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Modelling Temperature Dynamics and Pricing Temperature Derivatives: An Investigative Kenyan Example

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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ABBREVIATIONS

HDD – Heating Degree Day

CDD – Cooling Degree Day

GDP – Gross Domestic Product

CAT – Cumulative Average Temperature

CME – Chicago Mercantile Exchange

OTC – Over - The - Counter

GARCH - Generalized Autoregressive Conditional Heteroscedasticity

AR - Autoregressive

Abstract

The following research project will propose a mean-reverting stochastic process for modelling the daily average temperature in the Kenyan context. The proposed modelling framework will then be used to price a weather derivatives instrument. First, a general description of weather derivatives is provided along with their applicability in the market. Different models postulated in modelling the dynamics of temperature are then outlined. The Alaton model is then highlighted; a model that prescribes an Ornstein – Uhlenbeck process for the modelling of temperature. The methodology is then outlined as well as a description of the calibration of the model. Thereafter, the aforementioned model dynamics are used in pricing a CDD call option.

CHAPTER I: INTRODUCTION

1.1 Background to the Study

Weather derivatives are financial instruments whose underlying variable is meteorological data such as temperature, wind, or precipitation. They enable corporations and other organisations to insure and/or hedge their business extensively against unfavourable weather (Schiller, Siedler & Wimmer, 2010). Temperature derivatives are financial instruments in which the underlying variable is temperature. They are financial instruments which make prespecified pay-outs if prespecified temperature conditions occur (Campbell & Diebold, 2005).

The first transaction in the weather derivatives market took place in the US in 1997 (Alaton, Djehiche, & Stillberger, 2002). In 1998, however, the market was estimated at $500 million (Campbell & Diebold, 2005). This rapid growth in weather derivatives was driven by the energy sector. The El Nino phenomenon pushed multiple companies to hedge their seasonal weather risk (Alaton, Djehiche, & Stillberger, 2002). This market, as with any OTC (Over-The-Counter) contract, was prone to counterparty risk; the risk of default. Consequently, to increase the size of the market and to remove credit risk from the trading of contracts, the Chicago Mercantile Exchange started an electronic market place for weather derivatives in September 1999 (Schiller, Siedler, & Wimmer, 2010). Among the major market makers were firms such as Aquila Energy and Koch Energy Trading.

This derivatives portal has since evolved to offer a diverse set of derivative contracts. Temperature-based heating degree days (HDD)/cooling degree days (CDD) index is the most popular index for weather derivatives (Goncu, 2013). The specifications of these contracts are outlined in section 2.1.

The underlying variables in weather derivatives are not tradeable in the market and as such, the Black Scholes Merton pricing model is not applicable.

There have been multiple models proposed to form the basis of pricing the weather derivatives such as the Benth model provided by Benth and Saltyte-Benth (2005, 2007) or the generalized autoregressive conditional heteroscedasticity (GARCH) model provided by Campbell and Diebold (2005).

However, aside from weather derivatives instruments, insurance contracts may be used to hedge one against the adverse effects of unfavourable weather. The main difference between derivatives and insurance contracts is that the holder of the insurance contract has to prove that he has suffered a financial loss due to unfavourable weather in order to be compensated. Thus, they are designed to protect the holder from extreme weather events. Weather derivatives can be constructed to have pay outs in any weather condition. (Alaton, Djehiche, & Stillberger, 2002)

Furthermore, there is a higher chance of moral hazard or adverse selection effects in insurance contracts as compared to index contracts in which the value of the index is not dependent on the actions of the individual market participants (Dimitry & Barry, 2004).

However, index contracts have the disadvantage of basis risk. This results from the fact that the weather variables are measured at specific locations which may vary in other locations. Consequently, it may be inaccurate to design a derivative for investors who may be geographically separate from the locations used for data collection.

Despite the fact that the services sector contributes over half of Kenya's Gross Domestic Product (GDP), Kenya is still heavily reliant on agriculture. The success of the agriculture sector is heavily reliant on the weather. In the recent years, Kenyan weather has exhibited unpredictability. Currently, Kenya is undergoing a drought and the economy is experiencing significant inflation.

1.2 Problem Statement

The traded weather contracts in the CME are based on the Cumulative Average Temperature (CAT) index for European cities as well as Canadian cities (Benth & Saltyte Benth, 2011). The CAT index is the accumulated daily average temperatures over the measurement period. The weather dynamics in Kenya are different from those in European or Canadian cities and as a result, this index cannot be used as a basis for the payoffs.

The East African Region is characterized by two main seasons: the rainy season and the dry season. However, in recent years, these seasons have showed sporadic behaviour. Lindzen (2016) asserts that, as a consequence of global warming, there has been an increase in the frequency and intensity of some types of extreme weather.

This study models the temperature dynamics in Kenya using a two-factor stochastic process. In addition, the model will be calibrated for the seasonal aspect of temperature data in Kenya and, a temperature derivative will be designed thereafter.

1.3 Research Objectives

1) To structure a modelling framework for temperature dynamics in the Kenyan context using the Alaton Model
2) To price a CDD call option using the proposed modelling framework

1.4 Research Questions

1) Does the Alaton model provide a suitable modelling framework for Kenyan temperature data?
2) Can a CDD call option be priced using the proposed modelling framework?

1.5 Justification of the Study

The aim of the research is to price a CDD call option by modelling the temperature using the Alaton model. An appropriate temperature model will then enable the suitable pricing of temperature derivatives. This will then enable farmers to hedge themselves against temperature - related risks as well as speculation among investors.

