MODELING HEAT-RELATED MORTALITY BASED ON GREENHOUSE EMISSIONS IN OECD COUNTRIES

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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This Research Project has been submitted for examination with my approval as the Supervisor.

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Contents

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<u>ABSTRACT</u>

Greenhouse emissions by human activities are known to irreversibly increase global temperatures through the greenhouse effect. This study sought to propose a mortality model with sensitivity to heat-change effects as one of the underlying parameters in the model. As such, the study sought to establish the relationship between greenhouse gas emissions and mortality indices in five OECD[1] countries (USA, UK, Japan, Canada & Germany). Upon the establishment of the relationship using correlation analysis, an additional parameter that accounts for the sensitivity of heat-changes on mortality rates was incorporated in the Lee-Carter model. Based on the proposed model, new parameter estimates were calculated using iterative algorithms for optimization. Finally, goodness of fit for the original Lee-Carter model and the proposed model were compared using deviance comparison. The proposed model provides a better fit to mortality rates especially in USA, UK and Germany where the mortality indices have a strong positive correlation with the level of greenhouse emissions. The results of this study are of particular concern to actuaries and demographers and climate-risk experts who seek to use better mortality-modeling techniques in the wake of heat-effects caused by increased greenhouse emissions.

Key words: Lee-Carter model, greenhouse emissions, climate risk, OECD.

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[1] Organisation for Economic Co-operation and Development.

CHAPTER ONE: INTRODUCTION

1.1 Background

1.1.1 Trends in Mortality Modeling

(Excess mortality)[2] refers to the number of deaths caused by a specific disease, condition or exposure to harmful circumstances such as radiation, environmental chemicals or natural disaster. Mortality modeling has for a long time been of paramount interest to actuaries, demographers and epidemiologists. Epidemiologists have been at the forefront of identifying key risk factors that affect mortality trends. Demographers often rely on research from epidemiologists to determine future population patterns.

With increased longevity over time, actuaries have been keen to ensure nearly-accurate mortality modeling techniques are implemented as this has implications in the pricing of financial products contingent on survival or death (Bloom, D. E., Canning, D., & Moore, M., 2014). The need for accurate mortality models is necessitated by the stochastic nature of future mortality improvements.

One of the most successful stochastic mortality models was developed by (Lee & Carter, 1992). Ever since the inception of the Lee-Carter model, various improvements have been proposed to capture the uncertainty inherent in longevity improvements. (Cairns, A. J., 2013) highlights that robustness of models is key in order to maintain a level of trust with those who will benefit from the implementation of those models. (Cairns, A. J., 2013) further expounds that robustness could take the form of a better fit, a better forecast, or a better risk-management decision.

In spite of the root of all other mortality models the general trend is inclined towards multi-population mortality modeling. A notable example is the model proposed by (Li, N., & Lee, R., 2005) which is a multi-population extension of the Lee-Carter model. Their study proposed an additional common factor that drives mortality for multi-populations. In a recent research done by (Kleinow, T., 2015) the study proposed a common-age-effect multi-population model.

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[2] Definition from https://www.verywell.com/what-is-excess-mortality-2223377

The uniqueness of this model is that rather than having a common global period-effect, it has two country-specific period effects.

Another major trend in mortality modeling is the incorporation of an observable factor in extrapolative stochastic models. This extra complexity is deemed to have a better fit to historical data. For example, macroeconomic variables such as unemployment rate and gross domestic product have for a long time been incorporated in mortality models as an observable factor. This serves to improve the model fitting of mortality rates so that forecasts can be more accurate.

Despite the development of models that use an observable factor, there is scanty research on the use of heat-related observable factors in modeling mortality. (O'Hare, C., Seklecka, M., & Pantelous, A. A., 2015) excellently incorporated temperature-effects for older ages in their mortality model. However, for purposes of consistency across ages, this study focused on an underlying causative agent for temperature changes: Greenhouse gas emissions.

1.1.2 The Context of Greenhouse Emissions

Due to the fact that countries in the OECD are among the most industrialized, GHG emissions in these nations are among the highest in the world. Although the Kyoto protocol (United Nations, 1998)[3] provided a comprehensive outline of the necessary steps needed to reduce global emission of greenhouse gases in order to curb global climate change, implementation has been poor especially in terms of participation from developing countries (Frankel, J. A., 1999). It is worth recognizing that the Paris agreement on climate change in 2015 and the Kigali amendment to the Montreal protocol in 2016 could overcome these hindrances and lead to sustainable levels of greenhouse emissions.

