SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE IN INFORMATION AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBERSECURITY

END OF SEMESTER EXAMINATIONS

ICS 1205/CNS 1205: LINEAR ALGEBRA

DATE: 13th December 2024 TIME: 2 Hours

INSTRUCTIONS

1. Answer Question **ONE** (COMPULSORY) and any other **TWO** questions

QUESTION ONE (30Marks)

1. (a) Show that the solutions (4 marks)

$$\begin{aligned} x &= 19s - 35 \\ y &= 25 - 13s \\ z &= s \end{aligned}$$

is a solution of

$$\begin{aligned} 2x + 3y + z &= 5 \\ 5x + 7y - 4z &= 0 \end{aligned}$$

(b) Write the system of linear equations in row echelon form and solve the system (5 marks)

$$
\begin{aligned}
3x + y - 4z &= -1 \\
x + 10z &= 5 \\
4x + y + 6z &= 1
\end{aligned}
$$

(c) Write a simple algorithm in R-studio that can be used to solve the system of linear equations (5marks)

$$
\begin{aligned}
x_2 + x_3 - 2x_4 &= -3 \\
x_1 + 2x_2 - x_3 &= 2 \\
2x_1 + 4x_2 + x_3 - 3x_4 &= -2 \\
x_1 - 4x_2 - 7x_3 - x_4 &= -19
\end{aligned}
$$

(d) Find the inverse of the matrix A using Gaussian -Jordan elimination method (5 marks)

$$
\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{pmatrix}
$$

(e) Using Cramer's rule to solve the system of linear equations (5 marks)

$$
\begin{aligned}
4x_1 - 2x_2 &= 10 \\
3x_1 - 5x_2 &= 11
\end{aligned}
$$

(f) Write the vector **W**$=(1,1,1)$ as a linear combination of vectors in the set **S** . (6 marks)

$$
S = \{(1,2,3),(0,1,2),(-1,0,1)\}
$$

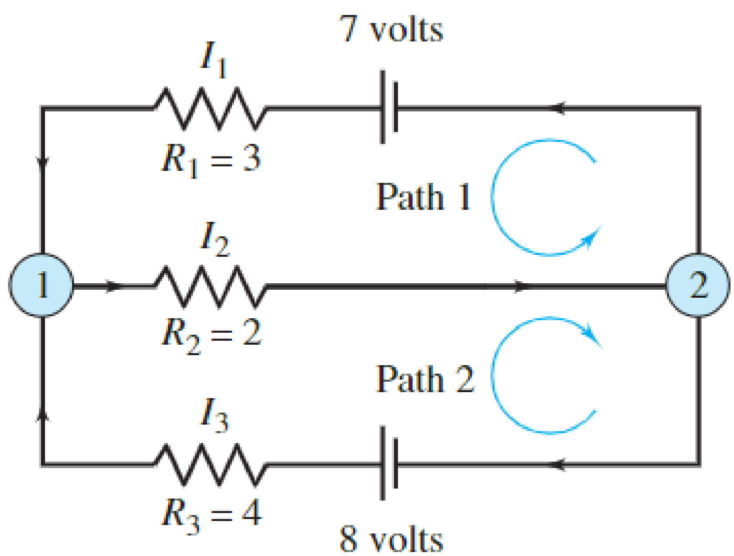
QUESTION TWO (20 MARKS)

2. (a) Solve the system of linear equations using Gaussian elimination method (6marks)

$$x_1 - 3x_2 + x_3 = 1$$
$$2x_1 - x_2 - 2x_3 = 2$$
$$x_1 + 2x_2 - 3x_3 = 1$$

(b) Determine the polynomial $p(x) = a_0 + a_1x + a_2x^2$ whose graph passes through the points (1,4),(2,0)and (3,12) (7 marks)

(c) Determine the currents I_1, I_2 and I_3 for the network shown below . (7 marks)



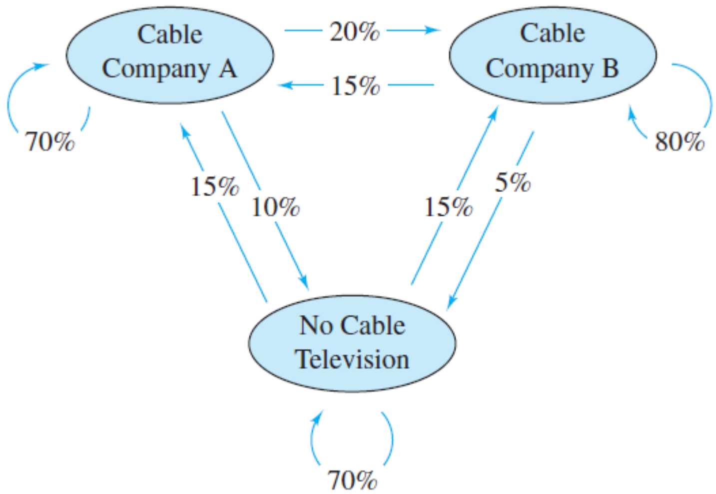
QUESTION THREE (20 MARKS)

3. (a) Find the determinant of $|AB|$ (6 marks)

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & -2 \\ 3 & 1 & -2 \end{bmatrix}$$

(b) Find the least squares regression line for points (1,1),(2,2),(3,4),(4,4)and (5,6) (7 marks)

(c) Two competing companies offer cable television service to a city of 100,000 households.The changes in cable subscriptions each year . Company A now has 15,000 subscribers and Company B has 20,000 subscribers. How many subscribers will each company have 1 year from now? (7 marks)



QUESTION FOUR (20 MARKS)

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix (7marks)

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

(b) i Write the uncoded row matrices of size (1×3) for the message **MEET ME MONDAY** (3 marks)

ii Use the matrix

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$$

to encode the message **MEET ME MONDAY**.(6 marks)

(c) Use Cramer's Rule to solve the system of linear equations for x, y, z (7 marks)

$$\begin{aligned} -x + 2y - 3z &= 1 \\ 2x + z &= 0 \\ 3x - 4y + 4z &= 2 \end{aligned}$$

QUESTION FIVE (20 MARKS)

5. (a) Provided that $X = (-1, -2, -2)$, $u = (0, 1, 4)$,$v = (-1, 1, 2)$ and $w = (3, 1, 2)$ in R^3.Find scalars a, b and c such that (5 marks)

$$X = au + bv + cw$$

(b) Show that the set of points on the line $x + 2y = 0$ is a subspace of R^2 (5 marks)

(c) Show that set $S = \{(1, 2, 3), (0, 1, 2), (-2, 0, 1)\}$ spans R^3 (5 marks)

(d) Determine whether the set of vectors in $R^3$is linearly independent or linearly dependent (5 marks)

$$S = \{(1, 2, 3), (0, 1, 2), (-2, 0, 1)\}$$