Previous works (Benth & Saltyte-Benth, 2007) have modelled temperature derivatives in the context of four main seasons: summer, spring, autumn and winter. The approach taken in this research will model the temperature in the context of the two main seasons experienced in Kenya: the rainy season and the dry season.

CHAPTER II: LITERATURE REVIEW

Temperature has a considerable effect on the activities of large scale farmers. Unfavourable temperature may result in financial losses due to reduced harvest. Consequently, farmers may seek to find financial instruments that may help them hedge these risks.

The following section examines the mechanism of the weather derivatives market as well as some models used in measuring temperature dynamics.

2.1 Weather Derivatives Contracts

Weather derivatives contracts are usually structured as swaps, futures and call/put options based on different underlying weather indices. Some commonly used indices, for temperature derivatives specifically, are heating - degree days and cooling - degree days (Alaton, Djehiche, & Stillberger, 2002).

The heating degree days is defined by the expression:

$$HDD_i \equiv \max\{C - T_i, 0\} \tag{2.1}$$

Whereas the Cooling Degree Days is defined as

$$CDD_i \equiv \max\{T_i - C, 0\} \tag{2.2}$$

Where T_i is defined as the temperature for day I and C is defined as some threshold temperature level.

The threshold C is, in the market, given as 18□ C and is the trigger point when people would turn on their air conditioners (in the case that temperature went above 18□ C). Conversely, when the temperature was below 18□ C people would use more energy to heat their homes.

2.2 Modelling Weather Derivatives

In the Black and Scholes framework (Black & Scholes, 1973), derivatives are assumed to be perfectly replicable; a natural condition for tradeable assets such as stocks. For derivatives contracts written on temperature indices, however, the underlying variable is non-tradeable and thus other frameworks have been proposed to model temperature dynamics.

Jewson and Brix (2005) outline three approaches for valuing weather derivatives; namely, burn analysis, index modelling and daily simulation.

2.2.1 Burn Analysis

Under this approach, the future expected payoffs of a given derivative are estimated by considering the payoffs yielded in the past. The price is then calculated as its fair value plus a possible risk premium (Schiller et al., 2012).

Assuming a derivative, for measurement period $[T_1, T_2]$, should be priced for the year $n+1$, the fictive indices of same derivatives in the year $n, n-1, n-2$, etc., would be estimated. This yields a series $Y_1, Y_2, \ldots, Y_n$ of n indices for the past n years.

Using the linear model

$$Y_i = B_0 + B_i \, x \, i + e_i, \quad i = 1, \ldots, n,$$

where Y_i is the payoff for a given year. The parameters B_0 and the trend (slope) parameter B_i are then estimated as

$$\hat{B}_1 = \frac{\sum_{i=1}^{n}(i - \frac{n+1}{2})(Y_1 - \bar{Y})}{\sum_{i=1}^{n}(i - \frac{n+1}{2})^2}$$

$$\widehat{B_o} = \bar{Y} - \frac{n1}{2}\hat{B}_1.$$

$\bar{Y}$ is the mean of the calculated indices over the past n years.

The index of the next year, $\hat{Y}_{n+1}$, can then be denoted as

$$\hat{Y}_{n+1} = \hat{B}_0 + B_1(n+1)$$

2.2.2 Index Modelling

Under this approach, the Burn Analysis is further extended by estimating the underlying distribution of the weather index. This yields a more stable derivatives price estimation than the Burn Analysis if the index distribution can be estimated relatively well. This is because index modelling smoothens the distribution and extrapolates the tails appropriately. Jewson and Brix (2005) state that for seasonal temperature modelling, a normal distribution would be appropriate; for monthly contracts, other standard distributions such as skew-normal distribution, gamma distribution or the log-normal distribution should be tested to find their goodness of fit.

2.2.3 Daily Simulation

Daily Simulation employs stochastic methods to model the daily temperatures observed.

2.2.3.1 The Dornier & Querel Model

Dornier and Querel (2000) the temperature dynamics of the form

$$dT_t = ds_t + k(T_t - s_t)dt + \sigma_t dB_t$$

where $s_t = A + Bt + C\sin(\omega t + \varphi)$ describes the mean seasonal volatility and the constant k represents the speed with which temperature reverts to its mean. The volatility, $\sigma_{t,}$ is assumed to be a measurable and bounded function and B_t is a Wiener process

The model regresses the change in de-seasonalized temperature against de-seasonalized temperature to establish the autocorrelation in the two time series.

2.2.3.2 The Alaton Model

Alaton et al. (2002) postulate similar temperature dynamics as Dornier and Querel. However, $\sigma_{t,}$ in the Alaton model is taken to be a constant function representing a monthly variation in volatility. This was obtained by fitting the model over 40 years of daily average temperature from Bromma (Stockholm, Sweden). They prescribed a Wiener process as the driving noise in the Ornstein-Uhlenbeck process. This was a result of the observation that the temperature differences were close to normally distributed. However, they did not provide a statistical test for normality.

2.2.3.4 Benth Model

The Benth Model (Benth & Saltyte-Benth, 2005) uses an Ornstein-Uhlenbeck process of the form

$$dT_t = ds_t + k(T_t - S_t)dt + \sigma_t dL_t.$$

The Benth model differs from the Dornier & Queler model only in that it includes the Levy noise L_t.The Levy process is a continuous-time stochastic process with independent, stationary increments. They suggest the use of a Levy process with marginal following the class of generalized hyperbolic distributions. This is flexible family of distributions which can model skewness and semi - heavy tails. Furthermore, its density and moment generating functions are explicitly known. Benth & Saltyte-Benth show that for Norwegian temperature data an Ornstein-Uhlenbeck process driven by a generalised hyperbolic Levy process with time - dependent variance fits reasonable well.