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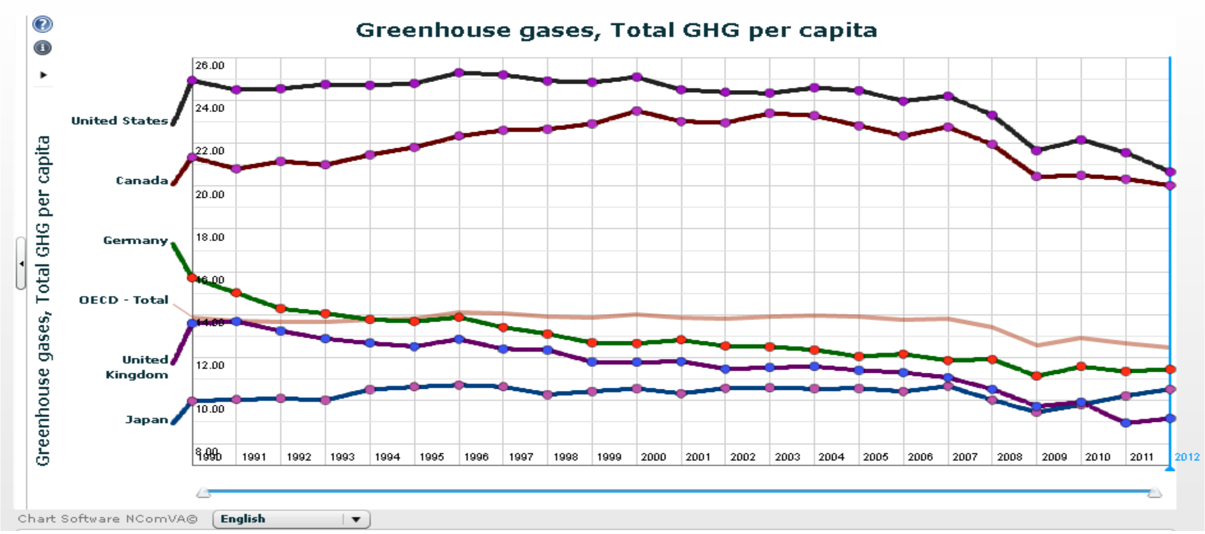
[3] United Nations (1998). Kyoto Protocol to the United Nations Framework Convention on Climate Change. Retrieved from http://unfccc.int/resource/docs/convkp/kpeng.pdf

Figure 1 below shows the trends in greenhouse emissions in selected OECD countries from 1990 to 2012.

Figure 1: (Source: http://stats.oecd.org/)



It is worth noting that the intensities of greenhouse emissions in USA and Canada are higher than the OECD total. There is need to stabilize these intensities to sustainable levels. Although there is a steady decline of greenhouse emission intensities between 2007 and 2012 for all the five countries, future emission uncertainties prove the need to ascertain the temperature-related impact of greenhouse emissions.

According to (Oreskes, 2004)[4] there is a scientific consensus that the mean surface temperature of the earth has warmed in recent decades. This global warming is largely attributed to emission of greenhouse gases; mainly (CO_2)[5] emission. Greenhouse gases exist naturally in the

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[4] Oreskes, N. (2004). Beyond the Ivory Tower. The Scientific Consensus on Climate Change. Retrieved from http://science.sciencemag.org/content/306/5702/1686

[5] Carbon dioxide.

atmosphere and they’re helpful in maintaining global temperatures to sustainable levels through the greenhouse effect.

However; human activities that have led to deforestation and burning of fossil fuels have intensified the greenhouse effect. If human activities that contribute to the increased greenhouse emissions are not controlled, then continued warming is expected to have tremendous effects among them being: increase in magnitude and frequency of natural hazards such as floods (Huntington, 2006) and droughts (Mason and Goddard, 2001).

1.1.3 The Context of Heat-related Mortality

The European heat wave in 2003 that resulted to more than 30000 deaths (De Bono, A., Peduzzi, P., Kluser, S., & Giuliani, G., 2004) revealed but one thing; there is need for robustness of mortality models in capturing shocks arising from heat changes. Projections of climate change in Europe show that in the next century, heat waves will become more frequent, intense and will last longer not only in regions with a heat-wave history but also in the regions not previously characterized by heat-wave events.

Significant temperature-mortality relationships have been widely documented. (Gosling, N., Lowe, A., McGregor, R., Pelling, M. & Malamud D., 2009) expounds on the impact that temperature-changes has on mortality. The temperature-effect of mortality has been overlooked especially when it comes to incorporating heat-effects in mortality models. This is a cause of concern particularly because global warming is steadily intensifying.

1.2 Problem Statement

Various studies have been conducted linking temperature-changes to excess mortality. (Armstrong, B., 2006). However, there has been increased concern of how future mortality trends will evolve and what the main drivers of mortality will be[6]. The problem that this study seeks to address is the impact of the dynamics of heat-conditions on mortality. In light of this, it is highly pivotal in ensuring mortality models are robust enough to fit the dynamics of heat-related deaths (O'Hare, C. et al., 2015).

It is evident that in the recent past, there has been enormous emission of greenhouse gases causing global temperatures to soar (Lashof, D. A., & Ahuja, D. R., 1990). Global warming, if not controlled to sustainable levels, will be a major threat to human life not only in relation to morbidity but also in relation to mortality (Doyon, B., Belanger, D. & Gosselin, P., 2008).

Even though (O'Hare, C. et al., 2015) developed the first mortality model to capture temperature-effects on mortality, there is no documented evidence of a mortality model that explicitly captures the underlying causative agent for the temperature changes. In fact, there is no consensus on what temperature measure is the best predictor of mortality (Barnett, A. G., Tong, S., & Clements, A. C. A., 2010). Therefore this study also seeks to address the gap of consistency when modeling heat-related mortality. As such, greenhouse emissions were used as a proxy for measuring heat-changes.

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[6] Institute & Faculty of Actuaries (2015). Longevity Bulletin: Modeling Edition, Issue 7. Retrieved from https://www.actuaries.org.uk/sites/default/files/field/document/Longevity%20Bulletin_7.PDF

1.3 Research Objectives

1. To establish the relationship between GHG[7] emissions and mortality rates.
2. To determine the robustness of the proposed mortality model after incorporating a heat-change parameter.in comparison to the original Lee-Carter model.

