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Stochastic modeling of electricity prices and option pricing.

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Declaration and Approval

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Abstract

Volatility and abrupt price changes is a problem that has marred the electricity market for decades. This problem is especially observed in deregulated markets whose prices are influenced by supply and demand factors. Another consideration is the fact that electricity is non-storable which means that its prices are quite difficult to control. In an effort to address these problems, the current study was developed to price electricity and options used to hedge against volatility and unexpected price jumps. The mean reverting jump diffusion was applied by taking into account day ahead spot prices derived from the Nordic electricity market or the Nord Pool. To price spread options, I applied the Monte Carlo simulation model. The analysis of the data was undertaken through R programming undertaken within the Anaconda software. The need to price electricity options was to furnish market participants with instruments to manage the financial risks that come with price volatility due power failure and demand factors. The analysis shows the complex nature of electricity pricing, hence there is no closed form solution for pricing these derivatives. While the study findings were not directly applicable to the Kenyan and East African context, it provided a robust context for future research especially as the need for a deregulated market grows in the country.

Key words: Electricity prices, Mean-reverting, Diffusion, Monte-Carlo simulation, Spread Option

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List of Abbreviations

DA	Day Ahead
EEX	European Energy Exchange
GBM	Geometric Brownian Motion
HEP	Over Hydro-Electric Power
HJM	Heath-Jarrow-Morton
IID	Independent and Identically Distributed
KenGen	Kenya Electricity Generating Company
MC	Monte Carlo
MLE	Maximum Likelihood Estimate
MRJD	Mean-Reverting Jump Diffusion
MWh	Mega-Watt hours
NASDAQ	National Association of Securities Dealers Automated Quotations
OTC	Over The Counter
O-U	Ornstein Uhlenbeck
SDE	Stochastic Differential Equation

Acknowledgment

First, I thank God for his presence and provision, I thank my family especially my sister for being my support system, my classmates for the unity and the numerous group discussions, and all the lecturers for their guidance and dedication.

Dedication

"... there is one elementary truth that ignorance of which kills countless ideas and splendid plans: that the moment one definitely commits oneself, then Providence moves too. All sorts of things occur to help one that would never otherwise have occurred. A whole stream of events issues from the decision, raising in one's favor all manner of unforeseen incidents and meetings and material assistance, which no man could have dreamed would have come his way. Whatever you can do, or dream you can do, begin it. Boldness has genius, power, and magic in it. Begin it now."

W. H. Murray

This thesis is dedicated to my family for all their support, love and care.

Chapter 1

Introduction

1.1 Background to the study

The energy sector in Kenya has undergone vast reforms in the past few decades. The first wave of reform according to Godinho and Eberhard (2019) occurred in 1996, as captured in the Electric Power Act of 1997 Kiiru (2002). This reform was externally motivated and driven by donors that favored the separation of commercial activities from policy and regulatory functions. These reforms also saw the uncoupling of the power generation function from the transmission and distribution function. The second wave of reform occurred amidst the adoption of a new political order which oversaw the partial privatization of the Kenya Electricity Generating Company, KenGen Godinho and Eberhard (2019). The reforms contributed significantly towards opening up the country as one of the most favored private energy investment destinations in the region. Partial privatization came with added advantages of operational efficiency but resulted in perceived problems that come with movement from cost-reflective tariffs to market driven cost dynamics.

Before these reforms were integrated, electricity prices in Kenya had been controlled by government backed regulators. These intermediaries ensured that the price of electricity was relatively stable with minimal variations. At the time, the prices of electricity were largely determined by the cost of production and the fact that the industry was monopolize, there were no excessive market factors influencing the price of electricity. These changes created a situation where the spot prices were determined by supply and demand forces. This situation came with unforeseen challenges of price controlled and undesirable effects for the final consumer Gil-Alana et al. (2017). The challenges underlying the electricity market are compounded by the fact that electricity cannot be stored but has to be transformed into other forms of energy. As a result, the cost of electricity has become excessively volatile and unstable largely due to market fluctuations, an aspect that has to be addressed going forward.

1.2 Problem statement

The overarching problem of the Kenyan electricity industry is the high cost of electricity coupled with fluctuating prices. This problem is an effect of market factors of supply and demand. Kenya predominantly relies on Hydro-Electric Power (HEP) which is dependent on seasonal weather patterns. While Kenya is working towards diversifying its power sources, its challenges range from the power rationing of the early 2000s and power crises of the 2010s Mutiso and Taneja (2018). All of these problems link back to one problem, effective financial modelling of supply and demand factors to create a better planning framework to address issues with access, profitability and management.

The issues plaguing the electricity market have resulted in a wide range of problems including stagnating demand, the fallout with large industrial power users, consumer dissatisfaction with pricing and unreliability of the service. Demand has been stagnating because of the high upfront costs of connection and reduced consumption by existing customers which is a big threat to the electricity business model. Industrial power users account for more than 50 per cent of consumption Mutiso and Taneja (2018). Dissatisfaction with cost has led some corporations to set up their power generation plants either using coal, HEP or solar to create better control systems for their production costs. In periods of low supply, the power generation companies use generators which depend on oil prices, an external variable that comes with added price fluctuation and currency exchange risks. Efforts at harmonizing prices have failed as these risks are aggregated and bundled into the electricity price without due consideration of the effect on the final customer. Some strategies like reduced night usage tariffs have also failed because of their short-term effects.

1.3 Objectives of the study

This study was developed with the aim of developing a more effective pricing strategy for the electricity market. It narrowed down to the mathematical modelling process looking to address inconsistencies and uncertainties surrounding the pricing of electricity in the country. As such, the objectives of the study were outlined as follows;

1. To examine stochastic modelling techniques used for energy market price dynamics;
2. To calibrate electricity market prices using a mean-reverting stochastic model with jumps;
3. To price an electricity aggregate price spread option using Monte Carlo (MC) simulation given underlying mean-reverting seasonal stochastic model with jumps;

1.4 Significance of study

The Kenyan electricity market is largely a monopoly. This means that electricity users have to depend on one entity for their power needs. The downside of this situation is that any adverse outcomes cannot be addressed by seeking a better electricity provider. The cost of electricity trickles down to the cost of other goods and services, given that electricity is essential for the operation of different sectors including the manufacturing, processing companies, educational institutions, commercial and residential establishments Gil-Alana et al. (2017). The problems with large fluctuations can be addressed by identifying a framework for stabilizing prices to make them more predictable and friendly to the dependent industries and the larger public. Botterud et al. (2010) observed that financial risk management has become an essential tool for reducing risk exposure in volatile electricity markets. The understanding of the short term and long term prices of electricity have the effect of furnishing market players with the right information to plan for adequate development of supply and demand. This enables them to develop sustainable electricity markets both in terms of customer satisfaction and cost-control Weron et al. (2004). My project is aimed at applying a stochastic model that enhances the understanding of electricity spot prices and pricing derivatives that could be strategically applied to manage risks and devise market development strategies for the future.

Chapter 2

Literature review

In the literature review section, I reviewed a range of sources in an effort to deconstruct different aspects of electricity pricing and the models used within those contexts. One of the key outcomes was gleaned from the analysis of the Nord Pool, an entity that runs the main power market in Europe. From that analysis, the review delved into the fundamental aspect of the electricity market, the electricity options market and various approaches to electricity price modelling. Summing up this knowledge would give a theoretical and practical basis for the application of stochastic models in the pricing of electricity in Kenya to address problems of exorbitant pricing and customer dissatisfaction.

Risk management has become an essential part of the electricity market. Companies have to apply effective hedging strategies to protect their profitability going forward and allow them make better decisions as regards to provision of services Souhir et al. (2019). There are various hedging instruments that have been developed over the years to facilitate the risk management efforts of the different stakeholders of the electricity market. Investors and key actors in the electricity market have taken to adopting hedging strategies that employ the use of financial instruments like forward contracts, options and futures contracts Canelas et al. (2020) and Castro (2017) . These derivatives will form an essential part of the discussion of the financial instruments that could be used to improve electricity pricing in Kenya. Forward contracts are basically derivative contracts made between two parties to buy or sell an asset or commodity at a future date. Options contracts, on the other hand, are created based on the perceived underlying value of an asset or commodity Zhao and Huchzermeier (2017). Depending on the outlook of an investor, they could either opt for a call option or a put option. A call option gives the holder the right but the obligation to buy the specific asset or commodity at the stated price within a specified time frame. A put option gives them the right to sell the asset or commodity at the stated price within a specified time frame. A futures contract is an agreement backed by a legal framework to buy or sell a particular commodity or asset at a predetermined price at some point (specified) in the future. While forward contracts are traded over the counter and are customizable, futures contracts are standardized and traded in exchanges Carbonneau and Godin (2021). Despite the differences, these instruments are used to reduce risk exposure electricity markets Botterud et al. (2010). Within the scope of this study, I will be looking to discuss the extent to which financial models have been applied in advanced markets to address problems of spikes in electricity prices and shortage.

2.1 The Nord Pool

The Nordic region is an example of a market that has a long experience with a restructured electricity market Botterud et al. (2010) and Uribe Gil et al. (2020). The nordic electricity market, also referred to as the Nord Pool was established in 1992 in the wake of deregulation of the energy markets in Norway Weron and Zator (2014). Within the next few years, other countries including Denmark, Finland and Sweden joined in to form the first internationally known power exchange. Over time, this market grew both in volume and efficacy in stabilization of power rates and hedging strategies for the larger electricity market of the countries that joined it. Subsequently, the Nord Pool has been able to facilitate the efficient and secure delivery of power across Europe from a reliability standpoint Janke et al. (2020). Fundamentally, the Nord Pool works by the creation and development of a range of Nordic power derivatives traded through quarterly, semi-annual and annual forwards, daily and weekly futures, options and contracts Uribe Gil et al. (2020). The Nord Pool defines its future contracts as standardized and exchange traded. They however do not consider these agreements marked-to-market for the future delivery of electricity at some point in the future Beigaite et al. (2018). Spot prices within the Nord Pool are determined by the intersection between supply and demand factors. Because of these features, the Nord Pool was acquired by the NASDAQ and undertaking that led to its change of name to NASDAQ OMX Commodities Europe especially as more European countries like Germany and the UK joined in Weron and Zator (2014). The market has grown much wider to accommodate Dutch, German and UK power futures and forwards, and even UK Natural Gas futures and carbon products Weron and Zator (2014). Today, an estimated 360 companies from 20 countries in the European region trade on the Nord Pool market, a feat that lends credence to its effectiveness in stabilizing energy prices and protecting investors from excessive risk.

2.1.1 Characteristics of the Electricity Market

Developing models for pricing electricity forwards, futures and options require, first, the understanding of the structure of electricity markets. The spot prices in the electricity market are fraught with fluctuations influenced by perceived patterns and seasonal factors. These dynamics are fundamentally explained from a theoretical standpoint. All markets show varying patterns of price changes and these patterns depend on the various aspects of the primary market Uribe Gil et al. (2020). One key aspect of electricity markets that cannot be understated is the need to balance supply and demand to avoid rationing, unexpected blackouts and abrupt price changes. This need is required to bring a sense of certainty through risk management, and as a result, stability in electricity prices.