2.2.3.3 Campbell and Diebold Model

Campbell and Diebold (2002) propose an autoregressive time series of the form

$$T_t = m_t + s_t + \sum_{i=1}^{L} p_{r-1} T_{t-1} + \varepsilon_t, \quad t = 1,2,\ldots$$

to model the dynamics of temperature. The trend, m_t, is linear and the seasonality, s_t, is modelled by a finite sum of sines and cosines. The model regresses today's de-seasonalized temperature against the temperature observed over the last L days.

2.3 Prior Work in the Kenyan context

Njoroge (2011) models Kisumu (Kenya) temperature using a mean - reverting stochastic process. He concludes that the forecasted temperature was reasonably accurate but highlights that the temperature model consistently overpriced the options. He attributes this to the limited dataset as well as the method of simulation used in estimating future temperature.

According to Ngare, Kweyu and Huka (2015) the first index-based weather insurance launched in 2009 was developed to cover livestock mortality due to drought in Marsabit Distirict. Syngenta Foundation partnered with UAP Insurance to launch their first index-based insurance product, Kilimo Salama. The product covered loss of inputs and targeted small scale farmers in Laikipia district. Approximately 200 maize farmers insured their farm inputs against drought in the long rains of 2009. The insurance product had reached up to 47,000 farmers, implying scalability of these products.

In this regard, Ngare et al. (2015) suggest a rainfall derivative contract pricing mechanism based on the rainfall index. The payout is denoted by

$$Payout\ (R) = Max(K - R, 0)xA$$

where R is the value of the index evaluated on the last day of the contract period. A is the slope of the payout function or tick payment or liability and K is the value of the index at which payments are initiated or the strike value.

2.4 Model Calibration

Model calibration may be done through Discrete Fourier Transform, averaging method, the regression method as well as maximum likelihood estimation. The averaging method computes the average daily temperature and smoothens them to estimate the model parameters. This method is simple but is less accurate when compared to the other techniques.

In discrete Fourier transform (DFT), the power spectrum of the variance process is estimated. DFT converts a finite sequence of equally-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete-time Fourier transform.

In ordinary least squares (OLS) the total of squared vertical distances between the observed responses within the dataset and the responses predicted by the linear approximation is minimized. With regards to temperature data, regression is done in harmonics of 365 days.

The Maximum Likelihood Estimation, or MLE, is a method used in estimating the parameters of a statistical model by finding the parameter values that maximize the likelihood of obtaining the observed values.

This study makes use of regression models.

CHAPTER III: RESEARCH METHODOLOGY

The study will capture the temperature dynamics via the Alaton model proposed by Alaton et al. (2002). The temperature dynamics are defined by a deterministic component as well as two mean reverting processes with seasonal short term volatility.

3.1 Research Design

The research design in this project will be exploratory as the aim is to observe the behaviour of historical temperature and thereafter suitably price a derivative instrument.

3.2 Population and Sampling

The population to be used in this research will be Kenyan temperature data. Purposive sampling will be used, as the locations selected will be chosen based on their economic activities. Thus temperature data from Nyeri was utilized for this study.

3.3 Data Collection

Temperature data will be obtained from the Kenya Meteorological Department. The data collected will be quantitative in nature.

3.4 Data Analysis

3.4.1 Temperature Dynamics

3.4.1.1 The Driving Noise Process

The temperature index *T(t)* is taken to a be a mean – reverting process and is driven by the SDE

$$dT(t) = a(T_t^m - T_t)dt + \sigma_t dW_t \tag{3.1}$$

where a determines the speed of the mean – reversion and $T_t^m$is the long-term mean.

-The problem with the prespecified equation is that it does not revert to T_t^m in the long run (Dornier & Querel, 2000). To obtain a process that reverts to the mean the term

$$\frac{dT_t^m}{dt} = B + \vartheta C cos(\vartheta t + \gamma) \quad (3.2)$$

is added to the drift term in Eq. (3.1). The resulting mean temperature, T_t^m, then contains a sinusoidal form which will adjust the drift so that the solution of the SDE has the long-run mean T_t^m.

Starting at $T_s = x$ we now get the following model for the temperature

$$dT_t = \left\{\frac{dT_t^m}{dt} + a(T_t^m - T_t)\right\} dt + \sigma_t dW -_t, \quad t > s \quad (3.3)$$

whose solution is

$$T_t = (x - T_s^m)e^{-a(t-s)} + T_t^m + \int_s^t e^{-a(t-T)}\sigma_t dW_T \text{ -} \quad (3.4)$$

where

$$T_t^m = A + Bt + Csin(\vartheta t + \gamma) \quad (3.5)$$

3.4.1.2 The Mean Temperature

The mean temperature is assumed to follow a sinusoidal model. This function would have the form

$$\sin(\vartheta t + \gamma) \quad (3.6)$$

where t denotes the time, measured in weeks. Since we know that the period of the oscillations is one year (neglecting leap years) we have $\vartheta = 2\pi/365$. As the yearly minimum and maximum mean temperatures do not usually occur at January 1 and July 1 respectively, a phase angle, γ, is added to the sine function.