1.4 Research Questions

1. What is the relationship between GHG emissions and mortality rates?
2. How does the goodness of fit of the extended model incorporating a heat-change parameter compare to the original Lee-Carter model?

1.5 Justification

The results of this study are of paramount importance to actuaries and demographers. Actuaries will have a more informed perspective of accurately pricing financial products such as pensions and insurance which are contingent on survival or death of individuals. Demographers will also be able to determine global population estimates and consequently advise governments on economic planning.

1.6 Scope of the Study

This study is limited to testing the robustness of the extended Lee-Carter model in fitting the dynamics of heat changes in an attempt to reveal the impact of global warming on mortality. The data that was used in this study is mortality rates and greenhouse emissions from five OECD countries: USA, UK, Canada, Germany and Japan. Therefore, this study focused on parameter estimation of both the original Lee-Carter model and the extended Lee-Carter model that captures heat-change effects. Goodness of fit tests between the two models was also carried out using deviance comparison.

Mortality forecasting based on the model proposed in this study is beyond the scope of this study. Also, temperature-change-projection uncertainties are beyond the scope of this study. Temperature-change-projection uncertainties may arise from various sources; for example, emission uncertainties and adaptation uncertainties (Gosling, N. et al., 2009).

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[7] Greenhouse gas(es).

CHAPTER TWO: LITERATURE REVIEW

The objective of this section is to underpin the conceptual foundations of the study. Section 2.1 surveys the existing literature that establishes the relationship and impact of heat changes on mortality. Section 2.2 discusses some of the existing mortality models. Section 2.3 explores the essence of incorporating an additional relevant parameter to a model in order to obtain a better fit.

2.1 Relationship and Impact of Heat-Changes on Mortality

Different studies have shown that high temperatures are directly related to mortality. Furthermore, many studies on the effects of climatic change on future mortality are being conducted. Temperature-changes are largely attributable to emission of greenhouse gases by human activities.

(Gosling, N. et al., 2009) comprehensively and critically analysed vast literature comparing temperature-mortality relationships. Their research annotated that present studies use calculations of excess mortality, an epidemiological or a synoptic climatological approach to establish the association between temperature and mortality. The general result of such calculations depict a strong association between temperature and mortality. However, their research also notes of the uncertainty inherent in future temperature-related mortality; especially uncertainty pertaining to temperature-mortality relationship modeling. The findings of their research are relevant in this study because they reveal what was missed in modeling temperature-mortality relationships: Modeling mortality based on the largely attributable factor to temperature changes, that is greenhouse emissions.

According to the fifth report of the Intergovernmental Panel on Climate Change[8], global warming is indeed a real threat. In fact, Climate Change & Sustainability Committee unravelled that GHG emissions is primarily the main cause for global temperature rise.[9] The committee further espoused that if GHG emissions is not controlled to sustainable levels then the effect of

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[8] IPCC 2013 Climate Change (2013). Working Group 1 Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. *The Physical Science Basis.* Retrieved from http://www.ipcc.ch/report/ar5/wg1/

[9] Climate Change and Sustainability Committee (2015). Climate Change and Resource Sustainability: An Overview for Actuaries. Retrieved from https://www.cia-ica.ca/docs/default-source/2015/215068e.pdf?sfvrsn=2

global warming will have direct impact on human mortality in the form of frequent severe occurrences such as heat waves.

Estimated projections of future greenhouse emissions reveal that global temperatures will most likely increase by more than 1.5 degrees celsius by the end of the 21st century under all scenarios. While incorporating these estimations, (Lowe, R., Ballester, J., Creswick, J., Robine, J. M., Herrmann, F. R., & Rodó, X., 2015) established a case that air-pollution effects tends to act in synergy with heat-changes thus accelerating mortality. While conducting a similar study to deduce the impact of temperature-changes on mortality, (Roldán, E., Gómez, M., Pino, M. R., Pórtoles, J., Linares, C., & Díaz, J., 2016) established a case of increasing mortality in Spain. (Roldán, E. et al., 2016) attributed the increasing mortality to climate-change-related heat waves in Spain.

2.2 A Review of Mortality Models

There are generally three broad approaches of modeling mortality: expectation methods, explanatory methods and extrapolation methods (Booth, H., & Tickle, L., 2008).

2.2.1 Expectation methods

Expectation methods rely heavily on expert judgement whereby an assumed mortality forecast is specified and accompanied by upside and downside scenarios. Due to the subjectivity and non-incorporation of stochasticity in expectation methods, actuaries have progressed towards sophisticated extrapolative methods in modeling and forecasting mortality (Booth & Tickle, 2008).

2.2.2 Explanatory methods

Explanatory methods of mortality modeling involve an approach whereby mortality rates are decomposed against causal risk factors such as macroeconomic variables and socio-economic indicators (e.g. changing lifestyle patterns). Process-based methods are almost similar to explanatory methods although process-based methods focus on modeling mortality from an epidemiological point of view using health-risk factors (e.g. Coronary heart disease).

2.2.3 Extrapolative methods

Extrapolative methods assume that future mortality trends will essentially be a continuation of past mortality trends. (Lee R., & Carter L., 1992) made the first attempt in incorporating a stochastic element to modeling longevity by fitting historical mortality data and modeling the time trend as a stochastic process.

(Lee R., & Carter L., 1992)'s seminal work postulated that the logarithm of the central mortality rate has a linear form:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \qquad (1)$$

Where: α_x is a constant age-dependent factor, κ_t is the time dependent mortality index, β_x is the sensitivity at age x to the mortality index and $\varepsilon_{x,t}$ is the error term at age x and time t. The model assumes that the errors are IID variables with normal distribution of mean 0 and variance δ^2.