2.1.2 Supply and demand dynamics

The deregulation of energy markets inadvertently results in the dependence on market factors like supply and demand. Supply within electricity markets is measured in terms of generation units within a specified region. Supply is measured in terms of marginal cost of production and the response. These two aspects basically depend on the efficacy of the generation company in producing power and the ability to transmit it to the required markets on time Weron and Zator (2014) and Souhir et al. (2019). There are a range of factors that affect the supply of electricity. Some of them include fluctuation of fuel prices (in case generators are used), and in some instances outages due to planned maintenance. These dynamics vary within the Kenyan context, with one of the pertinent issues being seasonal weather issues. During the dry season, low volume of water results in lower production units hence inability to meet demand.

Other problems that have been observed include transmission constraints, mismanagement and unforeseen circumstances.

Demand varies depending on the region. Within the Nordic context, demand fluctuates due to seasonality (due to climatic conditions). During winter, there is more demand for electricity as opposed to summer. This is due to the need for heating to keep families and other establishments warm. In regions like the US, summer signifies peak demand to increased use of air conditioning to address high temperatures and humidity Weron and Zator (2014). These dynamics also vary considerably in the Kenyan and east African context. Here, the main issues come with daily and weekly demand. At day time, there is more demand for power due to industrial operations and business operations. In households, demand peaks hourly with the highest consumption seen in the evenings after work or during the weekends, when most individuals spend their time at home. Some of the interventions that the government has tried to institute to address the demand issue is to incentivize companies to extend their operational frameworks to night time. That was partially the inspiration behind the perceived movement of the Kenyan economy to a 24 hour economy. Since HEP cannot be stored, increasing consumption at night would work as a means of creating additional demand to balance these market factors. The failure of these interventions has resulted in calls for more deregulation to make the electricity market more competitive. Meanwhile, the Nord Pool has long applied financial models that are tailored for their electricity market and that have been incredibly effective in addressing problems faced within the industry.

2.1.3 Day ahead market

The Nord Pool works in terms of a day-ahead market due to the impossibility of having a classical spot market Janke et al. (2020). System operators essentially need advanced notice to verify the viability of a contract in terms of feasibility of production and transmission amidst existing constraints Weron and Zator (2014). These Transmission System Operators (TSO) are tasked with ensuring that the energy sold within the exchange market is delivered to the buyer. Any shortfalls have to be subsequently bought from the spot markets once again capturing the supply and demand factors of the market Weron et al. (2004). This feature allows half-hour market changes hence enabling demand and supply to match instantaneously as retailers acquire electricity from wholesalers Lundin and Tangerås (2020). The analysis of the day ahead Nord Pool prices show an aspect of spikes that are fundamentally influenced by the fact that electricity cannot be stored and has to rely on market factors for price determination.

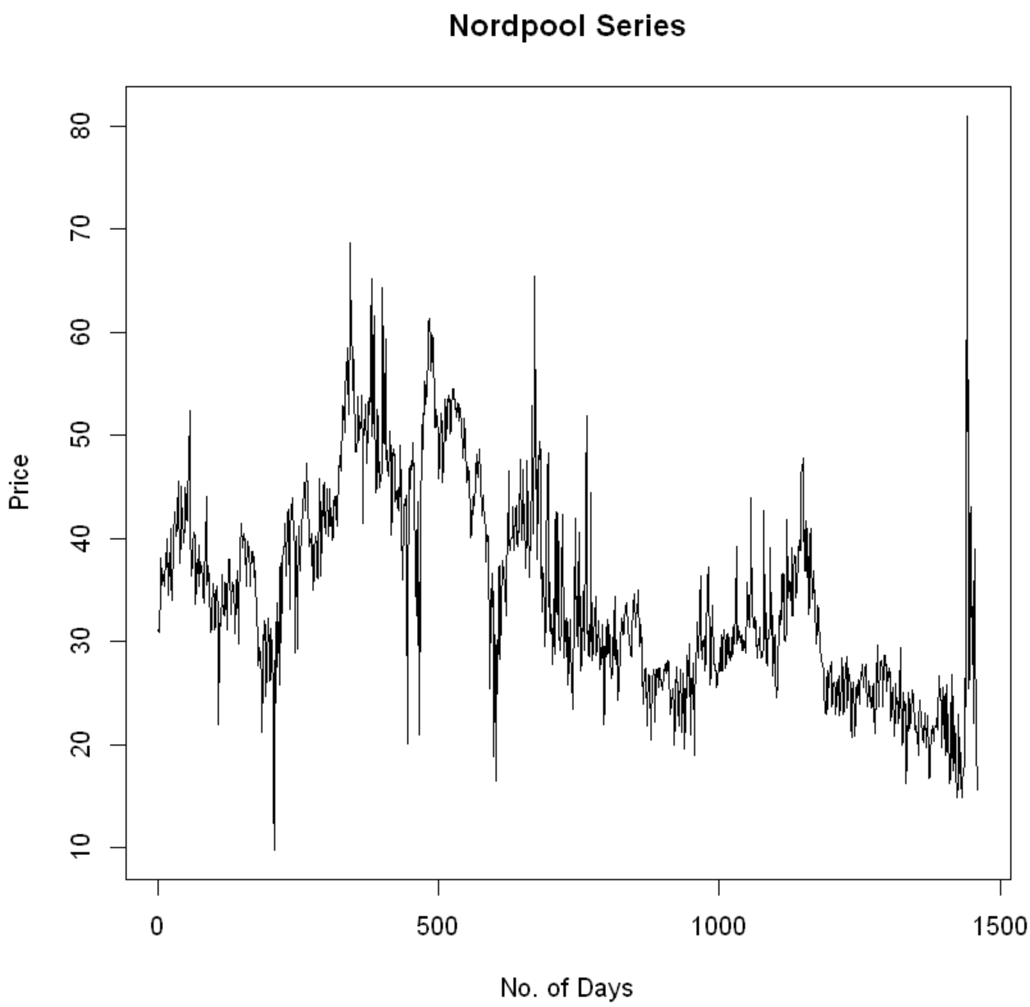

Figure 2.1: Day-Ahead NordPool Prices

Spikes are also caused by increased demand when supply is not at maximum due to power plant failures. Figure 2.1 exhibits both positive and negative prices. Electricity prices move rapidly to a mean level reverting towards the cost of production making it apt for the mean-reverting models to be used. Long term market analysis has shown the tendency of prices to revert to a mean level when the supply issues are addressed.

This is the trend that underscores the application of the mean-reverting model.

Close scrutiny of price changes have shown how correlated price levels are to the degree of volatility in the market Janke et al. (2020). Once again, these changes are influenced by the fundamental nature of the electricity market which is impacted by non-storability of electricity, and the variation in demand and supply factors. These changes are temporal and occur on a daily, weekly and seasonal basis Benini et al. (2002). This form of uncertainty informed the integration of the non-stationary model to reflect the perceived long term price fluctuations of the market Beigaite et al. (2018). This is another feature that captures the extent to which the Nord Pool is in touch with the underlying dynamics of the electricity market. The aggregate of these dynamics show the need for contracts that can help institute price control mechanisms and calm the volatile electricity market.

2.1.4 Electricity Options Market

A range of contracts are offered within the Nord Pool exchange. These contracts introduced exchange trading activities for market participants allowing them to seek arbitrage opportunities and maximize the optionality of their contracts Ottesen et al. (2018). The universality of these contracts and inclusion of the various market players are effective in price control hence they are preferred to over-the-counter private contracts that have an aspect of exclusion. The different electricity market stakeholders including generation companies, transmission and distribution entities were thus afforded an opportunity of using electricity derivatives (spinned off from financial derivatives) for risk management given the volatility of spot prices in the market Rios et al. (2019) and Piao et al. (2019). Each day is broken into hourly spot contracts, which are as aforementioned, determined by the intersection between supply and demand. This pricing mechanism is effective as it ensures there is no congestion at any point in time, and it reflects an efficient market since all the available information is perceived to be included in the price Alshehri et al. (2017). These dynamics are in line with the efficient market hypothesis which hypothesizes the fair market value of an asset or commodity when market players are perceived to have included all factors to be considered when getting into the available contracts Skånberg (2012).

The Nord Pool has a diversified market that incorporates both standardized and private contracts within its options pool Uribe Gil et al. (2020). These instruments are designed as a hedging mechanism and as a means of speculating different outcomes in the spot and forward markets. Apart from traditional (vanilla) options, exotic options are allowed to give the market participants a wider range of instruments that can be used depending on their market needs and preferences. One such contract, as explained by Eydeland and Wolyniec (2003) is the spread option. Spread options are contracts whose value is determined by the differences in price between two or more assets. These contracts are mostly sold over-the-counter due to their niche nature. Depending on a market participant's outlook of the market, they can either buy a put spread option or a call spread option to hedge against price differences at a perceived lower cost than actually buying the commodity and depending on its prices Beigaite et al. (2018). An example of this situation could be seen in a power producing company that uses natural gas to produce and sell electricity. To protect its operational margins and protect their bottom line, the company can buy a call option on the spark-spread which is essentially the differences between the natural gas (raw material) and electricity (product) multiplied by a factor of, say, 0.68. This factor is calculated by taking into account production factors like the efficiency of the plant in converting units of natural gas into electricity units. An assumption of 50 per cent efficiency and the inclusion of other internal variables would lead to such a result Borovkova et al. (2007).

Within the European market, there are different entities and countries that produce, sell and distribute electricity. Having all these players within one market allows market participants to get into regional spread contracts given the discordance of supply and demand factors in different regions Lundin and Tangerås (2020). As such a call option would give the holder the right but not the obligation to purchase a unit of electricity in one region and sell it in a different region at specified price. Such a contract allows for the exploitation of price difference, an undertaking that goes a long way in instituting balance in the market. These kinds of regional (locational) spread contracts are commonplace in the Nord Pool given the inherent desire of market participants to use their knowledge of market dynamics to hedge perceived risks and speculate within the markets.

2.2 Approaches to electricity market modelling

As aforementioned, the electricity market is different from the stock market due to its non-storability nature and volatility in demand and supply. This section aimed at discussing the different models applied within the electricity market with the final analysis reviewing the option pricing. This section will be reiterating the intrinsic market factors and their effect on the models and overarching goal of balancing prices on a real-time basis. The various models and characteristics will be found to be somehow correlated and included within the models to emphasize suitability in the electricity market. Some of these models will be reflective of models used in pricing traditional assets and commodities but will be specially applied to incorporate special aspects of the energy market that are not replicable with any other asset.

One such model that underscored this relationship is the Geometric Brownian Motion (GBM), one of the most common models used in finance to analyze stock prices Stojkoski et al. (2020). The GBM model is reflective of a weak form of the efficient market hypothesis as it assumes that the spot price reflects the totality of information on traded stocks. Price movements would thus be somehow independent of past price movements. These trends change when there are other factors influencing price movements. The GBM model works in the stock market because of the perceived stability in production factors of companies within a specified period Stojkoski et al. (2020). It is because of this predictability that the GBM model takes the form of a stochastic differential equation (SDE) that takes into account drift and volatility parameters of the stock.