Further, we assume mean temperature increases each year. Lindzen (2016) asserts that temperatures have been rising due to the greenhouse effect. Alaton et al (2002) attempt to depict this weak trend from data by assuming, as a first approximation, that the warming trend is linear. They observed that the effect on the overall dynamics of the mean temperature were weak and as such, only the linear term of this polynomial that will dominate.

The deterministic model for the mean temperature at time t, is of the form

$$T_t^m = A + Bt + Csin(\vartheta t + \gamma) \tag{3.7}$$

where the parameters A, B, C, γ are estimated so that the curve fits the data well.

3.5 Parameter Estimation

This section will highlight the estimation of the unknown parameters A, B, C, γ, a and σ based on the temperature data.

To estimate the constants in equation 3.7, we fit the function

$$Y_t = a_1 + a_2 t + a_3 sin(\vartheta t) + a_4 cos(\vartheta t) \tag{3.8}$$

to the temperature data using the method of least squares. The goal is to find a function that best describes the historical trend of the temperature data. The constants in the mean temperature model are then obtained by

$$A = a_1 \tag{3.9}$$

$$B = a_2 \tag{3.10}$$

$$C = \sqrt{a_3^2 + a_4^2} \tag{3.11}$$

$$\gamma = arctan\frac{a_4}{a_3} - \pi. \tag{3.12}$$

Basawa and Prasaka Rao (1980) specify a quadratic variation of temperature in estimating σ .Given a specific month μ of N_μ days, denote the outcomes of the observed temperatures during the month μ by Tj, j = 1,..., N_μ.

$$\sigma_\mu^2 = \frac{1}{N_\mu}\sum_{j=0}^{N_\mu-1}(T_{J+1} - T_j)^2 \tag{3.13}$$

The mean reversion parameter can be estimated via the following expression:

$$a_n = -\log\left(\frac{\sum_{i=1}^{n} Y_{i-1}\{T_i - T_i^m\}}{\sum_{i=1}^{n} Y_{i-1}\{T_{i-1} - T_{i-1}^m\}}\right) \tag{3.14}$$

Where

$$Y_{i-1} \equiv \frac{T_i^m - T_{i-1}}{\sigma_{i-1}^2} \tag{3.15}$$

CHAPTER IV: FINDINGS

4.1 ANALYSIS

The monthly data depicted average monthly data from January 1988 to February 2013. This was interpolated non - linearly to yield weekly average temperature data by using a spline function.

The output is depicted in the graph below:

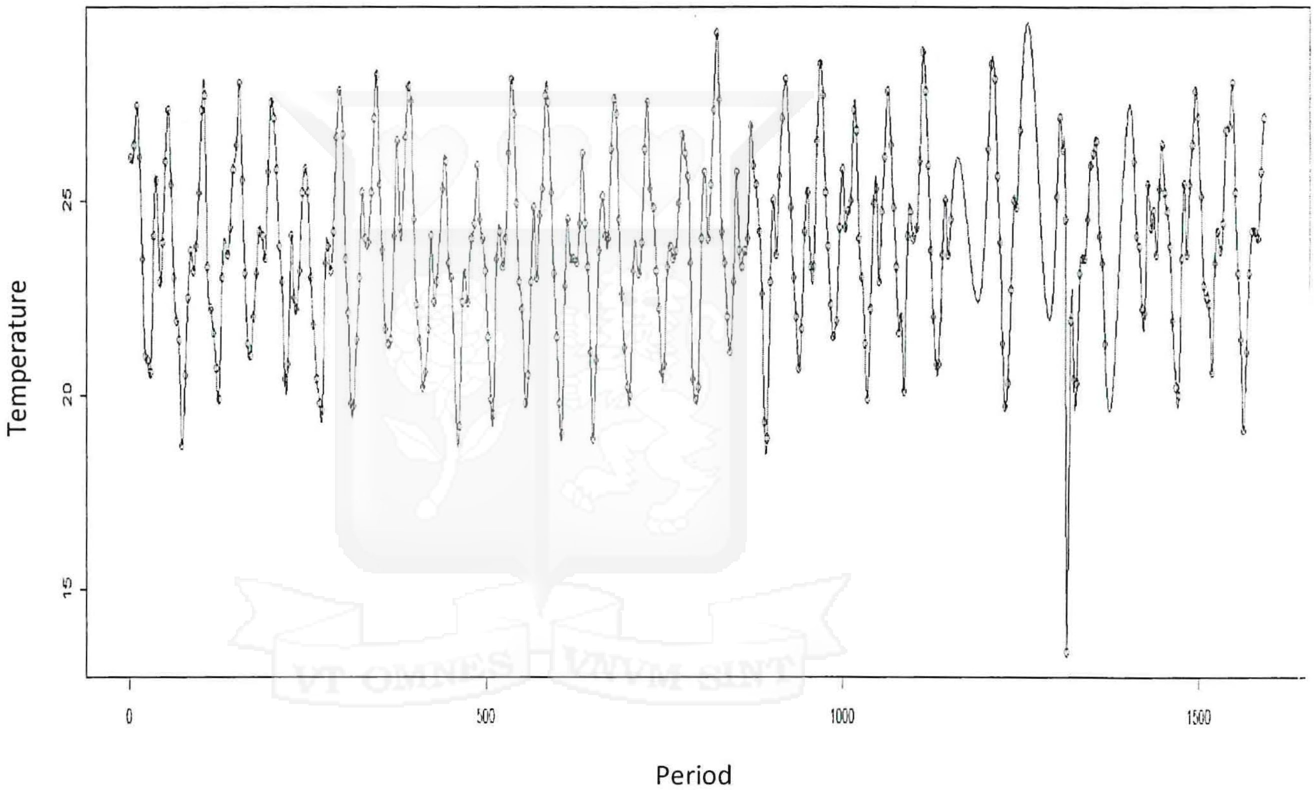

Figure 1: The weekly average interpolated data

The spline interpolation yields a reasonable temperature trajectory. The temperature oscillates from about 18□C to 27□C.