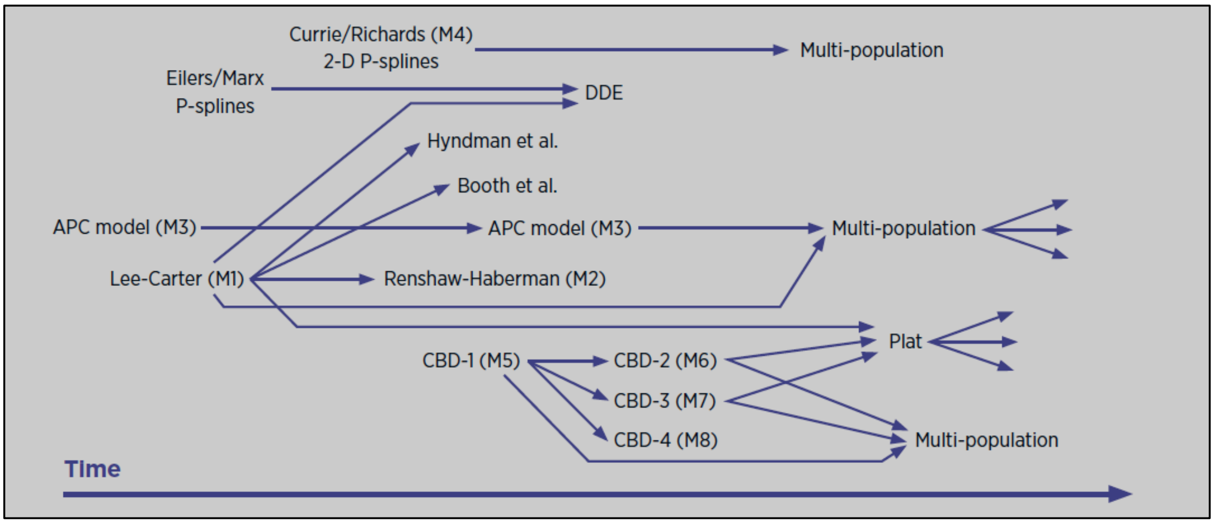
Over the years, attempts to improve the performance of the Lee-Carter model have been proposed among them include: (Booth, H., Maindonald, J. & Smith, L., 2002) who addressed the non-linearity of the time component of the Lee-Carter model particularly for Australian data, (Renshaw. A. E., & Haberman., S., 2003) and (Renshaw, A. E., & Haberman, S., 2006) who included factors that capture age, period and cohort effects.

There exists other classes of extrapolative stochastic mortality models other than the Lee-Carter class. Penalised spline (P-Spline) models; for example, allow the modeller to achieve a balance of smoothness and fit to historical data. In P-Spline models, mortality rates are taken to be a linear combination of functions at each defined segment of the data; for instance, age could be defined by simple polynomial functions (Bruce, R., Chris, R., Francisco, O., Michael, C., Lynn X., Benjazia, Z., & Yang, G., 2013).

Other classes involve a model- integrating approach. (Plat, 2009) and (O'Hare, C., & Li, Y., 2012) combine both the Lee-Carter class of models and the model proposed by (Cairns, A. J., Blake, D., & Dowd, K., 2006). (Plat, 2009) developed a mortality model of four factors which capture the influence of younger-age mortality to older-age mortality experience. (O'Hare & Li, 2012) extended the Plat model to capture the non-linear properties of mortality at younger ages.

In 2015, The Institute & Faculty of Actuaries (UK) tabulated the genealogy of mortality models as explained by (Cairns, A. J., 2013) to show the inclination towards multi-population modeling. Among all the seminal models, the Lee-Carter model has undergone more several improvements.

Figure 2: Genealogy of mortality models.



Note: Figure 2 above shows the various mortality models since their inception and their evolution towards multi-population modeling.

Extrapolative models discussed above have a stochastic element and; therefore, this study focused on them. In particular the Lee-carter model was considered due to its parsimony.

2.3 Model-extension Techniques

Due to the uncertainty pertaining to future mortality trends, recent modeling techniques have extended extrapolative models by having a blend of explanatory risk factors. For example research has shown that when the economy is performing well, then such a scenario is associated with high mortality (Granados, J. A. T., 2005). While conducting a similar study, (Gerdtham, U. G., & Ruhm, C. J., 2006) proved that after long-term declining trends are excluded, mortality rates especially for industrialized countries tends to rise in economic expansions and decline in economic recessions. The major critic of the study is that the impact of macroeconomic factors on mortality may vary in different countries.

(Clemens, T., Popham, F., & Boyle, P., 2014) analysed whether there is a causal relationship between unemployment and mortality. In their analysis, (Clemens et. al., 2014) found out that a strong causal relationship exists between unemployment and mortality, especially for males. In the quest for analysing the effect of macroeconomic variables on mortality, (Niu, G. & Melenberg, B., 2014) also conducted a similar study by incorporating GDP[10] per capita in an extended Lee-carter model that outperformed the original Lee-Carter in terms of fitting mortality data as well as forecasting future mortality rates. This proves the need to incorporate exogenous causal factors in order to obtain a better fit for the Lee-Carter model as was also proven by (French, D., & O'Hare, C., 2014).

