STRAHMORE INSTITUTE OF MATHEMATICAL SCIENCES

BACHELOR OF BUSINESS SCIENCE IN ACTUARIAL SCIENCE

EXAMINATION

BSA 3218 ACTUARIAL MODELING II

DATE: **March 25, 2025** Time: **2 HOURS**

Instruction: Answer question ONE and Any other TWO questions

Question One (30 Marks)

a)

i. Write down the equation of the Cox proportional hazards model in which the hazard function depends on duration t and a vector of covariates z. You should define all the other terms that you use.

ii. Explain why the Cox model is sometimes described as "semi-parametric" **(8 Marks)**

b) Show that if the force of mortality $\mu_{x+t} = \dfrac{q_x}{1-tq_x}$, this implies that deaths between exact ages x and x+1 are uniformly distributed. **(6 Marks)**

c) An investigation of mortality over the whole age range produced crude estimates of q_x for exact ages x from 2 years to 93 years inclusive. The actual deaths at each age were compared with the number of deaths which would have been expected had the mortality of the lives in the investigation been the same as English Life Table 15 (ELT15). 53 of the deviations were positive and 39 were negative.
Test whether the underlying mortality of the lives in the investigation is represented ELT15. **(8 Marks)**

d) A life insurance company has investigated the recent mortality experience of its male term assurance policy holders by estimating the mortality rate at each age, t . It is proposed that the crude rates might be graduated by reference to a standard mortality table for male permanent assurance policy holders with forces of mortality $\mu^{s}_{x+\frac{1}{2}}$, so that the forces of mortality $\overset{0}{\mu}_{x+\frac{1}{2}}$ implied by the graduated rates $\overset{0}{q}_{x}$ are given by the function:

$\overset{0}{\mu}_{x+\frac{1}{2}} = \mu^{s}_{x+\frac{1}{2}} + k,$ where k is a constant.

i. Describe how the suitability of the above function for graduating the crude rates could be investigated.

ii. .
- Explain how the constant k can be estimated by weighted least squares
- Suggest suitable weights

iii. Explain how the smoothness of the graduated rates is achieved. **(8 Marks)**

Question Two (20 Marks)

a) A study of the mortality of 12 laboratory-bred insects was undertaken. The insects were observed from birth until either they died or the period of study ended, at which point those insects still alive were treated as censored.

The following table shows the Kaplan-Meier estimate of the survival function, based on data from the 12 insects.

t (weeks)	*S(t)*
$0 \le t < 1$	1.0000
$1 \le t < 3$	0.9167
$3 \le t < 6$	0.7130
$6 \le t$	0.4278

i. Calculate the number of insects dying at durations 3 and 6 weeks.

ii. Calculate the number of insects whose history was censored. (10 Marks)

b) An investigation into mortality collects the following data:

θ_x = total number of policies under which death claims are made when the policyholder is aged x last birthday in each calendar year

$P_x(t)$ = number of in-force policies where the policyholder was aged x nearest birthday on 1 January in year t

i) State the principle of correspondence. **(2 Marks)**

ii) Obtain an expression, in terms of the $P_x(t)$, for the central exposed to risk, E^c_x, which corresponds to the claims data and which may be used to estimate the force of mortality in year t at each age x, μ_x. State any assumptions you make. **(4 Marks)**

iii) Comment on the effect on the estimation of the fact that the θ_x relate to claims, rather than deaths, and the $P_x(t)$ relate to policies, not lives. **(4 Marks)**

Question Three (20 Marks)

An investigation took place into the mortality of pensioners. The investigation began on 1 January 2023 and ended on 1 January 2024. The table below gives the data collected in this investigation for 8 lives.

Date of birth	*Date of entry into observation*	*Date of exit from observation*	*Whether or not exit was due to death (1) or other reason (0)*
1 April 1952	1 January 2023	1 January 2024	0
1 October 1952	1 January 2023	1 January 2024	0
1 November 1952	1 March 2023	1 September 2023	1
1 January 1953	1 March 2023	1 June 2023	1
1 January 1953	1 June 2023	1 September 2023	0
1 March 1953	1 September 2023	1 January 2024	0
1 June 1953	1 January 2023	1 January 2024	0
1 October 1953	1 June 2023	1 January 2024	0

The force of mortality, μ_{70}, between exact ages 70 and 71 is assumed to be constant.

a) i. Estimate the constant force of mortality, 70, using a two-state model and the data for the 8 lives in the table.

ii Hence or otherwise estimate q_{70}. **(10 Marks)**

b) Show that the maximum likelihood estimate of the constant force, μ_{70}, using a Poisson model of mortality is the same as the estimate using the two-state model. **(5 Marks)**

c) Outline the differences between the two-state model and the Poisson model when used to estimate transition rates. **(5 Marks)**

Question 4 (20 Marks)

a) Describe three advantages and disadvantages of graduating a set of observed mortality rates using a parametric formula. **(6 Marks)**

b) A lecturer at a university gives a course on Survival Models consisting of 8 lectures. 50 students initially register for the course and all attend the first lecture, but as the course proceeds the numbers attending lectures gradually fall.

Some students switch to another course. Others intend to sit the Survival Models examination but simply stop attending lectures because they are so boring. In this university, students who decide not to attend a lecture are not permitted to attend any subsequent lectures.
The table below gives the number of students switching courses and stopping attending lectures after each of the first 7 lectures of the course.

Lecture number	No. of students switching courses	No. of students ceasing to attend lectures but remaining registered for survival models
1	5	1
2	3	0
3	2	3
4	0	1
5	0	2
6	0	1
7	0	0

The university's Teaching Quality Monitoring Service has devised an Index of Lecture Boringness. This index is defined as the Kaplan-Meier estimate of the proportion of students remaining registered for the course who attend the final lecture. In calculating the Index, students who switch courses are to be treated as censored after the last lecture they attend.

i) Calculate the Index of Lecture Boringness for the Survival Models course. **(10 Marks)**

ii) Explain whether the censoring in this example is likely to be non-informative **(4 Marks)**

Question Five (20 Marks)

a) A mortality investigation has been carried out over the three calendar years, 2002, 2003 and 2004.

The deaths during the period of investigation, θ_x have been classified by age x at the date of death, where

x = calendar year of death- calendar year of birth.

Censuses of the numbers alive on 1 January in each of the years 2022, 2023, 2024 and 2025 have been tabulated and denoted by

$$P_x(2022), P_x(2023), P_x(2024) \text{ and } P_x(2025)$$

respectively, where x is the age last birthday at the date of each census.

i) State the rate year implied by the classification of deaths, and give the ages of the lives at the beginning of the rate year. **(4 Marks)**

ii) Derive an expression for the exposed to risk in terms of the $P_x(t)$ (t = 2022, 2023, 2024, 2025) which corresponds to the deaths data and which may be used to estimate the force of mortality, μ_{x+f} at age $x+f$. **(4 Marks)**

iii) Determine the value of f, stating any assumptions you make. **(4 Marks)**

b).

i) Describe the general form of the polynomial formula used to graduate the most recent standard tables produced for use by UK life insurance companies.

ii) Show how the Gompertz and Makeham formulae arise as special cases of this formula. **(8 Marks)**