The descriptive statistics of the temperature data was then obtained

Weekly Average Temperature	
Mean	23.87
1st Quantile	22.33
Median	23.90
3rd Quantile	25.52
Min	13.40
Max	29.55
Standard Deviation	2.35
Skewness	-0.15
Kurtosis	-0.09

The mean temperature is slightly less than the median temperature which explains why the temperature data is slightly negatively skewed. The descriptive statistics were computed from 1,589 temperature observations. The range between the 1st Quantile and the 3rd quantile is relatively small implying that the minimum temperature of 13.4□C is an outlier.

4.2 Parameter Estimation

In this section the parameters of the long – term mean were obtained as well the parameters of the driving noise process.

4.2.1 Fitting the mean temperature model to data

The parameters in equation (3.7) were derived by estimating the values of equations 3.8 – 3.13.

This yielded the equation of the form

$$T_t^m = 23.52 + 4.492 \cdot 10^{-3}t + 2.195\sin(\frac{2\pi}{52} - 1.617).$$

The output of this model alongside the historical temperature data is shown in the following figure:

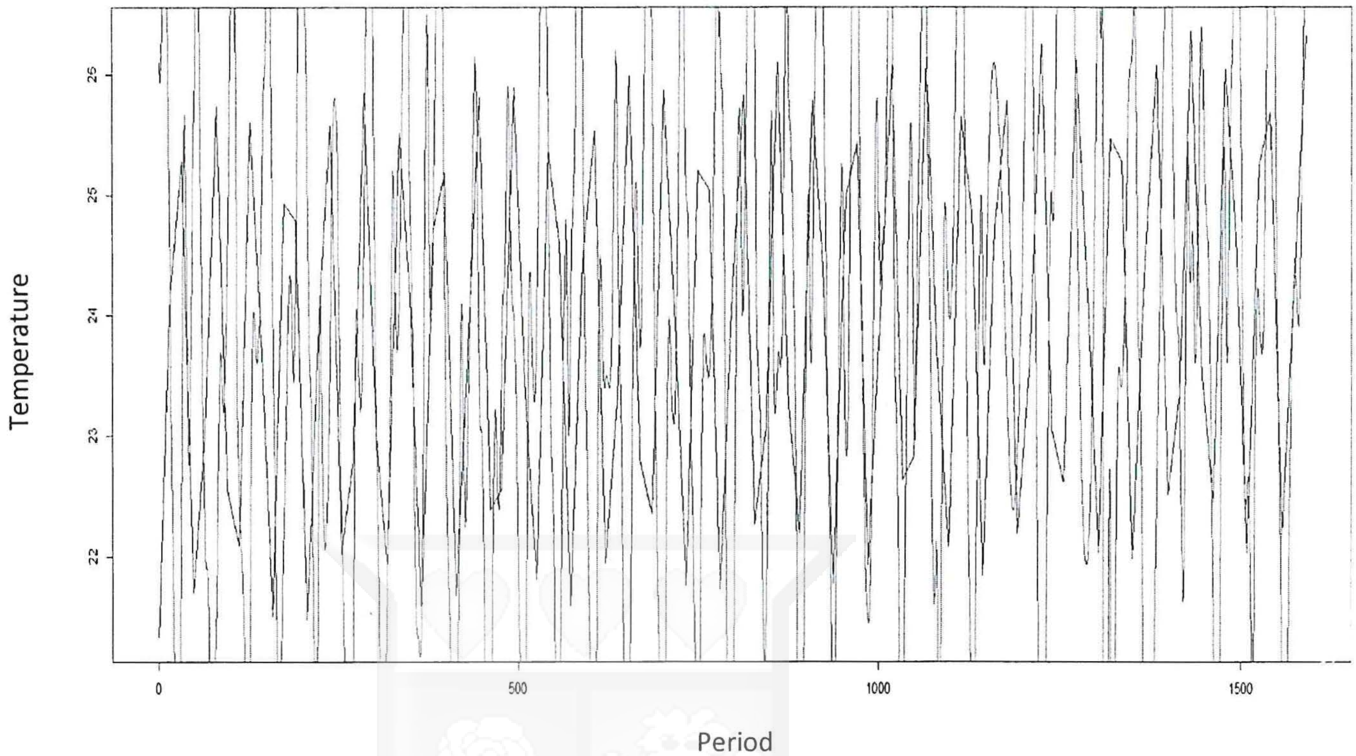

Figure 2: The weekly average mean temperature graphed alongside historical temperature

The estimated temperature fits reasonably well compared to the original temperature dataset. The estimated temperature has a narrower range, oscillating from 21□ C to 25.5□C.

4.2.2 Estimation of mean reversion parameter,a, as well as σ

The variance of the process was estimated via the quadratic variation

$$\sigma_{\mu}^{2} = \frac{1}{N_{\mu}} \sum_{j=0}^{N_{\mu}-1} (T_{J+1} - T_{j})^{2}.$$

The estimation yielded $\sigma_t = 0.226847$.