Even though (O'Hare, C. et al., 2015) incorporated an additional parameter that accounts for heat changes, the issue of contention with using temperature as a parameter is with regard to what measure of temperature is the best predictor of mortality. (Barnett, A. G. et. al., 2010) highlights that new studies tend to use a temperature-measure that it easily accessible even though it is not clear cut on which measure is best suited to predict mortality. Although it is assumed that different temperature measures have the same predictive ability, this study seeks to address the gap of such inconsistency by using GHG emissions as a proxy for measuring heat-change effects.

Figure 3: Conceptual Framework

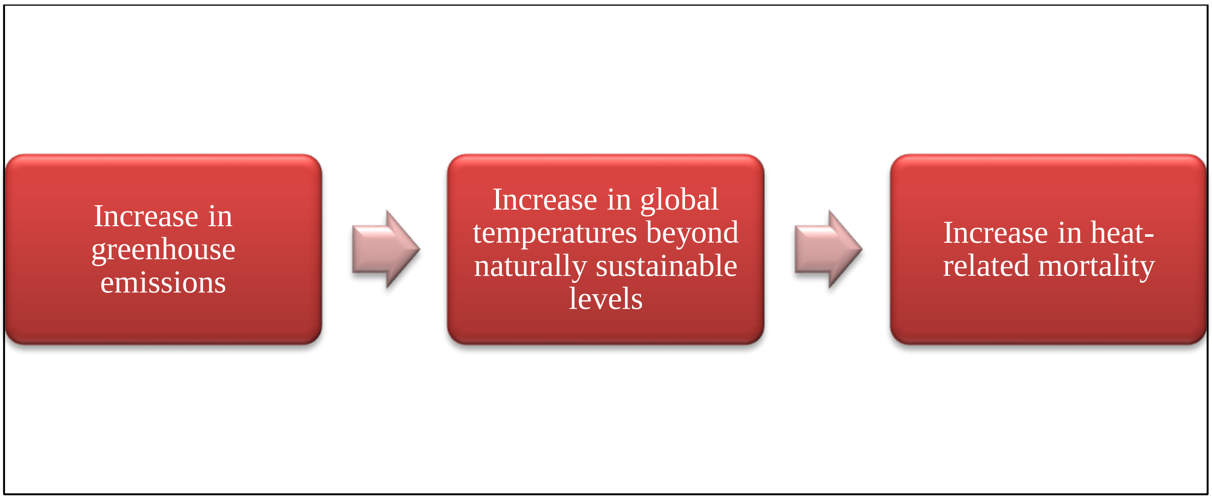


Figure 3 above shows the conceptual framework underpinning the study.

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[10] Gross Domestic Product

CHAPTER THREE: METHODOLOGY

This section sets out the methods and procedures used to conduct the study.

3.1 Research Design

The research approach used in this study was purely quantitative. This study involved fitting parameter estimates into the original Lee-carter model as well as the newly proposed extension of the model, given data for mortality rates together with an observable factor that explains heat-changes. The purpose for using quantitative data was to suffice the parametrization of the models to be used herein and carry out deviance comparison to determine the robustness of the models.

3.2 Population & Sampling Design

The population that was covered in this study comprises of death rates of males and females from five OECD countries: USA, Canada, UK, Germany and Japan. The sampling units are males and females mortality rates for the respective countries. This sample is fully representative of the mortality trends of all other developed countries. Model fitting for males and females was implemented separately since it is assumed that their mortality at any particular age differs.

3.3 Data Collection

Secondary data was used in this study. The data required for this study is mortality rates at individual ages from 1990 to 2012 for both males and females in USA, UK, Germany, Japan and Canada. In addition, total GHG emissions data in kilograms per capita from 1990 to 2012 is also required for all the five countries. The five selected countries were particularly used for this study because they are the highest emitters of greenhouse gases in OECD.

Mortality data is available in the (Human Mortality Database website)[11]. GHG emissions data is also available in the (OECD Statistics website)[12]. This data was downloaded from the

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[11] http://www.mortality.org/

[12] http://stats.oecd.org/

respective websites. The data obtained from these sites is adequate and credible, hence this enhances the appropriateness for its use in this study.

3.4 Data Analysis

Data analysis in this study was carried in four systematic steps. Firstly, parameter estimates for the original Lee-Carter model were computed for all individual ages across the five countries. Secondly, correlation tests were conducted across all the five countries to determine the extent of relationship between mortality indices and log-totals of GHG emissions in kilograms per capita. Here, the GHG emissions data was treated as the parameter that accounts for heat changes. The correlation tests were conducted separately for males and females.

This was followed by computing the parameter estimates of the extended Lee-Carter model that includes an additional parameter to account for heat changes. Due to inaccessibility of raw multidimensional datasets of mortality rates associated with greenhouse emissions, artificially stratified mortality experiences were created. The observed mortality rates were used as the base mortality experience. The levels of greenhouse emissions from 1990 to 2012 were used as the additive factors. As such, the output from the extended model is for testing purpose.

Lastly, the deviance output was used to compare the improvement of fit for the proposed model.