The GBM is a continuous-time stochastic process used in price modeling. It has an additional feature of generating positive prices, an important aspect in financial application. A stochastic process S_t is said to follow a GBM if it satisfies the following SDE:

$$dS_t = \mu S_t dt + \sigma S_t dW_t. \tag{2.1}$$

where $S(0) = s_0$, the constants μ and σ represent the drift and volatility parameters respectively and dW_t denotes the standard Brownian motion increments over the same period of time.

The GBM is effective in pricing spread options, but this only applies when the no-arbitrage argument is applied. In the stock market, it is virtually impossible to exploit arbitrage opportunities between exchanges because of its entrenchment in the efficient market hypothesis. The fundamentals of a given stock are inherently similar regardless of market participants analyzing. Any differences in their perspectives, if any, will be immediately captured in the supply and demand trends in the market. In the electricity market, the non-storability aspect makes it practically impossible to apply this model. The fact that electricity cannot be stored means that there will always be price differences between regions based on production parameters and demand aspects Eydeland and Wolyniec (2003). The easiest way to think of this dynamic is, say, when comparing the electricity produced in different regions using different mechanisms like HEP, geothermal plants and fuel-driven generators. While the commodity produced is inherently the same, the production costs are higher in some regions resulting in a higher price. Actions of the participants will result in the property of spot prices reflecting a mean-reverting characteristic. The volatility structure of electricity prices is thus inherently different from those of stock prices. The GBM model cannot thus be used to model for the fat tails in electricity price distributions (high kurtosis) which cripples it as a means of reflecting the complex structure of the electricity market Stojkoski et al. (2020) and Abdellah et al. (2019). The shortcomings of the GBM model created the need for specialized models that can effectively price energy products like electricity.

There are two main types of models considered when seeking to take into account the characteristics of the electricity spot prices. These models include the spot-based models and the forward-based models. The relationship between spot and forward based models is explained by the extensive theory that considers aspects like the convenience yield and the storage cost. This is however impossible to apply when considering electricity prices given that electricity cannot be stored. This relationship was thus explained by applying the market price of risk which entails the specification of the risk-neutral probability, or alternatively using the Heath-Jarrow-Morton (HJM) framework which was traditionally developed to model forward interests rates by taking into account the aspect randomness Harrison and Pliska (1981). The HJM framework has also been applied in some instances in option pricing Cap (2021). This is done such that the HJM framework is used to determine the fair market value of a derivative contract enabling market participants to identify undervalued or overvalued options. These models would be applicable within the electricity market because their forward approach uses interest rate concepts to calibrate and specify dynamics of the future electricity prices Benth et al. (2007); Prazeres (2017).

The spot models, on the other hand, use the dynamics of the exogenously-given day ahead electricity price which is practically non-tradable but could be used to derive prices of future contracts, or more exotic contracts negotiated between interested parties. According to Thornton and Schwab (2010), the forward price approach was different from the spot models because they applied future and forward prices which ended up accounting for the complicated dynamics of the forward curve. The downside of such models was however seen in the existence of a mispricing which is virtually impossible to detect once data has been entered into the model. These differences made the spot models more appropriate in giving an accurate reflection of the electricity prices which would allow for accuracy in describing the problem in question mathematically and coming up with an acceptable solution for the same Cartea and Figueroa (2005). As such, the spot price approach was selected for application within the current context.

2.2.1 The spot price model

Spot price models are preferred because they take into account the fundamental characteristics of electricity prices. They are also effective in distinguishing between seasonal components of supply and demand from mean-reverting spikes (caused by short term factors). Benth et al. (2007) structured a typical spot price model as follows;

$$S_t = \Lambda(t) + \sum_{i=1}^{n} X_t^i, i = 1, 2, ..., n, \tag{2.2}$$

$$dX_t^i = a_i(m_i - X_i)dt + \sigma_i(t)dL_t^i. \tag{2.3}$$

For all $\mathbb{R}_+ \to$R, Λ is a deterministic function capturing daily and seasonal variations whereas X^i = $X_t^i : t > 0$,i = 1,...,n are the O-U processes.

2.2.2 Structural models

Structural models attempt to mimic the electricity market price formations as a factor of demand and supply. They derive inelastic demand as follows;

$$S_t = \bar{S}_t + X_t, \tag{2.4}$$

$$dX_t = (\mu + \lambda X_t)dt + \sigma dW_t. \tag{2.5}$$

where $X(0) = x_0$, $\bar{S}_t$ and X_t represents the seasonal and stochastic components respectively, and dW represent an increment to a standard Brownian motion. The price is obtained by getting the equilibrium of the level of demand and a deterministic supply function that should be nonlinear to cater for the presence of spikes.

Essentially, these equations obtain price by matching the demand with the deterministic supply function. Based on the volatility of these aspects, the supply and demand functions have to be non-linear to cater for the resulting price spikes. Another model that was considered was non-linear pure diffusion as captured in the Ornstein-Uhlenbeck (OU) model for spot power prices. The utility of this model is in the fact that it provides a series of prices with jumps that are reflective of spikes Barlow (2002). The OU process was especially fitting within the current analysis because it addresses the shortcomings of the GBM model. It does this by allowing for the transformations of time variables, which within the electricity price market basically includes the various inconsistencies in price influenced by market factors. This also takes into account the tendency to revert to the mean price of a commodity, hence it is considered mean-reverting Chaiyapo and Phewchean (2017).

There are various studies that have underscored the utility of these models and how they could be applied in the process of pricing commodities. Spot models were applied by Schwartz (1997) in a bid to account for mean reversion. These spot models were later extended by Lucia and Schwartz (2002) to extend these applications to introduce deterministic seasonality but these efforts failed to capture jumps. Structural models were found to work when reproducing price spikes on a qualitative level but were found to be too restrictive when they only explained spikes on account of demand factors and supply constraints in some cases. Kanamura and Ōhashi (2007) used the structural model to generate spikes in prices that fit data observed and compared to other models like the Box-Cox transformation model and the Jump Diffusion model. Conceptualizing the effect of that model would require delving deeper into these additional models.

2.2.3 Jump Diffusion Models

The jump-diffusion models also represent a modified version of the GBM. It is models electricity prices by applying the log-price as a one-factor Markov jump diffusion as captured in the following equation;

$$dP_t = \theta(\mu_t - P_t)dt + \sigma dW_t + h(t)dJ_t. \tag{2.6}$$

The equation can be elaborated by observing how the spikes are conceptualized. The jump direction and intensity are considered level dependent and correlated. In terms of actual prices in the market, this would mean that in the event that the price is high (with a corresponding high jump intensity), then jumps that are directed downward are more likely Chen et al. (2017) and Goswami et al. (2018). On the other hand, low prices are expected to yield jumps that are both rare and directed upwards. These trends are succinctly aligned to the mean reverting property as prices are expected to jump in the opposite direction to restore balance Fu et al. (2017). This is a realistic trajectory in capturing intensity patterns that revert to a deterministic mean level rather than moving towards the stochastic pre-spike value. This equation was modeled by Branger et al. (2009) but was later improved by Benth et al. (2007) when they introduced a gamma process to take into account the day to day market variations and the spikes that come with them.

In this one-factor Markov model, the differences in the spike regime is distinguished from the base (fundamental) regime by considering the deterministic threshold on the price process. The spike regime here is observed when the price is higher than a base value determined by market factors Fu et al. (2017). This is the best way to capture deviation of prices from the mean which fundamentally represents market discordance, something that when addressed will revert the price to its mean value (based on historical overall market factors). There is however the risk of difficulty in determining the threshold value. The factors that influence the direction of prices vary and have a level of uncertainty as regards to when they will be addressed to restore a sense of balance in the market. Thornton and Schwab (2010) also identified inconsistencies in the one-factor model because of the perceived zero volatility for futures contracts derived from the spot model especially if it has a perceived long term outlook (a delivery time which is more than a few weeks). These features end up explaining the stochastic nature of the models, because of the perceived difficulty (impossibility) of pricing futures contracts using the spot model.

2.2.4 Regime switching models

The regime switching models were introduced in a bid to separate price spikes from mean reverting pricesHuisman and Mahieu (2003). According to the regime switching model, the price movements are not dualistic but include a third unquantifiable regime Biswas et al. (2018) and He and Zhu (2017). As such, there is a normal regime, an initial jump regime and a third regime. What this means is that jump processes do not, in the fundamental sense, capture the short-lived characteristics of the power spikes which in most instances, cannot be effectively determined through mathematical models because they are qualitative in nature but are reflected quantitatively in the electricity price He and Zhu (2017). This makes the transition structure restrictive and infrequent, hence unforeseen. The Nord Pool prices, according to Weron et al. (2004) could be determined by fitting the jump diffusion and the regime switching model. This would work because the only thing that distinguishes them is the spike formation mechanism. Weron (2009) would later solve the problems in these models by introducing a two-state unobservable Markov chain that models the transitions from the aforementioned base and spike regimes by incorporating an aspect of faster mean reversion and greater volatility as captured in the following equations;

$$dP_t = \theta^1(\mu_t - P_t)dt + \sigma^1 dW_t \text{ (base regime)}, \tag{2.7}$$

$$dP_t = \theta^2(\mu_t - P_t)dt + \sigma^2 dW_t \text{ (spike regime)}. \tag{2.8}$$

While this model is effective, it comes with the additional need for the calibration of the hidden Markov chain that oversees the regimen switching dynamic Thornton and Schwab (2010). This model was found to have one setback which is in the fact that the stochastic base level is not considered in the model. This is because following the spike regime, the price is quickly reverted to the seasonal mean instead of taking it back to the pre-spike base level which would be more reflective of the market dynamics.

2.2.5 Multi-factor models

While attempting to develop a model that is more responsive to the electricity market, I analyzed the multi-factor model. The two factor model is the simplest form of the multi-factor model. The first factor in this model represents the base signal while the second factor represents the spikes Recchioni et al. (2021) and Balt (2019). There are different additions that have been made to the fundamental two factor model in order to improve it. Villaplana (2003) proposed the consideration of a two-factor jump-diffusion model that incorporates seasonality to improve the valuation of electricity future contracts Cai (2017). This was done as an extension of the effort made by Schwartz and Smith (2000) in commodity price modeling and Lucia and Schwartz (2002) inclusion of a jump component with a non-constant intensity process. The standard formation of the model was as follows;

$$dS_t = \lambda_X(Y_t - S_t)dt + S_t\sigma_S dB_t^1, \tag{2.9}$$

$$dY_t = \mu Y_t dt + Y_t\sigma_Y dB_t^2, \tag{2.10}$$

$$Y_0 = y_0. \tag{2.11}$$

Here λ_X is the rate of mean reversion. This process is non-stationary with the long-term mean Y_t following a GBM with drift μ and therefore the initial value y_0 is significant.