The mean reversion parameter was estimated via the equation

$$a_n = -\log(\frac{\sum_{i=1}^{n} Y_{i-1}\{T_i - T_i^m\}}{\sum_{i=1}^{n} Y_{i-1}\{T_{i-1} - T_{i-1}^m\}}$$

where

$$Y_{i-1} \equiv \frac{T_i^m - T_{i-1}}{\sigma_{i-1}^2}$$

obtaining $a = 0.007446$.

4.3 Trajectory of Temperature

The trajectory of the temperature over the next year was simulated using Monte Carlo Simulation. The temperature is graphed below:

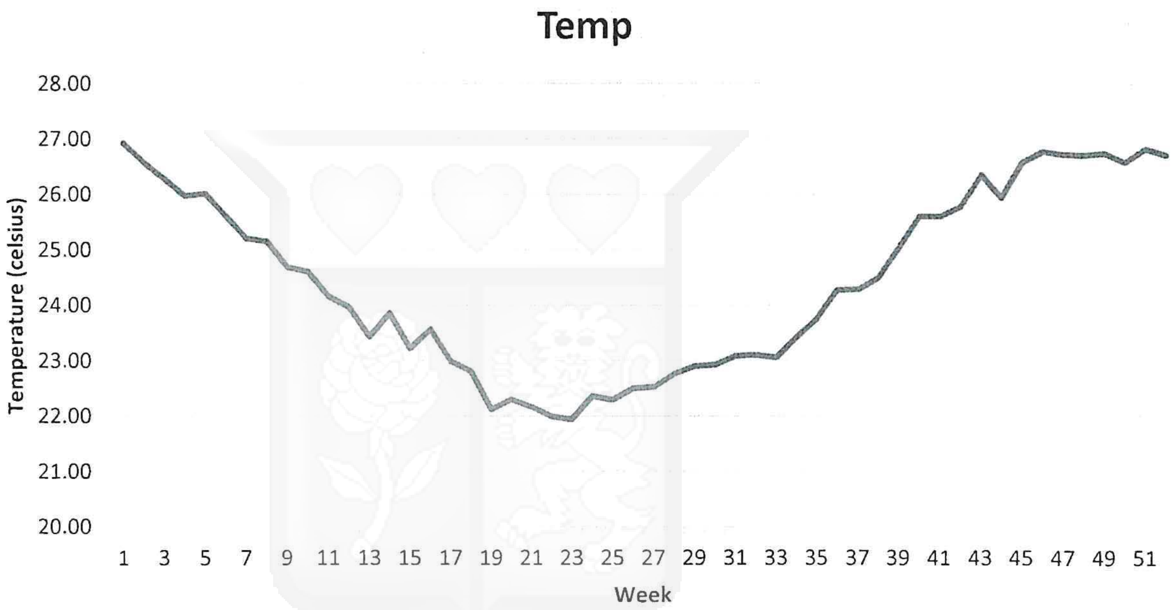

Figure 3: Simulated temperature over the next 52 weeks

4.4 Pricing the Temperature Derivatives

As mentioned in section (2.1), most weather derivatives involving temperature are based on heating or cooling degree days. We price a CDD option using the forecasted temperature.

Recall that the payout of the CDD call option is of the form

$$H_n = \sum_{i=1}^{n} \max\{T_{t_i} - C, 0\}.$$

The threshold temperature, *C*, is taken to be 24.63□ C. This threshold was obtained by averaging the monthly average Kenyan temperature. An assumed hypothetical multiplier of KES 200 for every additional degree of temperature above 24.63□C.

The price for any given derivative is calculated as

$$Price = \{E[Payoff]\}e^{rt} \quad (4.1)$$

where r is the risk-free interest rate. The expected payoff is calculated as

$$\max\{\ (T_t - C)\ x\ \lambda x\ P(T_t > C)\ ,0\} \quad (4.2)$$

where T_t is the expected temperature at time, t, C is the threshold temperature of 24.63□ C and λ is the tick payment of KES 200. The probability estimate, $P(T_t > C)$, was estimated by dividing the number of instances the expected temperature, T_t, exceeded the threshold temperature, C, by the total number of weeks.

The following table is displays the expected payoffs for weeks 5 – 13 using Eq. (4.2)

Forecast Period (t)	Expected Temp (T_t)	Threshold Temperature (C)	Payoff
5	26.01101	24.63	148.7244
6	25.62134	24.63	106.7602
7	25.21432	24.63	62.92686
8	25.16377	24.63	57.48324
9	24.69016	24.63	6.479049
10	24.61511	24.63	0
11	24.16132	24.63	0
12	23.97362	24.63	0
13	23.43158	24.63	0

Table 1: Expected payoff of CDD call option.

The price of the derivative is then obtained by discounting the expected payoffs at the risk-free rate (taken as the current interest rate on the 365 Day Treasury Bill) of 11.019%.

The following table displays the present value of the payoffs for weeks 5 - 13.

Forecast Period (t)	Payoff	Present Value
5	148.7244	147.1569
6	106.7602	105.4114
7	62.92686	62.00034
8	57.48324	56.51698
9	6.479049	6.356655
10	0	0
11	0	0
12	0	0
13	0	0

The price for one such derivative contract was obtained as KES 3,410.74

CHAPTER V: CONCLUSION

This thesis aims at modelling temperature dynamics using the Alaton model and thereafter, pricing a weather derivative with respect to this temperature model.

We postulated a mean – reverting stochastic process to model the temperature. We then used least squares regression to estimate the model parameters. The resulting model fit reasonably well alongside the historical temperature data; this illustrates that it suitably depicts the temperature dynamics. The CDD call option was then priced based on the expected payoff with respect to the threshold temperature.