To compute parameter estimates for the original Lee-Carter model, this study used the ‘ilc’ package in R software. The package is specifically designed for a class of Lee-Carter models and their extensions to perform an iterative fitting algorithm technique that minimizes the deviance function as proposed by (Renshaw & Haberman, 2006). The model fitting steps for the original Lee-Carter model as explained by (Butt & Haberman, 2009) are as follows:

3.5 The Models

3.5.1 Parameter Estimates for the Original Lee-Carter Model

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \qquad (1)$$

Where: $m_{x,t}$ is the central death rate of individuals aged x at time t, α_x is a constant age-dependent factor, κ_t is the time dependent mortality index, β_x is the sensitivity at age x to the mortality index and $\varepsilon_{x,t}$ is the error term at age x and time t. The model assumes that the errors are IID variables with normal distribution of mean 0 and variance δ^2.

Step 1: Estimate the appropriate initial values

$$\hat{\alpha}_x = \frac{1}{n}\sum_t \log(\hat{m}_{x,t}) \qquad (2)$$

$$\hat{\beta}_x = \frac{1}{k} \qquad (3)$$

$$\hat{\kappa}_t = 0 \qquad (4)$$

Where n denotes the number of dimensions across all ages at time t.

Step 2: Update the estimate of the age-effect parameter

$$\hat{\alpha}_x = \hat{\alpha}_x + \frac{\sum_t 2w(y-\hat{y})}{\sum_t 2w\hat{y}} \qquad (5)$$

Where w is a weighting factor, y is the observed death and $\hat{y}$ is the death estimate.

Step 3: Update the estimate of the mortality index parameter

$$\hat{\kappa}_t = \hat{\kappa}_t + \frac{\sum_x 2w(y-\hat{y})}{\sum_x 2w\hat{\beta}_x^2\hat{y}} \qquad (6)$$

Step 4: Update the estimate of the parameter that explains the sensitivity to the mortality index

$$\hat{\beta}_x = \hat{\beta}_x + \frac{\sum_t 2w(y-\hat{y})}{\sum_t 2w\hat{\kappa}_t^2\hat{y}} \qquad (7)$$

In each of the above four steps, the fitted values $\hat{y}(\hat{\alpha}_x, \hat{\beta}_x, \hat{\kappa}_t)$ and the deviances $D_i(y_{x,t}\ ,\ \hat{y}_{x,t})$ are calculated. The subscript i refers to the ith step.

Step 5: Checking the deviance convergence

The final step involves checking the deviance convergence between the third and the fourth step.

$$\Delta D = D_3 - D_4$$

This iteration process continues until $\Delta D \approx 0$

When convergence is attained, the parameters $\hat{\beta}_x$ and $\hat{\kappa}_t$ are rescaled so as to satisfy the model conditions that $\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$. Rescaling is achieved by setting:

$$\hat{\beta}_x = \frac{\widehat{\beta}_x}{\sum_x \widehat{\beta}_x} \qquad (8)$$

and

$$\widehat{\kappa}_t = \widehat{\kappa}_t \times (\textstyle\sum_x \widehat{\beta}_x) \quad (9)$$

3.5.2 Parameter Estimates for the Extended Lee-Carter Model

In this section, the study covers a stochastic mortality model that includes both the latent and observable factors; in this case, GHGs is the observable factor. The model in this regard can be considered as an extension of the original Lee-Carter model. The model has a similar structure to the poisson log-bilinear model proposed by (Brouhns, N., Denuit, M., & Vermunt, J. K., 2002). Their research proposed a model whereby the model parameters are essentially the same as the classical Lee-Carter model (Equation 1). The only difference was that the number of deaths was assumed to follow a poisson distribution.

However; for the purpose of this research, this study considered a stratified Lee-Carter model to include an observable factor. This study posits that the logarithm of the central mortality rate $\{\ln(m_{x,t})\}$ has a linear form:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \alpha_g + \ln(\varepsilon_{x,t}) \qquad (10)$$

Where: α_x is a constant age-dependent factor, κ_t is the time dependent mortality index, β_x is the sensitivity at age x to the mortality index, α_g is a factor that accounts for the effects of greenhouse gas emissions on heat-changes and $\varepsilon_{x,t}$ is the error term at age x and time t. It is assumed that the model has a poisson error structure as was also assumed by (Brouhns, N., Denuit, M., & Vermunt, J. K., 2002).

The model fitting approach that is used for this model is almost similar to the one used in the original Lee-Carter model in section 3.5.1. (Butt & Haberman, 2009) explains the iterative fitting technique for this class of the extended Lee-Carter model as follows:

Step 1: Estimate the appropriate initial values

$$\hat{\alpha}_x = \frac{1}{n \times l} \sum_{t,g} \log(\hat{m}_{x,t,g}) \qquad (11)$$

$$\hat{\alpha}_g = 0 \qquad (12)$$

$$\hat{\beta}_x = \frac{1}{k} \qquad (13)$$

$$\widehat{\kappa}_t = 0 \qquad (14)$$

l accounts for the extra dimensions that relates to the factor explaining heat-changes. In this study, it is greenhouse gas emissions in kilograms per capita.

Step 2: Update the estimate of the age-effect parameter

$$\hat{\alpha}_x = \hat{\alpha}_x + \frac{\sum_{t,g} 2w(y-\hat{y})}{\sum_{t,g} 2w\hat{y}} \qquad (15)$$

Where w is a weighting factor, y is the observed death and $\hat{y}$ is the death estimate.