There were later improvements of the same model with Barlow et al. (2004) including Kalman filters for calibration. They later concluded that the two-factor model was applicable in the modelling of electricity spot prices in a more reliable way. The OU mean reverting model was once again proposed by Benth et al. (2007) with the inclusion of seasonality and spike dynamics. After including these aspects, they concluded that the model would be sufficient in capturing the aforementioned stylized features that influence electricity spot prices. These models were also discussed and their application in the pricing of Europeans call and put options on electricity contracts in the Nord Pool. The two-factor model was stepped by Thornton and Schwab (2010) into a three-factor driven model that uses a 2-Levy driven OU process. This was done with the goal of capturing the day to day variation in spot prices which would be later used to price future contracts in closed form. Other researchers, Meyer-Brandis and Tankov (2008) developed a statistical procedure for estimating the sum OU model which represents the price as a sum of Levy-driven OU processes. The efficacy of this improved model was captured when EEX data was used, and the salient features of spot prices discussed. The aggregate of these models so far (including the multi-factor, mean-reverting, jump diffusion models) could be extended to be applied in the pricing of derivatives like futures and forwards as compared to the use of other additive models.

2.2.6 Electricity Option Pricing

Summing up these models, it is clear that they could be applied within the context of a more liberalized electricity market. As more markets accept deregulation of the energy and electricity markets, there is an inherent need to develop responsive pricing models to address the market problems and dynamics that will arise with volatility. There will also be a need to provide responsive risk management strategies for these market players to hedge their positions and institute price control mechanisms. There are studies that already acknowledge the existence of risk in the electricity market and suggest real solutions to these risks. Pineda and Conejo (2012) proposed options as the best financial instruments to address the problems that come with these different types of risk. They went on to discuss and elaborate a multi-stage stochastic model to develop an optimal strategy for an energy producing entity that aims at applying risk averse strategies. These suggestions cited the preference for option contracts as opposed to forward contracts. Another research effort that was undertaken to address these issues was done by Albanese et al. (2012) which involved the presentation of a numerical algorithm for pricing electricity derivatives based on the factors that underlie the generation of underlying prices. They further studied the rate of convergence of the algorithm based on the Merton jump-diffusion model and applied it to calculate the prices and sensitivities within the European and Bermudan markets. These prices and sensitivities were tested considering the fact that the prices followed both a mean-reverting paradigm and constant jumps of varying magnitudes.

Caporin et al. (2012), on the other hand, proposed a Monte Carlo (MC) pricing framework that was based on a bivariate time series model. The bivariate model applied two key variables including temperature (which is a function of weather/seasonality) and energy prices which made the methods more appropriate for pricing as opposed to traditional methods. The fact that there are no closed form techniques to predict the direction of electricity prices when they are assumed to follow a Mean Reverting Jump Diffusion (MRJD) process prompted Nobaza (2012) to propose the use of efficient MC methods for pricing electricity forward contracts. This not only reduced the variance but also improved the accuracy of the model.

2.2.7 The Monte Carlo (MC) method for options pricing

Given the analysis of the electricity market and the price dynamics, I was able to observe that spread options essentially belong to the class of path-dependent options. This basically means that the options do not depend on the price of the commodity at maturity but the price trajectory it takes from the time it is purchased until it matures Burger et al. (2008). It also emerged that there are no closed form solutions for path-dependent options and this is what brought about the need to calculate them using numerical methods like the Monte Carlo simulation. The Monte Carlo method is widely used in the financial realm for option pricing for electricity Sodhi (2018). Theoretically, this method relies on risk-neutral valuation that takes into account various possible price paths and subsequently calculates the anticipated payoffs for each path discounted to current prices Guo and Lapenkova (2010). The utility of these methods is in the fact that they can effectively work when pricing a commodity like electricity, which is marred by constant fluctuations that are in some cases completely unpredictable and complex. They are however termed slow given time constraints, to be considered competitive as a strategy for valuation.

Efforts at improving the MC method were seen first in Guo and Lapenkova (2010) application in pricing swing options in the electricity market, and later in the reiteration by other researchers. Cheng and Liu (2012), for instance presented a variance reduction technique that was combined with the MC stochastic simulation model to improve the pricing process under the risk neutral measure. This lent credence to this model as a more accurate model for pricing compared to finite differences and binary tree methods. Discrete barrier options as priced by YunJiao (2013) were also observed to improve efficiency through variance reduction.

The analysis of studies on the Nord Pool have captured the need for the link between theory and the market operations. Incorporating theory works effectively in determining whether a market is well-functioning. This was partially the reason behind the analysis of the Nord Pool and the Kenyan context, to identify problems to be encountered within the current research framework and how they would affect the goals of the current study. One of the setbacks identified within the current study was the inability to find data that could be applied in the analysis of prices and the testing of identified models. Over the years, more countries have taken to deregulating their power markets but very few power exchanges have been established to address the challenges of such an operational environment. The Nord Pool by far represents the most robust market around the world making it the perfect case study to apply when seeking to introduce its system to emerging markets like Kenya. Because of the lack of sufficient data on the Kenyan market and an environment that has not undergone full deregulation, I intend to use data from the Nordic region to capture the efficacy of my models within an electricity market set-up. I found this data reliable because the Nord Pool is currently the most stable electricity market in the world, and this is the goal that I aim to achieve in the application of their financial models within the East African set-up.

Chapter 3

Methodology

The methodology section captures the means applied to meet the objectives cited in the introduction section. Within this study, I derived my methodology from the analysis and discussion of various forms of literature aligned to my study. As such, I found that electricity had distinct features which set it apart from other commodities and even alternative investments like stocks. The models analyzed were thus required to take into account these special features to ensure more accurate reflection of prices. It was observed that the models incrementally attempted to factor in the nuances and behavior seen in electricity markets through the analysis of different markets from the supply and demand standpoint.

Considering these aspects, my modeling framework is designed to factor in both jump patterns and mean reversion because of their effects on electricity prices in the Nord Pool. I also recognize that commodity prices have traditionally been quoted in terms of log prices Schwartz (1997), an aspect that was disputed by Villaplana (2003) because log transformation affected the estimation of the jump component and skewness of the data. This would have to be addressed to eliminate the possibility of having negative prices in the analysis. As aforementioned, the data used within the current analysis was derived from the Nord Pool due to availability and reliability of the data. The total observations included within the current analysis amounted to 1460 (N-1460), collected in the time spanning the beginning of 2016 to the end of 2019. This data was consulted because it was both recent and acquired from trusted sources. Any inconsistencies that coils have arisen in the data over the stated period were ignored. These inconsistencies were likely to be reflected in the margin of error during the analysis process. The analytical tool used in the analysis of data, using the discussed models was R programming undertaken within the Anaconda software. This software was chosen because of the familiarity with its applications and perceived effectiveness in solving these types of complex mathematical models, and especially when taking into account the inherent nature of the electricity market in the Nord Pool.

3.1 The mean reverting jump diffusion model

As discussed by Edward (2014), the spot prices were modelled using the MRJD model. Prior to applying this model, the fit of the jumps were tested to verify their following of an exponential distribution to go with the model.

Given the filtered $\mathbb{P}$ $(\Omega, F, \mathcal{F}_{t\in[0,T]}, S)$ assuming that the filtration $\mathcal{F}_{t\in[0,T]}$ satisfies the conditions that the filtration $\mathcal{F}_t$ is right continuous and F_0 contains all S-null sets. On this given probability space, it is assumed that spot electricity prices are governed by the process given

$$S_t = exp(f_t + X_t + Y_t), \tag{3.1}$$

$$dX_t = -\alpha X_t dt + \sigma dW_t, \tag{3.2}$$

$$dY_t = -\beta Y_t dt + JdN_t. \tag{3.3}$$

where $S(0) = s_0$, f_t is the seasonal term, X_t is an O-U process and Y_t is the spike process. J_t is an IID process used to model the jump size, N_t is the time in between the jumps and β represents the mean-reversion speed. W_t, N_t, and J_t are assumed to be independent. We will hence assume positive jumps and hence exponentially distributed jump sizes, and a Poisson distribution for the jump frequency, N_t.

Based on the analysis of prior literature on these models and their application in the analysis of electricity prices in the Nord Pool, I applied them with the derived data without significant adjustment of the underlying parameters.

3.2 Calibration of the model

Historical spot prices already incorporate aspects of seasonality which give a glimpse into the seasonal dynamics surrounding the pricing of electricity. There was a need to remove the seasonality aspect of the prices so as to better reflect market dynamics and how they influence prices going forward. This would ensure that model parameters are better reflected and that the model is more effective in pricing the options contract.

From the mean reverting jump diffusion model, estimating f_t, X_t and Y_t from knowing the historical spot price values S_t makes the process of estimation significant. The deterministic seasonal component is first estimated and after the removal of the seasonal component the OU process, X_t, is estimated.

3.2.1 Deseasonalisation

As earlier highlighted, seasonality is caused by changes in climatic conditions such as temperatures hence causing regular variation in demand. Deseasonalisation is the stripping of the seasonal component from the spot prices for estimation of the model parameters. A linear sinusoidal function is adapted by Villaplana (2003), Benth et al. (2008) and Edward (2014) defined as

$$f(t) = c + \sum_{i=1}^{6} a_i cos(2\pi\Lambda_i t) + b_i sin(2\pi\Lambda_i t). \tag{3.4}$$

Other techniques are a constant piece-wise function, for instance Knittel and Roberts (2001), of a one year period, where for each month one tries to determine an average value out of the whole analyzed time series. This is a flexible method, however it does not have the smoothness therefore having a negative impact on the statistical inference of the deseasonalized price process.

In the model Eq.3.4 a, b and c are constants and the first term is interpreted as the cost linked to power production. $\Lambda_1 = 1, \Lambda_2 = 2, \Lambda_3 = 4, \Lambda_4 = 365/7, \Lambda_5 = 2x365/7$ and $\Lambda_6 = 4x365/7$ calculate the weekly and annual seasonality. For the parameters to be estimated we assume the model of the form

$$S_t = exp(f(t)). \tag{3.5}$$

3.2.2 Ornstein-Uhlenbeck Process

The zero O-U process is a stochastic process following the SDE

$$dX_t = -\alpha X_t dt + \sigma dW_t, \tag{3.6}$$

where $X(0) = x_0$, α represents the mean reversion rate, σ represents the volatility and and W_t represents a standard Brownian motion. I calculated the stochastic part of the equation after deseasonalising. The O-U process is first calculated and the outliers used to calculate the spike prices, Y_t.

$$ln(S_t) - f(t) = X_t + Y_t. \tag{3.7}$$

For α and σ to be calculated, the solution for Eq.3.6 is needed.

$$X_t = X_0 e^{-\alpha t} + \int_0^t \sigma e^{-\alpha(s-t)} dW_s, \tag{3.8}$$

for $0 \leq s \leq t$.

I derived this from using the Ito's lemma to the function; $f(X_t, t) = X_t e^{\alpha t}$ to get

$$df(X_t, t) = \alpha X_t e^{\alpha t} dt + e^{\alpha t} dX_t = \sigma e^{\alpha t} dW_t, \tag{3.9}$$

taking $\mu = 0$, and integration from 0 to t gives;

$$X_t e^{\alpha t} = X_0 + \int_0^t \sigma e^{-\alpha(s-t)} dW_s, \tag{3.10}$$

hence

$$X_t = X_0 e^{-\alpha t} + \int_0^t \sigma e^{-\alpha(s-t)} dW_s. \tag{3.11}$$

The model is later simulated using the solution of the O-U process, by generating multiple paths through the MC simulation.

