with zero mean and common variance σ^2. If $X_n = Y_1 + \cdots + Y_n$, then show that $X_n - n\sigma^2$ is a martingale. (8 marks)