Step 3: Update the estimate of the parameter that accounts for heat-changes

$$\hat{\alpha}_g = \hat{\alpha}_g + \frac{\sum_{t,x} 2w(y-\hat{y})}{\sum_{t,x} 2w\hat{y}} \qquad (16)$$

Step 4: Update the estimate of the mortality index parameter

$$\widehat{\kappa}_t = \widehat{\kappa}_t + \frac{\sum_{x,g} 2w(y-\hat{y})}{\sum_{x,g} 2w\widehat{\beta}_x^2 \hat{y}} \qquad (17)$$

Step 5: Update the estimate of the parameter that explains the sensitivity to the mortality index

$$\hat{\beta}_x = \hat{\beta}_x + \frac{\sum_{t,g} 2w(y-\hat{y})}{\sum_{t,g} 2w\widehat{\kappa}_t^2 \hat{y}} \qquad (18)$$

In each of the above five steps, the fitted values $\hat{y}\left(\hat{\alpha}_x, \hat{\alpha}_g, \hat{\beta}_x, \hat{\kappa}_t\right)$ and the deviances $D_i\left(y_{x,g,t}\ ,\ \hat{y}_{x,g,t}\right)$ are calculated.

Step 6: Checking the deviance convergence

The final step involves checking the deviance convergence between the third and the fourth step.

$$\Delta D = D_4 - D_5$$

This iteration process continues until $\Delta D \approx 0$

Similarly as in the previous procedure in section 3.5.1, the parameters $\hat{\beta}_x \; and \; \hat{\kappa}_t$ have to be rescaled when convergence is attained so as to satisfy the model conditions that $\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$. Rescaling is achieved by setting:

$$\hat{\beta}_x = \frac{\widehat{\beta}_x}{\sum_x \widehat{\beta}_x} \quad (19)$$

and

$$\hat{\kappa}_t = \hat{\kappa}_t \times (\textstyle\sum_x \hat{\beta}_x) \quad (20)$$

3.5.3 Deviance Comparison

Once model parameters for the original and extended Lee-Carter model were estimated, the deviances of the two models were computed. The deviances were then used to assess which of the two models gives a good fit to mortality rates.

CHAPTER FOUR: RESULTS AND ANALYSIS

4.1 Correlation Analysis

Lee-Carter regression was performed on mortality rates and the mortality indices for each year were used in the correlation analysis. The levels of greenhouse emissions were transformed into logarithmic values as it is assumed that the level of greenhouse emissions cannot be negative.

Table 1: Correlation coefficients between mortality indices and greenhouse emissions by country and gender.

COUNTRY	**CORRELATION COEFFICIENT**	
	Males	**Females**
USA	0.7887	0.8855
UK	0.9562	0.9601
JAPAN	0.0812	0.0106
GERMANY	0.9652	0.9754
CANADA	0.0490	0.1420

From the correlation analysis, the correlation coefficients are positive under all scenarios. This reveals that the level of greenhouse emissions and mortality indices move together in the same direction. The correlation is particularly strong in Germany, followed by UK and USA. The relationship between the level of greenhouse emissions and mortality indices was found to be insignificant in Japan and Canada. The proposed mortality model would therefore not be appropriate for use in these two countries. The fact that there exists a strong positive correlation between greenhouse emissions and mortality indices in Germany, UK and USA suggest that the level of greenhouse emissions has a significant impact on the mortality of individuals.

An in-depth examination from the analysis also suggests that when the level of greenhouse emissions is high, females are more likely to die than males. The only exception to this was in Japan. This insinuates that in the presence of greenhouse emissions, females' life expectancy will be lower than that of males all other factors kept constant.

4.2 Extended Lee-Carter Model

The extended Lee-Carter model that incorporates the greenhouse emissions data was run for USA, UK and Germany. The output for the proposed model as shown in figures 4, 5 & 6 unravels that from the observed mortality rates, the age-dependent factor with the emissions-effect incorporated is higher than in the original Lee-Carter model and depends on the specific level of greenhouse emissions in a particular year. This implies that the level of greenhouse emissions has an additive effect to the age-dependent factor. However, since (O'Hare, C. et al., 2015) found out that the impact of temperature-changes on mortality is more pronounced for older ages, there is need for a detailed critical observation of the impact of greenhouse emissions on the age-dependent-factor.

The outputs from the extended model that show the effect of greenhouse emissions on mortality are plotted below:

Figure 4(a)