The main properties of O-U process include conditional distribution and first moment and stationarity. Given an O-U process, Eq.3.6, the conditional distribution of change is given by

$$X_{t+\Delta t} - X_t e^{-\alpha \Delta t} N(0, \frac{\sigma^2}{2\alpha} - (1 - e^{-2\alpha\delta t}). \tag{3.12}$$

I later derived a Maximum Likelihood Estimate (MLE) from the above conditional distribution. Given a set of observations (S$_0, S_1, S_2, ..., S_n$),the MLE estimation for the O-U process, Eq.3.6, as stated by Edward (2014) is given as

$$\alpha = -\frac{1}{\delta} log(\frac{S_{xy}}{S_{xx}}), \tag{3.13}$$

$$\hat{\sigma^2} = \frac{1}{n}(S_{yy} - 2e^{-\alpha\delta} S_{xy} + e^{-2\alpha\delta} S_{xx}), \tag{3.14}$$

$$\sigma = \hat{\sigma^2} \frac{2\alpha}{1 - e^{-2\alpha\delta}}, \tag{3.15}$$

where δ is the time step and

$$S_{xx} = \sum_{i=1}^{n} S_{i-1}^2, \tag{3.16}$$

$$S_{xy} = \sum_{i=1}^{n} S_{i-1} S_i, \tag{3.17}$$

$$S_{yy} = \sum_{i=1}^{n} S_i^2. \tag{3.18}$$

3.2.3 The Spike Process

Spikes signify price jumps that go beyond three standard deviations from the mean prices. After deseasonalisation, spikes become integral in the effective consideration of the factors influencing future price changes. This aspect will be applied to further improve the model.

From Eq.3.1, after estimation of the O-U process, I used the outliers to estimate the spike process given as:

$$dY_t = -\beta Y_{t-}dt + J_t dN_t. \tag{3.19}$$

where $Y(0) = y_0$, J_t is distributed exponentially, N_t takes a Poisson distribution and β represents the rate of mean reversion. J_t is an Independent and Identically Distributed (IID) process used to show the jump size, N_t is the time in between the jumps.

I estimated the jump component from the deseasonalised log-prices d_t through a two-step procedure. First, all jumps—defined as price increments exceeding 3 standard deviations of all price changes are removed from d_t. Next, the intensity and the distribution of the magnitude J_t of the jumps is estimated from these few selected points Bierbrauer et al. (2004).

3.3 Option Pricing

To facilitate the option pricing process, I applied two key models, the Monte Carlo method and the Girsanov Transform. The various equations and components of these models are shown in the subsequent section.

3.3.1 The Monte Carlo Method

I introduce the MC method in this section to calculate the expectation

$$E_Q[exp{-}rT(exp\{S_T\} - K_+)]. \tag{3.20}$$

If X is a random variable with values in $\mathbb{R}$, then by MC E(f(X)) for each function f on X can be estimated by;

$$\overline{E}f(S) \simeq \frac{1}{n}\sum_{n=1}^{N} f(S_n). \tag{3.21}$$

where $S_1, S_2, ..., S_N$ are independent simulations of S.

With Nord Pool day-ahead trading is done across 21 bidding zones and in 14 countries. All the 21 bidding zones have different prices but the system price is used, which is calculated for every delivery hour. My approach consists of approximating the general averages of the 21 bidding zones after the Monte-Carlo simulation applying the known option pricing models to price an exotic spread option. Hence the payoff is represented as;

$$exp{-}rT(\{\frac{1}{n}\sum_{n=1}^{N} S_T^n\} - S_T^p, 0)^+), \tag{3.22}$$

where $S_t^1 ... S_t^n$, $0 \leq\ t\ \leq T$ represent the bidding areas' prices, S_t^p is the system price (strike price).

3.3.2 The Girsanov Transform

Pricing is done in the risk neutral measure, which assigns probabilities so that the discounted price (discounted at a risk free rate) is a martingale. Harrison and Pliska (1981) states that a market is complete if and only if its vector price process has a certain martingale representation property. According to Krishnan (2013), the martingale property states that given all known information about the prior events, the future expectation of a stochastic process is equal to the current value such that

$$V_t = \mathbb{E}_t[V_T e^{-rT} e^{-rt} | \mathcal{F}_t], \tag{3.23}$$

$$V_t e^{rt} = \mathbb{E}_t[V_T e^{-rT} | \mathcal{F}_t], \tag{3.24}$$

$$\mathbb{E}_t[V_T e^{-rT} | \mathcal{F}_t] = V_t e^{rt}, \tag{3.25}$$

$$t < T, \tag{3.26}$$

where V_t is the payoff at maturity, r represents the discounting rate and T represents the date of maturity.

Let W $=:$ $\{\mathrm{W}_t : 0 \leq t \leq T\}$ be a $\mathbb{P}-$ Brownian motion and $\lambda = \lambda_t : 0 \leq t \leq T$ an adapted process that satisfies the Novikov condition Shreve (2004) derives the Girsanov's process as

$$\mathbb{E}[e^{\frac{1}{2}\int_0^T \lambda_t^2 dt}] < \infty. \tag{3.27}$$

For $0 \leq t \leq T$ define the density process

$$Z(t) = exp\{\int_0^t \lambda_s dW_s - \frac{1}{2}\int_0^t \lambda_s^2 ds\}. \tag{3.28}$$

Then, a new probability measure, Q $\sim \mathbb{P}$, can be defined such that

$$\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}} = Z_t, \tag{3.29}$$

and

$$W_t^{\mathbb{Q}} = W_t - \int_0^t \lambda(s) ds, \tag{3.30}$$

such that $\mathbb{Q}$ is a Brownian motion.

The Girsanov's theory is used to make a change of measure in the levy process in order to reduce the initial problem into a pricing problem.

Chapter 4

Results and Discussion

4.1 Seasonality

The first part of my analysis involved the deseasonalisation of the time series data obtained from the Nord Pool. Seasonality refers to the fluctuation of prices based on seasonal effects like the weather among other aspects. These fluctuations in demand and supply bring about seasonality of spot electricity prices. The removal of the seasonal component allowed for the analysis of secular, cyclical and irregular variations. Identifying the component of seasonal variation in prices and nullifying the effects is very important when seeking to price an option. In this way, I was able to get rid of seasonal noise to improve my model and its final results.

I used the truncated Fourier Transform to perform this first part of the analysis. This is because when applied to a polynomial multiplication, the transform has the property of eliminating jumps which can be explained by seasonal effects as earlier explained. The truncated Fourier transform is defined by an elaborate equation which is defined as well as the calibration estimates that are applied in my model. The graph (Figure 4.1) effectively captures the exaggerated jumps due to seasonalisation and the actual prices determined following the deseasonalisation of the data analyzed.

The truncated Fourier Transform adapted by Benth et al. (2008) in Eq.3.4 can now be written as

$$\begin{aligned} f(t) = c &+ a_1 cos(2\pi\Lambda_1 t) + b_1 sin(2\pi\Lambda_1 t) + a_2 cos(2\pi\Lambda_2 t) + b_2 sin(2\pi\Lambda_2 t) \\ &+ a_3 cos(2\pi\Lambda_3 t) + b_3 sin(2\pi\Lambda_3 t) + a_4 cos(2\pi\Lambda_4 t) + b_4 sin(2\pi\Lambda_4 t) \\ &+ a_5 cos(2\pi\Lambda_5 t) + b_5 sin(2\pi\Lambda_5 t) + a_6 cos(2\pi\Lambda_6 t) + b_6 sin(2\pi\Lambda_6 t). \end{aligned} \tag{4.1}$$

We present the calibration estimates in Table 4.1.

Parameter	Estimate
c	4.2232696
a_1	-0.4647226
b_1	0.0795942
a_2	-0.2431276
b_2	-0.1512506
a_3	0.0621667
b_3	-0.0352138
a_4	-0.0001408
b_4	-0.0099973
a_5	0.0014018
b_5	-0.0043550
a_6	0.0006674
b_6	-0.0003895

Table 4.1: Estimated Seasonality Co-efficients

Plotting the daily average Nord Pool market system prices from January 1, 2016 until 31st December, 2019, Figure 4.1 shows the seasonality superimposed on the Nord Pool series. This shows that the analysed day-ahead prices exhibit strong seasonality as earlier discussed.

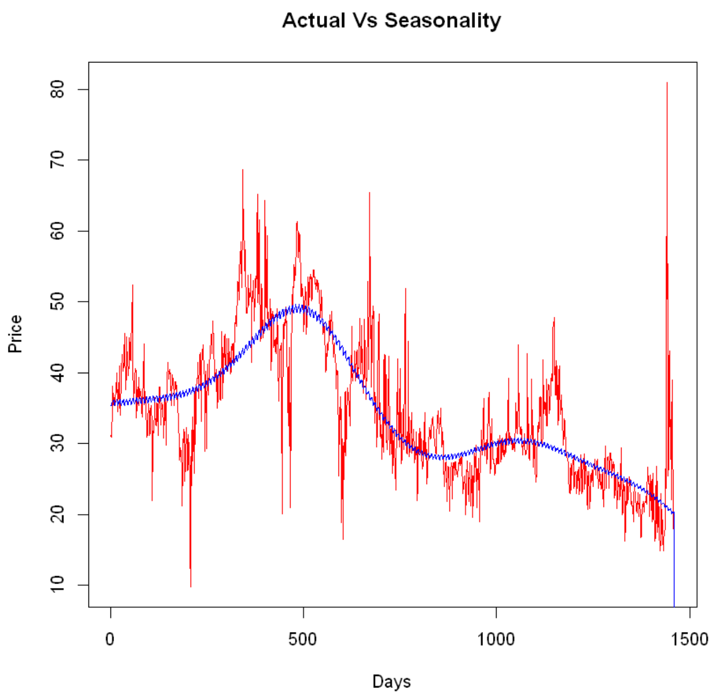

Figure 4.1: Actual Prices Vs Seasonality

4.2 The O-U and the Spike process

The deseasonalised log prices were applied in this section for parameter estimation. After removing the seasonality, the deseasonalised log prices in Figure 4.2 are used to calibrate the O-U process for parameter estimation.

$$ln(S_t) - f(t) = X(t) + Y(t). \tag{4.2}$$

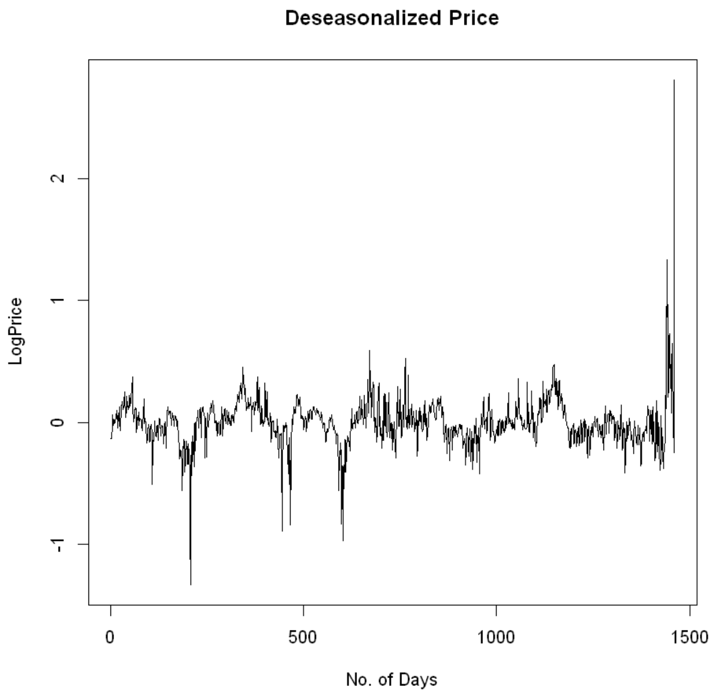

Figure 4.2: The change each time step for the deseasonalised logarithmic price

The X(t) process (which is an O-U process) is first estimated, ignoring the spike process Y(t) which will be later estimated. The estimated O-U parameters as shown in Table 4.2 are obtained using MLE Eq.3.13 and Eq.3.14.