The parameter estimation fit the data reasonably well; as such we conclude that the Alaton model is suitable for modelling temperature.

5.1 Limitation of study

The temperature data obtained was monthly data; to enhance parameter estimation, we used spline interpolation to convert the monthly data to weekly data. This rendered more data points which introduced some degree of error in the estimation. Daily temperature data would have allowed better parameter estimation.

The threshold temperature, C, used in calculating CDD's or HDD's was estimated as the average of Kenyan temperature as provided by World Bank information. In section (2.1) we mentioned that the threshold temperature was estimated, empirically, as the trigger point for use of air – conditioning/heating appliances; this was in reference to Europe and the United States of America. However, such an empirical study is yet to be carried out in Kenya.

5.1 Recommendations

The temperature model is based solely on Nyeri temperature; as a result, it fails to take into account other locations that may exhibit varying temperature dynamics. In section (1.1), we mention that index contracts have the disadvantage of basis risk resulting from the fact that the weather variables are measured at specific locations which may vary in other locations. Consequently, it may be inaccurate to design a derivative for investors who may be geographically separate from the locations used for data collection. As such, more locations should be used for data collection to estimate model parameters tailored to the respective locations of the investors.

The thesis priced solely a CDD call option. This may be limiting to investors who may prefer other temperature derivative. As such, other derivative instruments such as futures and swaps should be priced using the proposed temperature framework.

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Appendix

Appendix 1: Estimation of Forecasted Temperature

A	B	C	D	E	F	G	H	J
Forecast Period	DTmean /dt	Tmean (t)	T(t)	a(Tm-T)	=E+B	Error	σdB(t)	Expected Temp
1	-0.001	23.20	27.10	-0.03	-0.03	-0.63	-0.14	26.93
2	0.000	26.10	26.93	0.20	0.20	1.24	0.28	26.58
3	0.001	25.94	26.58	0.20	0.20	0.64	0.15	26.29
4	0.001	25.77	26.29	0.20	0.20	0.07	0.02	25.98
5	0.002	25.57	25.98	0.19	0.20	1.10	0.25	26.01
6	0.003	25.35	26.01	0.19	0.20	0.35	0.08	25.62
7	0.003	25.11	25.62	0.19	0.19	-0.39	-0.09	25.21
8	0.004	24.86	25.21	0.19	0.19	0.49	0.11	25.16
9	0.005	24.60	25.16	0.19	0.19	-0.46	-0.10	24.69
10	0.005	24.34	24.69	0.18	0.19	0.38	0.09	24.62
11	0.006	24.08	24.62	0.18	0.19	-0.45	-0.10	24.16
12	0.006	23.81	24.16	0.18	0.19	-0.11	-0.03	23.97
13	0.007	23.56	23.97	0.18	0.19	-1.37	-0.31	23.43
14	0.008	23.31	23.43	0.17	0.18	1.63	0.37	23.86
15	0.008	23.08	23.86	0.18	0.19	-0.18	-0.04	23.22
16	0.009	22.86	23.22	0.17	0.18	2.30	0.52	23.57
17	0.010	22.67	23.57	0.18	0.19	0.59	0.13	22.99
18	0.010	22.49	22.99	0.17	0.18	0.61	0.14	22.81
19	0.011	22.35	22.81	0.17	0.18	-1.79	-0.41	22.12
20	0.012	22.23	22.12	0.16	0.18	-0.45	-0.10	22.30
21	0.012	22.14	22.30	0.17	0.18	-0.65	-0.15	22.17
22	0.013	22.08	22.17	0.17	0.18	-1.16	-0.26	21.99
23	0.013	22.05	21.99	0.16	0.18	-1.26	-0.29	21.94

24	0.014	22.06	21.94	0.16	0.18	0.55	0.13	22.36
25	0.015	22.09	22.36	0.17	0.18	0.09	0.02	22.29
26	0.015	22.16	22.29	0.17	0.18	0.70	0.16	22.50
27	0.016	22.26	22.50	0.17	0.18	0.37	0.08	22.52
28	0.016	22.38	22.52	0.17	0.18	0.85	0.19	22.76
29	0.017	22.54	22.76	0.17	0.19	0.78	0.18	22.90
30	0.018	22.72	22.90	0.17	0.19	0.09	0.02	22.93
31	0.018	22.92	22.93	0.17	0.19	-0.10	-0.02	23.09
32	0.019	23.14	23.09	0.17	0.19	-1.01	-0.23	23.10
33	0.019	23.38	23.10	0.17	0.19	-2.22	-0.50	23.06
34	0.020	23.63	23.06	0.17	0.19	-1.70	-0.39	23.43
35	0.020	23.88	23.43	0.17	0.19	-1.39	-0.31	23.77
36	0.021	24.15	23.77	0.18	0.20	-0.33	-0.07	24.27
37	0.021	24.41	24.27	0.18	0.20	-1.45	-0.33	24.29
38	0.022	24.68	24.29	0.18	0.20	-1.70	-0.39	24.49
39	0.023	24.93	24.49	0.18	0.20	-0.44	-0.10	25.04
40	0.023	25.18	25.04	0.19	0.21	0.94	0.21	25.60
41	0.024	25.41	25.60	0.19	0.21	-0.13	-0.03	25.60
42	0.024	25.63	25.60	0.19	0.21	-0.31	-0.07	25.77
43	0.025	25.83	25.77	0.19	0.22	1.38	0.31	26.36
44	0.025	26.00	26.36	0.20	0.22	-1.25	-0.28	25.94
45	0.026	26.15	25.94	0.19	0.22	0.94	0.21	26.58
46	0.026	26.27	26.58	0.20	0.22	1.26	0.29	26.78
47	0.027	26.36	26.78	0.20	0.23	0.62	0.14	26.73
48	0.027	26.42	26.73	0.20	0.23	0.28	0.06	26.71
49	0.027	26.45	26.71	0.20	0.23	0.29	0.06	26.74
50	0.028	26.45	26.74	0.20	0.23	-0.43	-0.10	26.58
51	0.028	26.41	26.58	0.20	0.23	0.80	0.18	26.82

