



SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBER SECURITY
END OF SEMESTER EXAMINATION

CNS 1204: Calculus II

Date: 19th December, 2023

Time: 2 Hours

Instructions

1. This examination consists of FIVE questions.
2. Answer Question ONE (COMPULSORY) and any other TWO questions.

QUESTION ONE (30 MARKS)

- (a) The TexMechs IT Company determines that the marginal cost for their 64GB flash drive is given by the differential rate equation

$$\frac{dC}{dx} = -0.02x + 6$$

where x is the number of flash drives produced and C is the cost in dollars.

- (i) Determine the general solution of the cost function C . [2 Marks]
 - (ii) Find the particular cost function C , knowing that the cost of producing 10 64GB flash drive is \$400. [2 Marks]
- (b) The figure below shows the cubic curve with equation $y = x^3 - 4x$, $x \geq 0$. The curve meets the x -axis at the origin O and at the point where $x = 2$. The finite region R_1 is bounded by the curve and the x -axis, for $2 \leq x \leq \sqrt{8}$.

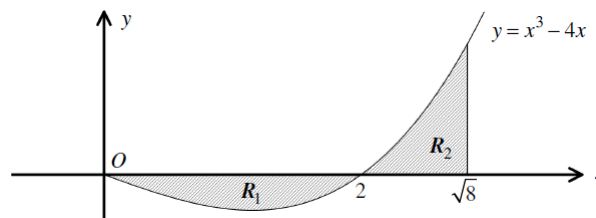


Figure 1

Show that the area of R_1 is equal to the area of R_2 .

[4 Marks]

- (c) Evaluate the following definite integral. [3 Marks]

$$\int x \sin x \, dx$$

- (d) If the population of a country is 50 million people and the population is increasing exponentially with a growth constant, $k = 0.59$, calculate precisely the population after 4 years. [Hint: $y(t) = y_0 e^{kt}$] [3 Marks]

- (e) Use the Midpoint Rule with $n = 4$ to estimate $\int_0^2 (2x^2 + 1) \, dx$. [5 Marks]

- (f) Determine the particular solution for the following initial value problem [5 Marks]

$$\frac{dy}{dx} = xy - 3x, \quad y(0) = 5y.$$

- (g) (i) Find the Maclaurin expansion of $f(x) = \cos x$ up and including the term in x^6 . [4 Marks]

- (ii) Hence find an approximate value for $\int_0^{0.1} \cos x \, dx$. [2 Marks]

QUESTION TWO (20 MARKS)

- (a) Find the total area between the curve $y = x^3$ and the x-axis between $x = -2$ and $x = 2$. Comment on your answer. [4 Marks]

- (b) Solve the following initial value problem (IVP) and find the interval of validity for the solution. [6 Marks]

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}, \quad r(\theta) = 2$$

- (c) Media consultants for the new local magazine *ITRave!* have projected that the number of subscriptions will grow during the first five years at a rate given by

$$S'(t) = \frac{1000}{(1 + 0.3t)^{3/2}}, \quad 0 \leq t \leq 60,$$

where t is the number of months since the magazine's first issue and $S'(t)$ is the rate of change in the number of subscriptions measured in *subscriptions/month*.

- (i) Evaluate $S'(12)$. [3 Marks]

- (ii) Evaluate and interpret $\int_0^6 S'(t) \, dt$. [7 Marks]

QUESTION THREE (20 MARKS)

- (a) Since running a series of first-come, first-served promotions, the FineHomes Furniture Store has found that its sales rate during its three-month sales drive is given by the function

$$s(t) = \frac{10}{t} + 2, \quad 1 \leq x \leq 12$$

where t represents the number of weeks that the promotion has been running and $s(t)$ is the sales rate measured in $\frac{\text{thousands of dollars}}{\text{week}}$.

- (i) Determine the total increase in sales generated from the first to the fifth week.
[4 Marks]
- (ii) Knowing that \$6000 was made during the first week, recover $R(t)$,
the revenue generated after t weeks. [5 Marks]
- (b) By letting $u = \sin x$, evaluate [5 Marks]

$$\int \sin^2 x \cos^3 x \, dx.$$

- (c) The Current (year 2020) population of the Earth is 7.5 billion people, and the yearly birth and death rates are $\beta = 0.021$ and $\delta = 0.009$ respectively. Assuming the birth and death rates remain constant, find the population of the Earth in the year 2120. [6 Marks]

QUESTION FOUR (20 MARKS)

- (a) A manager at the Black Box microprocessor manufacturing company finds through data gathered in research that the marginal cost function for a certain type of automobile computer chip made at the facility is given by

$$MC(x) = C'(x) = 6x\sqrt{x^2 + 11}$$

where x represents the number of auto computer chips produced each hour and $MC(x)$ represents the marginal cost. The manager also knows that it costs \$1932 to manufacture 5 chips.

- (i) Recover the cost function C . [7 Marks]
- (ii) Find the fixed costs. [To avoid confusion, call the arbitrary constant d .] [3 Marks]
- (b) Evaluate

$$\int_0^y \frac{dx}{7x}.$$

[6 Marks]

- (c) Given that the half-life T of radium is 1690 years, how much will remain of **ten** grams of radium after 200 years? [4 Marks]

QUESTION FIVE (20 MARKS)

- (a) Using the technique of partial fraction decomposition, evaluate [6 Marks]

$$\int \frac{3x - 11}{x^2 - x - 6} dx.$$

- (b) The marginal profit $MP(x)$ for producing x units of an electrical appliance is given by the linear function

$$MP(x) = 80 + 0.03x$$

where $MP(x)$ is in dollars per unit. Knowing that $P = 0$ when $x = 0$, recover the profit function P . [Hint: $P(x) = \int MP(x) dx$] [4 Marks]

- (c) Evaluate the following antiderivative. [5 Marks]

$$\int x \ln x dx.$$

- (d) Find the area bounded by the lines $y = 0$, $y = 1$ and $y = x^2$. [5 Marks]

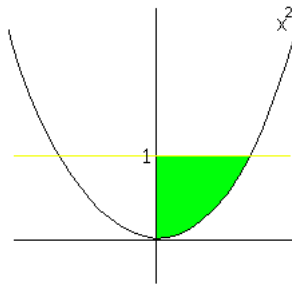


Figure 2

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