Parameter	Estimate
α	62.3890
β	0.0093
σ	2.3366

Table 4.2: Estimated O-U parameters

The spike process, also mean-reverting, is then estimated after the O-U process process and the estimated parameters are shown in Table 4.3.

Parameter	Estimate
α	62.3890
β	0.0093
σ	2.9151

Table 4.3: Estimated Spike parameters

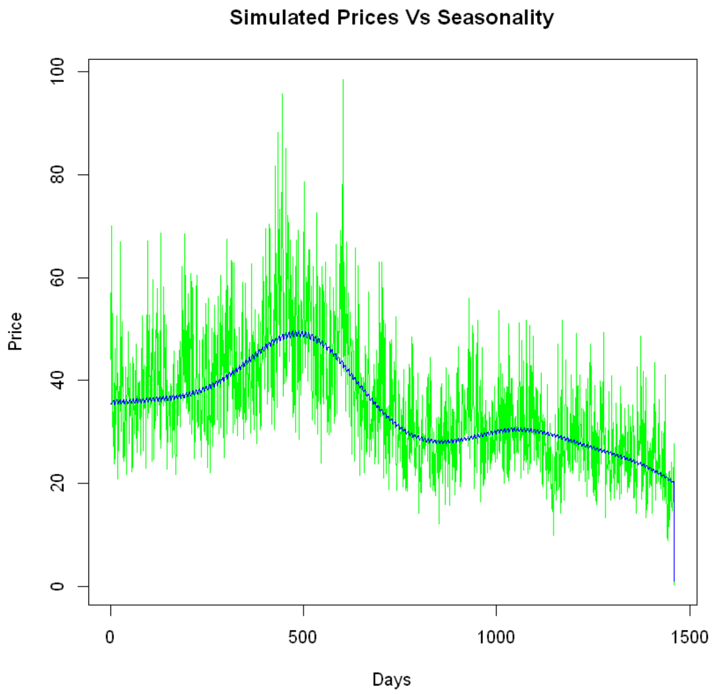

Figure 4.3: Simulated Prices Vs Seasonality

A fully calibrated model is hence obtained. A MC simulated model over four years spot prices is obtained, though not an exact fit of the old data. The seasonality and the jumps occurrence are observed. This can be seen by comparing the realisations to the historical data in Figure 4.4.

4.3 Pricing an Option

The spread options payoffs are significant to the suppliers and reflect the level to which the option will be applied for hedging or risk management depending on the side of the trade an individual is in. A spread option is a type of option that gets its value from the difference between the prices of two or more assets whose values at time t

Figure 4.4: Actual Prices Vs Calibrated Prices

we denote by $S_1(t)$ and $S_2(t)$ Carmona and Durrleman (2003). We consider only options of the European type for which the buyer has the right to be paid, at the maturity date T, the difference $S_2(T) - S_1(T)$, known as the spread. To exercise the option, the buyer must pay at maturity a pre-specified price K, known as the strike, or the exercise price of the option. The payoff of a spread option at maturity T is $(S_2(T) - S_1(T) - K)^+$.

I analysed an aggregate spread option, calculated by getting the difference between the average price of the n suppliers in the Nord Pool Market and the actual market price (System Price). I used different maturity periods, 1 month maturity, 3 months, 6 months and a year, for my pricing model in order to analyze the changes over time and the trends realized from my calculations. With the strike prices of 40 and 70 for call and put options respectively chosen randomly from the pricing data. I used a 1-year Libor (London Interbank Offered Rate) based on the fact that the prices had been recorded in Euros and my case study was in Europe (used Euros as the currency). I applied different strike price levels arbitrarily to test the models and their efficacy when pricing an option.

The spread call option payoff is defined as

$$e^{-rt}(\frac{1}{n}[S_t^1 + S_t^2 + ... + S_t^n] - S_t^p - K)^+, \tag{4.3}$$

where $S_t^1 ... S_t^n$ = suppliers electricity prices, for $n = 21$ (number of suppliers), r = 0.46%, S_t^p is the System Price, K representing the strike rice, and t is given as 1 month, 3 months, 6 months and 12 months.

We therefore obtain the call options payoff as

Maturity	K =40
1 month	0.8835192
3 months	0.5460755
6 months	6.242244
1 year	10.678

Table 4.4: Estimated Nord Pool Call Option Using Monte Carlo Simulation

For a put option we are going to have the payoff as

$$e^{-rt}(K - S_t^p - \frac{1}{n}[S_t^1 + S_t^2 + ... + S_t^n])^+. \tag{4.4}$$

Maturity	K =70
1 month	27.01648
3 months	19.28395
6 months	6.865424
1 year	4.20213

Table 4.5: Estimated Nord Pool Put Option Using Monte Carlo Simulation

From a theoretical standpoint, the price of an option rises as the underlying price of the asset exhibits movements towards the strike price. The prices of the call options fluctuated depending on the maturity specified. At the strike prices of 40 euros, the price of the option ranged from 0.88 euros for 1 month maturity to 10.678 euros for a 1 year maturity. The model showed that the longer the time horizon specified, the higher the amount of the option.

The put options had higher strike price because of the need to buy at lower prices and sell at higher prices. Wholesale distributors of electricity also preferred this option because it allows them to sell electricity at a higher price when the overall prices are lower. This hedging technique allows them to offset the cost of production and even earn profits. Because of the fundamental nature of a put option (buying electricity at a high price in a market with low prices is riskier), the put options turned out to be more expensive compared to the call options. The short term puts were also considerably more expensive as compared to the long term options. For the strike price of 70 euros, the 1 month put option was priced at 27.016, while the 1 year put option was priced at 4.202 euros. The model showed that the longer the time horizon specified, the lower the amount of the option.

Chapter 5

Conclusion

5.1 Conclusions and recommendations

The current study was developed with the aim of pricing electricity price options within a vibrant market. The spot prices were simulated by using data from the Nord Pool which is the most reliable electricity prices around the world. In the analysis, the electricity prices were modelled through the MC simulation of the MRJD model. Other models were also explained within the paper with a view to further deconstructing the nature of the price movements in an electricity market and how it changes over time. These models were important because they contributed to the understanding of the dynamics, and eventually contributed in the development of the final model. In the long run, the model was able to capture fundamental characteristics like seasonality, mean-reversion of price spikes and how they influence the pricing of options. The outcomes of the analysis pointed to the need for improvements that would be incorporated if stochastic volatility is integrated as well as separate mean reversion rates for spikes and the jump process.

5.2 Limitations of the study

This study had originally been inspired by the need to stabilize electricity prices and improve reliability of power transmission and distribution companies in Kenya and the larger East African context. In terms of meeting this goal, there are various constraints that were encountered in the process. In the initial analysis of the research, I indicated that the Kenyan market had not been completely deregulated to allow for healthy competition between firms seeking to transmit and distribute electricity. This resulted in a situation where there was no data specific to the region to be used in the financial modelling of electricity prices and options. Data from the Nord Pool which is more robust and reliable was thus used within this analysis. Because of these constraints, it was difficult to employ these results to the local context. The analysis and the outcomes seen in the study could however be considered exploratory research on how a more competitive electricity market could address the current problems faced within the Kenyan and larger East African context. As the country expands its power production efforts, there will be a need to improve efficiency in deployment and stability of prices. This will come in handy in the event that the production exceeds local supply and there is a need to sell to the neighboring East African countries. The models will be crucial for entities looking to hedge their production against unpredictable supply and demand factors.

5.3 Ideas for future research

The same models could be applied to model electricity prices within the local context but using a set of data that is more reflective of local conditions. The electricity market is regulated in the country but the production and connection costs undergo fluctuations which end up affecting prices within the larger context. Using historical data, these dynamics can be assessed with a view to planning for volatility and varying costs. This could be taken up as an idea to spur a research that is parallel to this study and is trained towards addressing the same problems this study had been designed to address.

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Appendix

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Document Information

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Entire Document

Stochastic Modeling of Electricity Prices and Option Pricing Philgonah A. Omungoh Proposal presented in ful llment of the academic requirement for the degree of Masters in Mathematical Finance of Strathmore University November 2021

Declaration and recommendation Declaration This proposal is my original work and has not been submitted or presented for assess- ment in any institution Signature .... Date Philgonah A. Omungoh Recommendation This proposal has been submitted for assessment with our approval as supervisors ac- cording to Strathmore University regulations. Approval The thesis of Philgonah A. Omungoh was reviewed and approved by the following: Dr. Samuel Chege Maina Senior Lecturer, Strathmore Institute of Mathematics Strathmore University Meleah Oleche Subject Group Leader, Mathematical Finance Strathmore University Ferdinand Othieno Acting Dean, Strathmore Institute of Mathematics Strathmore University ii

Abstract Volatility and abrupt price changes is a problem that has marred the electricity mar- ket for decades. This problem is especially observed in deregulated markets whose prices are in uenced by supply and demand factors. Another consideration is the fact that electricity is non-storable which means that its prices are quite di cult to con- trol. In an e ort to address these problems, the current study was developed to price electricity and options used to hedge against volatility and unexpected price jumps. The mean reverting jump di usion was applied by taking into account day ahead spot prices derived from the Nordic electricity market or the Nord Pool. To price spread options, I applied the Monte Carlo simulation model. The analysis of the data was undertaken through R programming undertaken within the Anaconda software. The need to price electricity options was to furnish market participants with instruments to manage the nancial risks that come with price volatility due power failure and demand factors. The analysis shows the complex nature of electricity pricing, hence there is no closed form solution for pricing these derivatives. While the study ndings were not directly applicable to the Kenyan and East African context, it provided a ro- bust context for future research especially as the need for a deregulated market grows in the country. Key words: Electricity prices, Mean-reverting, Di usion, Monte-Carlo simulation, Spread Option iii