52	0.029	26.34	26.82	0.20	0.23	0.60	0.14	26.71

Appendix 2: Monte – Carlo Simulation of future Temperature Values as well as Payoffs

Forecast Period	Exp Temp	StrikeTemp	Payoff	Present Value
1	26.92749	24	315.2685	314.601121
2	26.57946	24	277.7875	276.6127362
3	26.28797	24	246.3964	244.8350189
4	25.97952	24	213.1787	211.3793588
5	26.01101	24	216.5705	214.2880512
6	25.62134	24	174.6063	172.4003759
7	25.21432	24	130.773	128.8475404
8	25.16377	24	125.3294	123.2226751
9	24.69016	24	74.3252	72.92115212
10	24.61511	24	66.24311	64.85416253
11	24.16132	24	17.37334	16.97305643
12	23.97362	24	0	0
13	23.43158	24	0	0
14	23.86328	24	0	0
15	23.22423	24	0	0
16	23.56568	24	0	0
17	22.98721	24	0	0
18	22.81367	24	0	0
19	22.12261	24	0	0
20	22.30343	24	0	0
21	22.1702	24	0	0
22	21.9934	24	0	0
23	21.94173	24	0	0

24	22.35855	24	0	0
25	22.29395	24	0	0
26	22.49852	24	0	0
27	22.52495	24	0	0
28	22.76078	24	0	0
29	22.90108	24	0	0
30	22.925	24	0	0
31	23.08599	24	0	0
32	23.10054	24	0	0
33	23.06287	24	0	0
34	23.43114	24	0	0
35	23.76503	24	0	0
36	24.2719	24	29.28102	27.13038293
37	24.28686	24	30.89306	28.56343029
38	24.49409	24	53.20927	49.09263832
39	25.0383	24	111.8168	102.9475576
40	25.6033	24	172.6633	158.6312025
41	25.59932	24	172.2346	157.9023875
42	25.77415	24	191.0628	174.7930226
43	26.35634	24	253.7595	231.6594659
44	25.93781	24	208.6871	190.1091817
45	26.57942	24	277.784	252.5191191
46	26.77909	24	299.2865	271.4900668
47	26.72597	24	293.5665	265.7376285
48	26.70932	24	291.7726	263.5546763
49	26.73971	24	295.0461	265.9474269
50	26.57561	24	277.3732	249.48826
51	26.81744	24	303.4171	272.3362744

52	26.70885	24	291.7225	261.285349

Appendix 3: Solution of SDE

$$dT_t = \left\{\frac{dT_t^m}{dt} + a(T_t^m - T_t)\right\}dt + \sigma_t dW_t, \quad t > s$$

Let $U_t = T_t e^{at}$

$$U_t = f(t, T_t)$$

$$dU_t = df(t,T_t) = \left(aT_t^m \frac{\partial f}{\partial T_t} - aT_t \frac{\partial f}{\partial T_t} + \frac{\partial f}{\partial T_t} + \frac{\partial^2 f}{2\partial T_t^2}\sigma^2\right)dt + \frac{\partial f}{\partial T_t}\sigma dB_t$$

$$\frac{\partial f}{\partial T_t} = e^{at} \qquad \frac{\partial^2 f}{\partial T_t} = 0 \qquad \frac{\partial f}{\partial T_t} = aT_t e^{at}$$

$$dU_t = [aT_t^m e^{at} - aT_t e^{at} + aT_t e^{at}]dt + e^{at}\sigma dB_t$$

$$\int_0^t dU_s = \int_0^t (aT_t^m e^{as} ds + \int_0^t e^{as}\,\sigma dB_s$$

$$U_t = U_0 + T_t^m e^{at} - T_t^m + \int_0^t e^{as}\,\sigma dW_s$$

$$T_t e^{at} = T_0 + T_t^m(e^{at} - 1) + \int_0^t e^{as}\,\sigma dW_s$$

$$T_t = T_0 e^{-at} + T_t^m(1 - e^{-at}) + \int_0^t e^{a(t-s)}\,\sigma dW_t$$

Which can be re-written as

$$T_t = (T_0 - T_t^m)e^{-at} + T_t^m + \int_0^t e^{a(t-s)}\,\sigma dW_t$$

$$T_t = (x - T_s^m)e^{-a(t-s)} + T_t^m + \int_s^t e^{-a(t-T)}\sigma_t dW_T$$

where x = T_s.

Appendix 4: Spline Interpolation

```
"Naming of Variables"

temp_data<-Research_Dat

per<-Research_Dat$`Time Period`

temp<-Research_Dat$Temp


"Interpolate Data to find missing data points"

interpoldata<-spline(x=per, y=temp,n=1589)

plot(per,temp)

points(interpoldata$x,interpoldata$y,type = "l")
```