



**Strathmore**  
UNIVERSITY

STRATHMORE SCHOOL OF COMPUTING AND ENGINEERING SCIENCES  
BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBERSECURITY  
END OF SEMESTER EXAMINATION  
CNS 1103: DIFFERENTIAL CALCULUS

DATE: 31st October 2024

TIME: 2 Hours

**INSTRUCTIONS**

1. This examination consists of **FIVE** questions
2. Answer Question ONE (COMPULSORY) and any other **TWO** questions

**QUESTION ONE (30Marks)**

1. (a) Find the domain , range and sketch the graph of the function  $f(x) = \sqrt{9 - x^2}$   
(6 marks)
- (b) Determine whether the limit exist or not of the following functions
  - i.  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$  (3 marks)
  - ii.  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$  (3 marks)
- (c) Prove that

$$g(x) = \left\{ \begin{array}{ll} \frac{x^2 - x - 6}{x - 3}, & x \neq 3 \\ 5, & x = 3 \end{array} \right\}$$

is continuous at  $x = 3$  (6 marks)

(d) Find the derivative of the function  $f(x) = \frac{x}{x-1}$ , by using definition of derivative (first principle of differentiation) (6 marks)

(e) Suppose that it cost  $C(x) = x^3 - 6x^2 + 15x$  dollars to produce  $x$  radiators when 8 to 30 radiators are produced and that  $r(x) = x^3 - 3x^2 + 12x$  gives the dollar revenue from selling  $x$  radiators. Your shop currently produces 10 radiator a day. About how much extra will it cost to produce one more radiator a day, and what is your estimated increase in revenue for selling 11 radiator a day (6marks)

### QUESTION TWO (20Marks)

2. (a) Given that  $\lim_{x \rightarrow c} f(x) = 5$  and  $\lim_{x \rightarrow c} g(x) = -2$ , find the following limits

i.  $\lim_{x \rightarrow c} (f(x) + g(x))$  (2 marks)

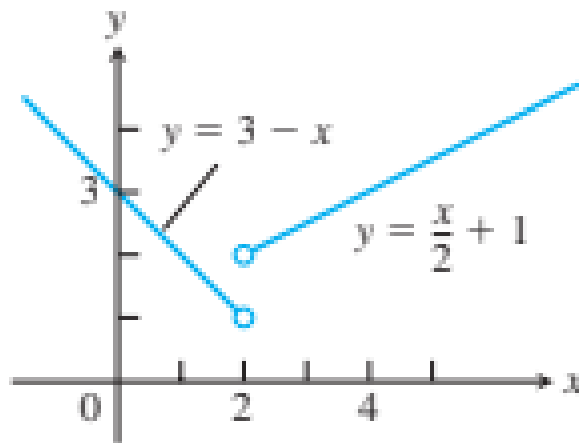
ii.  $\lim_{x \rightarrow c} (f(x) + 3g(x))$  (2 marks)

(b) i. State the formal definition of limit (3 marks)

ii. Show by using formal definition of the limit that  $\lim_{x \rightarrow 1} 5x - 3 = 2$  (5 marks)

(c) Let

$$f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & x > 2 \end{cases}$$



- i. find  $\lim_{x \rightarrow 2^+} f(x)$  and  $\lim_{x \rightarrow 2^-} f(x)$  (2 marks)
- ii. Does  $\lim_{x \rightarrow 2} f(x)$  exist? If so what is it? If not, why not (2 marks)
- iii. find  $\lim_{x \rightarrow 4^+} f(x)$  and  $\lim_{x \rightarrow 4^-} f(x)$  (2 marks)
- iv. Does  $\lim_{x \rightarrow 4} f(x)$  exist? If so what is it? If not, why not (2 marks)

**QUESTION THREE (20Marks)**

3. (a) For what value of  $b$  that the function

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

is continuous at  $x = -2$  (5 marks)

- (b) Let  $f(x) = 4x^3 - 6x^2 + 3x - 2$ , Use Intermediate Value Theorem to show that  $f(x)$  has roots between closed interval 1 and 2 (5 marks)

- (c) Sketch the graph of  $f$

$$f(x) = \begin{cases} x, & x < 1 \\ 3x^2 - 1, & x \geq 1 \end{cases}$$

Is  $f$  continuous at  $x = 1$ ? if not, state the nature of discontinuity of  $f$  at  $x = 1$  (5 marks)

- (d) Given  $h(x) = \sin(x^2)$ , show that  $h(x)$  is continuous on real number ( $\mathbb{R}$ ) (5 marks)

**QUESTION FOUR (20Marks)**

4. (a) Find the derivative of the following function

i.  $f(x) = 5e^x + \cos x$  (2 marks)

ii.  $f(x) = \frac{x}{x^2 + 1}$  (3 marks)

- (b) i. Find  $f'(x)$  of  $f(x) = \sqrt{x}e^x$  (3 marks)

ii. If  $f(x) = \sqrt{x}g(x)$ , where  $g(4) = 2$  and  $g'(4) = 3$ , find  $f'(4)$  (3 marks)

- (c) i. Find an equation of the tangent line to the curve  $f(x) = x^2$  at the point (1,1) ( 6 marks)
- ii. If another line PQ is normal to a tangent line at (1,1) .Find the equation of the line PQ (3 marks)

**QUESTION FIVE (20Marks)**

5. (a) Find the derivative of  $f(x) = (8x - 3)^5$  (5 marks)
- (b) Given  $f(x) = x^3 - 3x + 1$  on  $(-2 \leq x \leq 3)$  .Does f have absolute maximum vale and absolute minimum value (7 marks)
- (c) Let  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ . Find all the local maximum and Minimum values (8 marks)