Acknowledgment First I thank God for his presence and provision, I thank my family especially my sis- ter for being my support system, my classmates for the unity and all the lecturers for their guidance and dedication. viii

Dedication \... there is one elementary truth that ignorance of which kills countless ideas and splendid plans: that the moment one de nitely commits oneself, then Providence moves too. All sorts of things occur to help one that would never otherwise have occurred. A whole stream of events issues from the decision, raising in one's favor all manner of unforeseen incidents and meetings and material assistance, which no man could have dreamed would have come his way. Whatever you can do, or dream you can do, begin it. Boldness has genius, power, and magic in it. Begin it now." W. H. Murray This thesis is dedicated to my family for all their support, love and care. ix

Abbreviations DA Day Ahead EEX European Energy Exchange GBM Geometric Brownian Motion HEP Over Hydro-Electric Power HJM Heath-Jarrow-Morton IID Independent and Identically Distributed KenGen Kenya Electricity Generating Company MC Monte Carlo MLE Maximum Likelihood Estimate MRJD Mean-Reverting Jump Di usion MWh Mega-Watt hours NASDAQ National Association of Securities Dealers Automated Quotations OTC Over The Counter O-U Ornstein Uhlenbeck SDE Stochastic Di erential Equation x

Chapter 1 Introduction 1.1 Background to the study The energy sector in Kenya has undergone vast reforms in the past few decades. The rst wave of reform according to Godinho and Eberhard (2019) occurred in 1996, as captured in the Electric Power Act of 1997 Kiiru (2002). This reform was externally motivated and driven by donors that favored the separation of commercial activities from policy and regulatory functions. These reforms also saw the uncoupling of the power generation function from the transmission and distribution function. The sec- ond wave of reform occurred amidst the adoption of a new political order which over- saw the partial privatization of the Kenya Electricity Generating Company, KenGen Godinho and Eberhard (2019). The reforms contributed signi cantly towards opening up the country as one of the most favored private energy investment destinations in the region. Partial privatization came with added advantages of operational e ciency but resulted in perceived problems that come with movement from cost-re ective tar- i s to market driven cost dynamics. Before these reforms were integrated, electricity prices in Kenya had been controlled by government backed regulators. These intermediaries ensured that the price of elec- tricity was relatively stable with minimal variations. At the time, the prices of elec- tricity were largely determined by the cost of production and the fact that the indus- try was monopolize, there were no excessive market factors in uencing the price of electricity. These changes created a situation where the spot prices were determined by supply and demand forces. This situation came with unforeseen challenges of price controlled and undesirable e ects for the nal consumer Gil-Alana et al. (2017). The challenges underlying the electricity market are compounded by the fact that electric- ity cannot be stored but has to be transformed into other forms of energy. As a result, 1

the cost of electricity has become excessively volatile and unstable largely due to mar- ket uctuations, an aspect that has to be addressed going forward. 1.2 Problem statement The overarching problem of the Kenyan electricity industry is the high cost of electric- ity coupled with uctuating prices. This problem is an e ect of market factors of sup- ply and demand. Kenya predominantly relies on Hydro-Electric Power (HEP) which is dependent on seasonal weather patterns. While Kenya is working towards diversifying its power sources, its challenges range from the power rationing of the early 2000s and power crises of the 2010s Mutiso and Taneja (2018). All of these problems link back to one problem, e ective nancial modelling of supply and demand factors to create a better planning framework to address issues with access, pro tability and manage- ment. The issues plaguing the electricity market have resulted in a wide range of problems including stagnating demand, the fallout with large industrial power users, consumer dissatisfaction with pricing and unreliability of the service. Demand has been stag- nating because of the high upfront costs of connection and reduced consumption by existing customers which is a big threat to the electricity business model. Industrial power users account for more than 50 per cent of consumption Mutiso and Taneja (2018). Dissatisfaction with cost has led some corporations to set up their power gen- eration plants either using coal, HEP or solar to create better control systems for their production costs. In periods of low supply, the power generation companies use gen- erators which depend on oil prices, an external variable that comes with added price

uctuation and currency exchange risks. E orts at harmonizing prices have failed as these risks are aggregated and bundled into the electricity price without due consid- eration of the e ect on the nal customer. Some strategies like reduced night usage tari s have also failed because of their short-term e ects. 2

1.3 Objectives of the study This study was developed with the aim of developing a more e ective pricing strategy for the electricity market. It narrowed down to the mathematical modelling process looking to address inconsistencies and uncertainties surrounding the pricing of elec- tricity in the country. As such, the objectives of the study were outlined as follows; 1. To examine stochastic modelling techniques used for energy market price dy- namics; 2. To calibrate electricity

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market prices using a mean-reverting stochastic model with jumps; 3. To price an electricity aggregate price spread option using Monte Carlo (MC) simulation given underlying mean-reverting seasonal stochastic model with jumps; 1.4 Signi cance of study The Kenyan electricity market is largely a monopoly. This means that electricity users have to depend on one entity for their power needs. The downside of this situation is that any adverse outcomes cannot be addressed by seeking a better electricity provider. The cost of electricity trickles down to the cost of other goods and services, given that electricity is essential for the operation of di erent sectors including the manu- facturing, processing companies, educational institutions, commercial and residential establishments Gil-Alana et al. (2017). The problems with large uctuations can be addressed by identifying a framework for stabilizing prices to make them more pre- dictable and friendly to the dependent industries and the larger public. Botterud et al. (2010) observed that nancial risk management has become an essential tool for reducing risk exposure in volatile electricity markets. The understanding of the short term and long term prices of electricity have the e ect of furnishing market players with the right information to plan for adequate development of supply and demand. This enables them to develop sustainable electricity markets both in terms of cus- tomer satisfaction and cost-control Weron et al. (2004). My project is aimed at ap- 3

plying a stochastic model that enhances the understanding of electricity spot prices and pricing derivatives that could be strategically applied to manage risks and devise market development strategies for the future. 4

Chapter 2 Literature review In the literature review section, I reviewed a range of sources in an e ort to decon- struct di erent aspects of electricity pricing and the models used within those con- texts. One of the key outcomes was gleaned from the analysis of the Nord Pool, an entity that runs the main power market in Europe. From that analysis, the review delved into the fundamental aspect of the electricity market, the electricity options market and various approaches to electricity price modelling. Summing up this knowl- edge would give a theoretical and practical basis for the application of stochastic mod- els in the pricing of electricity in Kenya to address problems of exorbitant pricing and customer dissatisfaction. Risk management has become an essential part of the electricity market. Compa- nies have to apply e ective hedging strategies to protect their pro tability going for- ward and allow them make better decisions as regards to provision of services Souhir et al. (2019). There are various hedging instruments that have been developed over the years to facilitate the risk management e orts of the di erent stakeholders of the electricity market. Investors and key actors in the electricity market have taken to adopting hedging strategies that employ the use of nancial instruments like forward contracts, options and futures contracts Canelas et al. (2020) and Castro (2017) . These derivatives will form an essential part of the discussion of the nancial instru- ments that could be used to improve electricity pricing in Kenya. Forward contracts are basically derivative contracts made between two parties to buy or sell an asset or commodity at a future date. Options contracts, on the other hand, are created based on the perceived underlying value of an asset or commodity Zhao and Huchzermeier (2017). Depending on the outlook of an investor, they could either opt for a call op- tion or a put option. A call option gives the holder the right but the obligation to buy the speci c asset or commodity at the stated price within a speci ed time frame. A put option gives them the right to sell the asset or commodity at the stated price 5

within a speci ed time frame. A futures contract is an agreement backed by a legal framework to buy or sell a particular commodity or asset at a predetermined price at some point (speci ed) in the future. While forward contracts are traded over the counter and are customizable, futures contracts are standardized and traded in ex- changes Carbonneau and Godin (2021). Despite the di erences, these instruments are used to reduce risk exposure electricity markets Botterud et al. (2010). Within the scope of this study, I will be looking to discuss the extent to which nancial models have been applied in advanced markets to address problems of spikes in electricity prices and shortage. 2.1 The Nord Pool The Nordic region is an example of a market that has a long experience with a re- structured electricity market Botterud et al. (2010) and Uribe Gil et al. (2020). The nordic electricity market, also referred to as the Nord Pool was established in 1992 in the wake of deregulation of the energy markets in Norway Weron and Zator (2014). Within the next few years, other countries including Denmark, Finland and Sweden joined in to form the rst internationally known power exchange. Over time, this market grew both in volume and e cacy in stabilization of power rates and hedg- ing strategies for the larger electricity market of the countries that joined it. Subse- quently, the Nord Pool has been able to facilitate the e cient and secure delivery of power across Europe from a reliability standpoint Janke et al. (2020). Fundamentally, the Nord Pool works by the creation and development of a range of Nordic power derivatives traded through quarterly, semi-annual and annual forwards, daily and weekly futures, options and contracts Uribe Gil et al. (2020). The Nord Pool de nes its future contracts as standardized and exchange traded. They however do not con- sider these agreements marked-to-market for the future delivery of electricity at some point in the future Beigaite et al. (2018). Spot prices within the Nord Pool are de- termined by the intersection between supply and demand factors. Because of these features, the Nord Pool was acquired by the NASDAQ and undertaking that led to its change of name to NASDAQ OMX Commodities Europe especially as more European countries like Germany and the UK joined in Weron and Zator (2014). The market 6

has grown much wider to accommodate Dutch, German and UK power futures and forwards, and even UK Natural Gas futures and carbon products Weron and Zator (2014). Today, an estimated 360 companies from 20 countries in the European region trade on the Nord Pool market, a feat that lends credence to its e ectiveness in stabi- lizing energy prices and protecting investors from excessive risk. 2.1.1 Characteristics of the Electricity Market Developing models for pricing electricity forwards, futures and options require, rst, the understanding of the structure of electricity markets. The spot prices in the elec- tricity market are fraught with uctuations in uenced by perceived patterns and sea- sonal factors. These dynamics are fundamentally explained from a theoretical stand- point. All markets show varying patterns of price changes and these patterns depend on the various aspects of the primary market Uribe Gil et al. (2020). One key aspect of electricity markets that cannot be understated is the need to balance supply and demand to avoid rationing, unexpected blackouts and abrupt price changes. This need is required to bring a sense of certainty through risk management, and as a result, sta- bility in electricity prices. 2.1.2 Supply and demand dynamics The deregulation of energy markets inadvertently results in the dependence on mar- ket factors like supply and demand. Supply within electricity markets is measured in terms of generation units within a speci ed region. Supply is measured in terms of marginal cost of production and the response. These two aspects basically depend on the e cacy of the generation company in producing power and the ability to transmit it to the required markets on time Weron and Zator (2014) and Souhir et al. (2019). There are a range of factors that a ect the supply of electricity. Some of them include

uctuation of fuel prices (in case generators are used), and in some instances outages due to planned maintenance. These dynamics vary within the Kenyan context, with one of the pertinent issues being seasonal weather issues. During the dry season, low volume of water results in lower production units hence inability to meet demand. 7

