



Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN INFORMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN COMPUTER NETWORK AND SECURITY
END OF SEMESTER EXAMINATION
ICS 1103 \ CNS 1103: Differential Calculus

Date: 22nd July, 2024

Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) Evaluate the following limit algebraically $\lim_{x \rightarrow 3} (7x^3 - 10x)$ [2 Marks]

(b) Given the function $f(x) = 3x^3 - 2x^2 - 3x + 7$, find:

(i) $f(-2)$ [1 Marks]

(ii) $f(t - 1)$ [2 Marks]

(c) A manufacturing company produces and sells tables. The cost function is given by:

$$C(x) = 3x + 120x^{\frac{1}{2}} + 3000$$

where x is the number of tables. The tables are sold for \$ 200 each. Find the total profit from selling 25 tables. [3 Marks]

(d) Determine the derivative of the following functions:

(i) $f(x) = x^2 e^{4x}$ [2 Marks]

(ii) $f(x) = \sqrt{4x + 6}$ [2 Marks]

(e) Determine the continuity of the function $f(x) = \frac{1}{x-3}$ at $x = 3$. [5 Marks]

(f) Find an equation for the horizontal asymptote of the graph of $f(x) = \frac{3x+7}{2x-5}$ [3 Marks]

(g) Use the intermediate value theorem to show that $\frac{x^2+x-15}{x-8} = 0$ has at least one solution on the interval $[-5,1]$. [5 Marks]

(h) The annual per capita consumption of light and skim milk in the United States can be modelled by

$$f(x) = 9.12 + 2 \ln 2x, \quad 1 \leq x \leq 16$$

where x represents the number of years since 1979 and $f(x)$ represents the annual per capita consumption of light and skim milk in gallons.

(i) Determine $f'(x)$. [1 Marks]

(ii) Evaluate and interpret $f'(5)$. [2 Marks]

(i) Find the derivative of:

$$y = (3x^2 + 3x - 2)(5x + 3)$$

[2 Marks]

QUESTION TWO (20 MARKS)

- (a) State the three conditions that a function $f(x)$ should satisfy to be continuous at a point $x = a$, where a is any real number. [3 Marks]
- (b) Determine the interval(s) where the following functions are continuous:
- (i) $f(x) = x^2 + x - 3$ [2 Marks]
- (ii) $h(x) = \sqrt{x - 8}$ [3 Marks]
- (iii) $f(x) = \frac{x-1}{(x-1)(3x+1)}$ [3 Marks]
- (c) Use the squeeze theorem to determine the value of the indicated limits
Given $3 + 2x \leq f(x) \leq x - 1$ for all x find:
- (i) $\lim_{x \rightarrow -4} f(x)$ [3 marks]
- (ii) $\lim_{x \rightarrow 1} (x - 1)^2 \cos\left(\frac{1}{x - 1}\right)$ [4 Marks]
- (c) Determine the points of discontinuities of the given function
(where the function is not continuous). [2 Marks]

$$f(x) = \frac{x^3 - 64}{x^2 - 16}$$

QUESTION THREE (20 MARKS)

- (a) Given $f(x) = -3 - 4x$ and $g(x) = x - 7$, find
- (i) $(g \circ f)(x)$ [3 Marks]
- (ii) $(g \circ f)(5)$ [2 Marks]
- (b) Determine the inverse of the function $h(x)$, where [3 Marks]

$$h(x) = 3(2x - 5)$$

- (c) A peach grower takes steps to improve the quality of his product over several years. The annual profits follow the function

$$P(t) = \frac{25(t + 4)}{t + 5}$$

where t is the time in years and $P(t)$ is profit in thousands of dollars.

- (i) Draw the graph of P in the viewing window $[0,10]$ by $[0,30]$ [3 Marks]
- (ii) Find the annual profit 3 years after the start of the improvement program [2 Marks]
- (iii) Find and interpret $\lim_{t \rightarrow \infty} P(t)$. [2 Marks]

(d) The resale value R (in dollars) of a certain type of industrial equipment has been found to behave according to the function $R(t) = 900,000e^{-0.01t}$, where t is the number of years since original purchase.

(i) Determine the original value of the piece of equipment. [2 Marks]

(ii) Determine the number of years it will take for the value of the piece of equipment to reach \$ 600,000. [3 Marks]

QUESTION FOUR (20 MARKS)

(a) Determine the derivative of the following functions:

(i) $h(x) = (8x^2 + 3)(x^2 + 2x - 1)$ [2 Marks]

(ii) $h(x) = \frac{2-x}{x^2+1}$ [3 Marks]

(iii) $f(x) = (4x^2 + 3x)^8$ [3 Marks]

(b) Use the first principle to determine $f'(x)$ for $f(x) = 3x - 2$. [3 Marks]

(c) Evaluate the following limits:

(i) $\lim_{x \rightarrow 2} (x + 1)^2 \cdot (3x - 1)^3$ [2 Marks]

(ii) $\lim_{x \rightarrow -2} \frac{x + 2}{|x + 2|}$ [3 Marks]

(iii) $\lim_{x \rightarrow 4^+} \frac{3}{(4 - x)^3}$ [2 Marks]

(iv) $\lim_{x \rightarrow \infty} (2x^4 - x^2 - 8x)$ [2 Marks]

QUESTION FIVE (20 MARKS)

(a) Given $f(x) = x^3 - 12x^2 + 36x$ find:

(i) The critical values of f [3 Marks]

(ii) the local extrema [4 Marks]

(b) Given the following equations find $\frac{dy}{dx}$

(i) $y^2 - 12x^3 = 8y$ [3 Marks]

(ii) $\sin x + \cos y = e^{4y}$ [4 Marks]

(c) Acme Stuffed Animals, Inc, is introducing a new line of teddy bears. The total cost of producing Scare Bear (with glowing eyes) is projected to be $C(x) = 36000 + \sqrt{10000x}$, where x is the number of units made and $C(x)$ is the cost in dollars.

Find and interpret $\lim_{x \rightarrow 100} C(x)$ [3 Marks]

(d) Sketch the graph of a function that meets the following conditions :

- (i) Has at least one absolute maximum. [1 Mark]
- (ii) Has one relative minimum. [1 Mark]
- (iii) Has no absolute minimum. [1 Mark]

END OF PAPER