Other problems that have been observed include transmission constraints, misman- agement and unforeseen circumstances. Demand varies depending on the region. Within the Nordic context, demand uc- tuates due to seasonality (due to climatic conditions). During winter, there is more demand for electricity as opposed to summer. This is due to the need for heating to keep families and other establishments warm. In regions like the US, summer signi- es peak demand to increased use of air conditioning to address high temperatures and humidity Weron and Zator (2014). These dynamics also vary considerably in the Kenyan and east African context. Here, the main issues come with daily and weekly demand. At day time, there is more demand for power due to industrial operations and business operations. In households, demand peaks hourly with the highest con- sumption seen in the evenings after work or during the weekends, when most individu- als spend their time at home. Some of the interventions that the government has tried to institute to address the demand issue is to incentivize companies to extend their operational frameworks to night time. That was partially the inspiration behind the perceived movement of the Kenyan economy to a 24 hour economy. Since HEP cannot be stored, increasing consumption at night would work as a means of creating addi- tional demand to balance these market factors. The failure of these interventions has resulted in calls for more deregulation to make the electricity market more competi- tive. Meanwhile, the Nord Pool has long applied nancial models that are tailored for their electricity market and that have been incredibly e ective in addressing problems faced within the industry. 2.1.3 Day ahead market The Nord Pool works in terms of a day-ahead market due to the impossibility of hav- ing a classical spot market Janke et al. (2020). System operators essentially need ad- vanced notice to verify the viability of a contract in terms of feasibility of production and transmission amidst existing constraints Weron and Zator (2014). These Trans- mission System Operators (TSO) are tasked with ensuring that the energy sold within the exchange market is delivered to the buyer. Any shortfalls have to be subsequently bought from the spot markets once again capturing the supply and demand factors of 8

the market Weron et al. (2004). This feature allows half-hour market changes hence enabling demand and supply to match instantaneously as retailers acquire electricity from wholesalers Lundin and Tanger as (2020). The analysis of the day ahead Nord Pool prices show an aspect of spikes that are fundamentally in uenced by the fact that electricity cannot be stored and has to rely on market factors for price determi- nation. Figure 2.1: Day-Ahead NordPool Prices Spikes are also caused by increased demand when supply is not at maximum due to power plant failures. Figure 2.1 exhibits both positive and negative prices. Electricity prices move rapidly to a mean level reverting towards the cost of production making it apt for the mean-reverting models to be used. Long term market analysis has shown the tendency of prices to revert to a mean level when the supply issues are addressed. 9

This is the trend that underscores the application of the mean-reverting model. Close scrutiny of price changes have shown how correlated price levels are to the de- gree of volatility in the market Janke et al. (2020). Once again, these changes are in-

uenced by the fundamental nature of the electricity market which is impacted by non-storability of electricity, and the variation in demand and supply factors. These changes are temporal and occur on a daily, weekly and seasonal basis Benini et al. (2002). This form of uncertainty informed the integration of the non-stationary model to re ect the perceived long term price uctuations of the market Beigaite et al. (2018). This is another feature that captures the extent to which the Nord Pool is in touch with the underlying dynamics of the electricity market. The aggregate of these dy- namics show the need for contracts that can help institute price control mechanisms and calm the volatile electricity market. 2.1.4 Electricity Options Market A range of contracts are o ered within the Nord Pool exchange. These contracts in- troduced exchange trading activities for market participants allowing them to seek arbitrage opportunities and maximize the optionality of their contracts Ottesen et al. (2018). The universality of these contracts and inclusion of the various market play- ers are e ective in price control hence they are preferred to over-the-counter private contracts that have an aspect of exclusion. The di erent electricity market stake- holders including generation companies, transmission and distribution entities were thus a orded an opportunity of using electricity derivatives (spinned o from nancial derivatives) for risk management given the volatility of spot prices in the market Rios et al. (2019) and Piao et al. (2019). Each day is broken into hourly spot contracts, which are as aforementioned, determined by the intersection between supply and de- mand. This pricing mechanism is e ective as it ensures there is no congestion at any point in time, and it re ects an e cient market since all the available information is perceived to be included in the price Alshehri et al. (2017). These dynamics are in line with the e cient market hypothesis which hypothesizes the fair market value of an asset or commodity when market players are perceived to have included all factors to be considered when getting into the available contracts Sk anberg (2012). 10

The Nord Pool has a diversi ed market that incorporates both standardized and pri- vate contracts within its options pool Uribe Gil et al. (2020). These instruments are designed as a hedging mechanism and as a means of speculating di erent outcomes in the spot and forward markets. Apart from traditional (vanilla) options, exotic op- tions are allowed to give the market participants a wider range of instruments that can be used depending on their market needs and preferences. One such contract, as explained by Eydeland and Wolyniec (2003) is the spread option. Spread options are contracts whose value is determined by the di erences in price between two or more assets. These contracts are mostly sold over-the-counter due to their niche nature. Depending on a market participant's outlook of the market, they can either buy a put spread option or a call spread option to hedge against price di erences at a per- ceived lower cost than actually buying the commodity and depending on its prices Beigaite et al. (2018). An example of this situation could be seen in a power pro- ducing company that uses natural gas to produce and sell electricity. To protect its operational margins and protect their bottom line, the company can buy a call op- tion on the spark-spread which is essentially the di erences between the natural gas (raw material) and electricity (product) multiplied by a factor of, say, 0.68. This fac- tor is calculated by taking into account production factors like the e ciency of the plant in converting units of natural gas into electricity units. An assumption of 50 per cent e ciency and the inclusion of other internal variables would lead to such a result Borovkova et al. (2007). Within the European market, there are di erent entities and countries that produce, sell and distribute electricity. Having all these players within one market allows mar- ket participants to get into regional spread contracts given the discordance of supply and demand factors in di erent regions Lundin and Tanger as (2020). As such a call option would give the holder the right but not the obligation to purchase a unit of electricity in one region and sell it in a di erent region at speci ed price. Such a con- tract allows for the exploitation of price di erence, an undertaking that goes a long way in instituting balance in the market. These kinds of regional (locational) spread contracts are commonplace in the Nord Pool given the inherent desire of market par- ticipants to use their knowledge of market dynamics to hedge perceived risks and speculate within the markets. 11

2.2 Approaches to electricity market modelling As aforementioned, the electricity market is di erent from the stock market due to its non-storability nature and volatility in demand and supply. This section aimed at dis- cussing the di erent models applied within the electricity market with the nal anal- ysis reviewing the option pricing. This section will be reiterating the intrinsic market factors and their e ect on the models and overarching goal of balancing prices on a real-time basis. The various models and characteristics will be found to be somehow correlated and included within the models to emphasize suitability in the electricity market. Some of these models will be re ective of models used in pricing traditional assets and commodities but will be specially applied to incorporate special aspects of the energy market that are not replicable with any other asset. One such model that underscored this relationship is the Geometric Brownian Mo- tion (GBM), one of the most common models used in nance to analyze stock prices Stojkoski et al. (2020). The GBM model is re ective of a weak form of the e cient market hypothesis as it assumes that the spot price re ects the totality of information on traded stocks. Price movements would thus be somehow independent of past price movements. These

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trends change when there are other factors in uencing price move- ments. The GBM model works in the stock market because of the perceived stability in production factors of companies within a speci ed period Stojkoski et al. (2020). It is because of this predictability that the GBM model takes the form of a stochastic di erential equation (SDE) that takes into account drift and volatility parameters of the stock. The GBM is a continuous-time stochastic process used in price modeling. It has an additional feature of generating positive prices, an important aspect in nancial ap- plication. A stochastic process S t is said to follow a GBM if it satis es the following SDE:

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dS t = S t dt + S t dW t : (2.1) where S(0) =

s 0 , the constants and represent the drift and volatility parameters respectively and dW t denotes the standard Brownian motion increments over the same 12

period of time. The GBM is e ective in pricing spread options, but this only applies when the no- arbitrage argument is applied. In the stock market, it is virtually impossible to exploit arbitrage opportunities between exchanges because of its entrenchment in the e cient market hypothesis. The fundamentals of a given stock are inherently similar regard- less of market participants analyzing. Any di erences in their perspectives, if any, will be immediately captured in the supply and demand trends in the market. In the electricity market, the non-storability aspect makes it practically impossible to apply this model. The fact that electricity cannot be stored means that there will always be price di erences between regions based on production parameters and demand aspects Eydeland and Wolyniec (2003). The easiest way to think of this dynamic is, say, when comparing the electricity produced in di erent regions using di erent mechanisms like HEP, geothermal plants and fuel-driven generators. While the commodity produced is inherently the same, the production costs are higher in some regions resulting in a higher price. Actions of the participants will result in the property of spot prices re-

ecting a mean-reverting characteristic. The volatility structure of electricity prices is thus inherently di erent from those of stock prices. The GBM model cannot thus be used to model for the fat tails in electricity price distributions (high kurtosis) which cripples it as a means of re ecting the complex structure of the electricity market Sto- jkoski et al. (2020) and Abdellah et al. (2019). The shortcomings of the GBM model created the need for specialized models that can e ectively price energy products like electricity. There are two main types of models considered when seeking to take into account the characteristics of the electricity spot prices. These models include the spot-based models and the forward-based models. The relationship between spot and forward based models is explained by the extensive theory that considers aspects like the con- venience yield and the storage cost. This is however impossible to apply when con- sidering electricity prices given that electricity cannot be stored. This relationship was thus explained by applying the market price of risk which entails the speci ca- tion of the risk-neutral probability, or alternatively using the Heath-Jarrow-Morton (HJM) framework which was traditionally developed to model forward interests rates by taking into account the aspect randomness Harrison and Pliska (1981). The HJM 13

framework has also been applied in some instances in option pricing Cap (2021). This is done such that the HJM framework is used to determine the fair market value of a derivative contract enabling market participants to identify undervalued or overvalued options. These models would be applicable within the electricity market because their forward approach uses interest rate concepts to calibrate and specify dynamics of the future electricity prices Benth et al. (2007); Prazeres (2017). The spot models, on the other hand, use the dynamics of the exogenously-given day ahead electricity price which is practically non-tradable but could be used to derive prices of future contracts, or more exotic contracts negotiated between interested par- ties. According to Thornton and Schwab (2010), the forward price approach was dif- ferent from the spot models because they applied future and forward prices which ended up accounting for the complicated dynamics of the forward curve. The down- side of such models was however seen in the existence of a mispricing which is virtu- ally impossible to detect once data has been entered into the model. These di erences made the spot models more appropriate in giving an accurate re ection of the elec- tricity prices which would allow for accuracy in describing the problem in question mathematically and coming up with an acceptable solution for the same Cartea and Figueroa (2005). As such, the spot price approach was selected for application within the current context. 2.2.1 The spot price model Spot price models are preferred because they take into account the fundamental char- acteristics of electricity prices. They are also e ective in distinguishing between sea- sonal components of supply and demand from mean-reverting spikes (caused by short term factors). Benth et al. (2007) structured a typical spot price model as follows; S t = (t) + n X i=1 X i t ;i = 1; 2;:::; n; (2.2) dX i t = a i (m i * X i)dt + i (t)dL i t : (2.3) 14