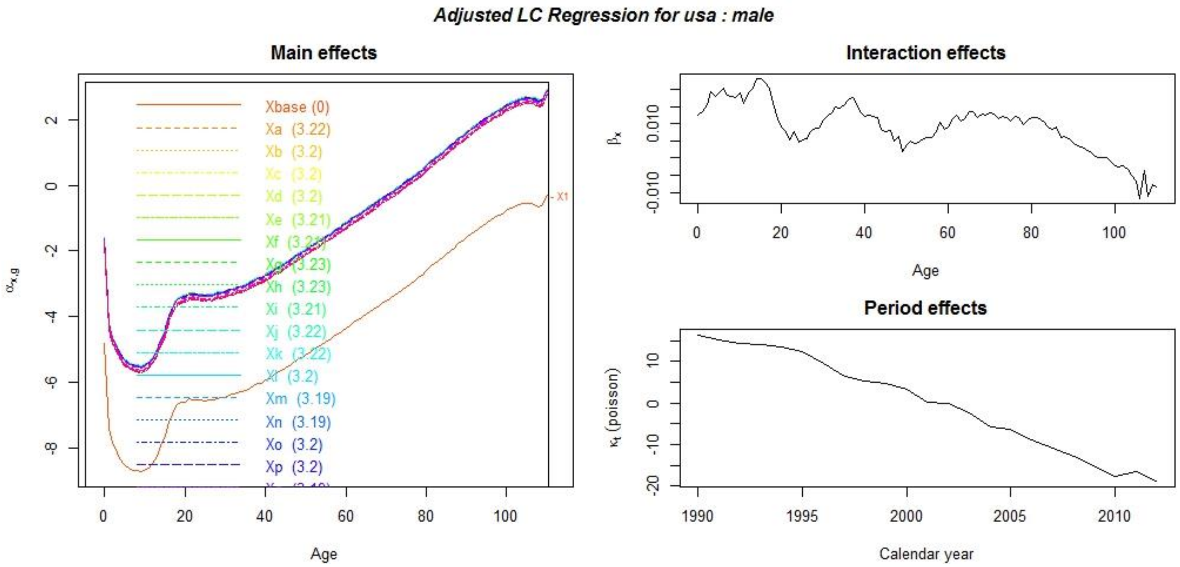


Figure 4(b)

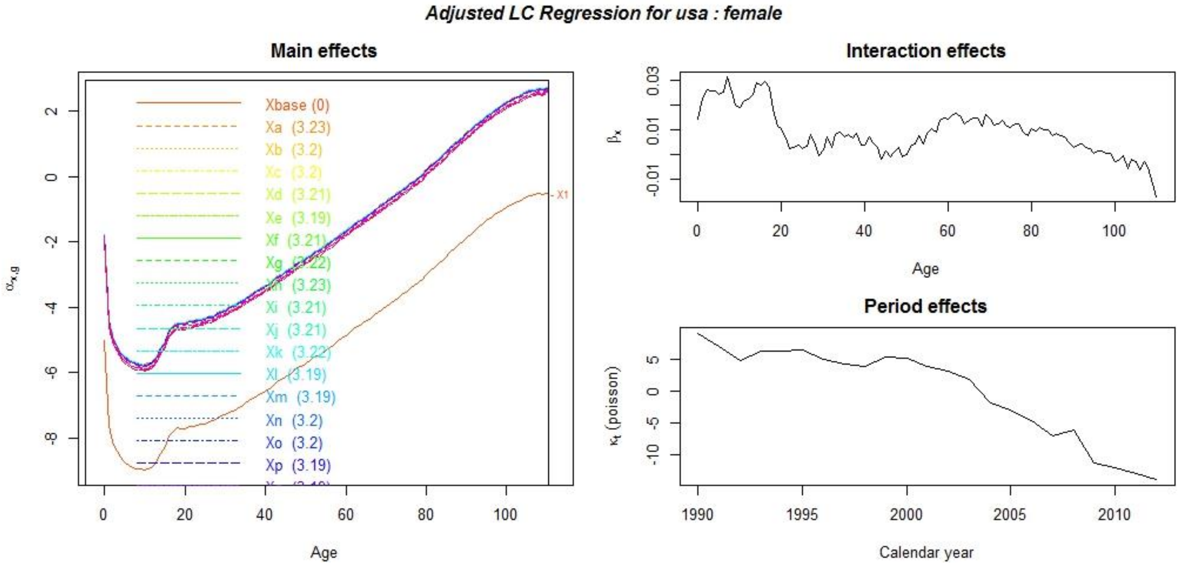


Figure 5(a)

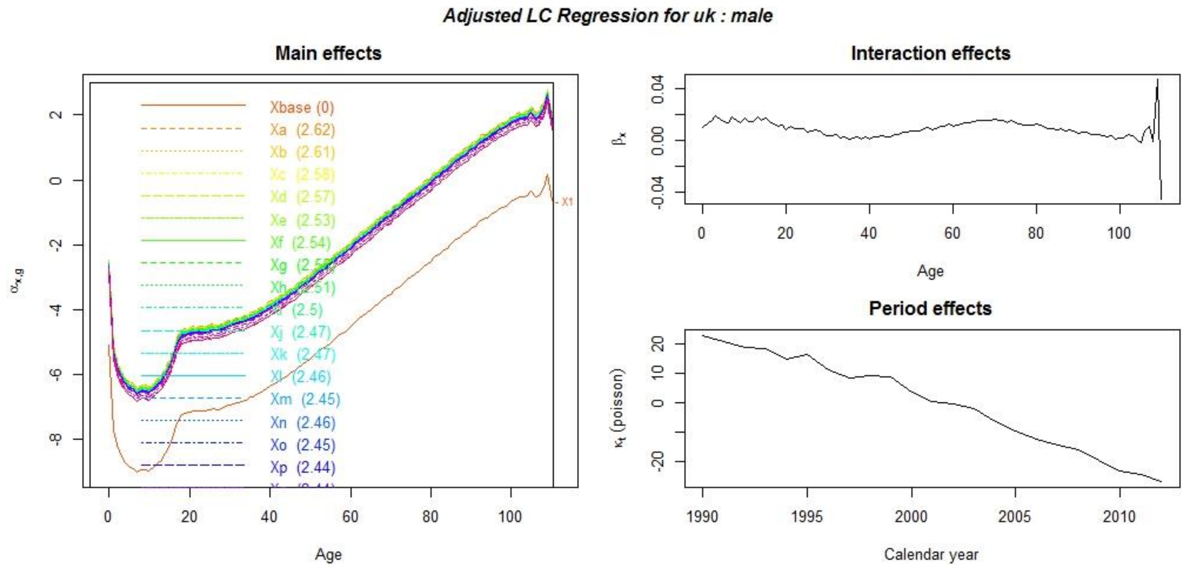


Figure 5(b)

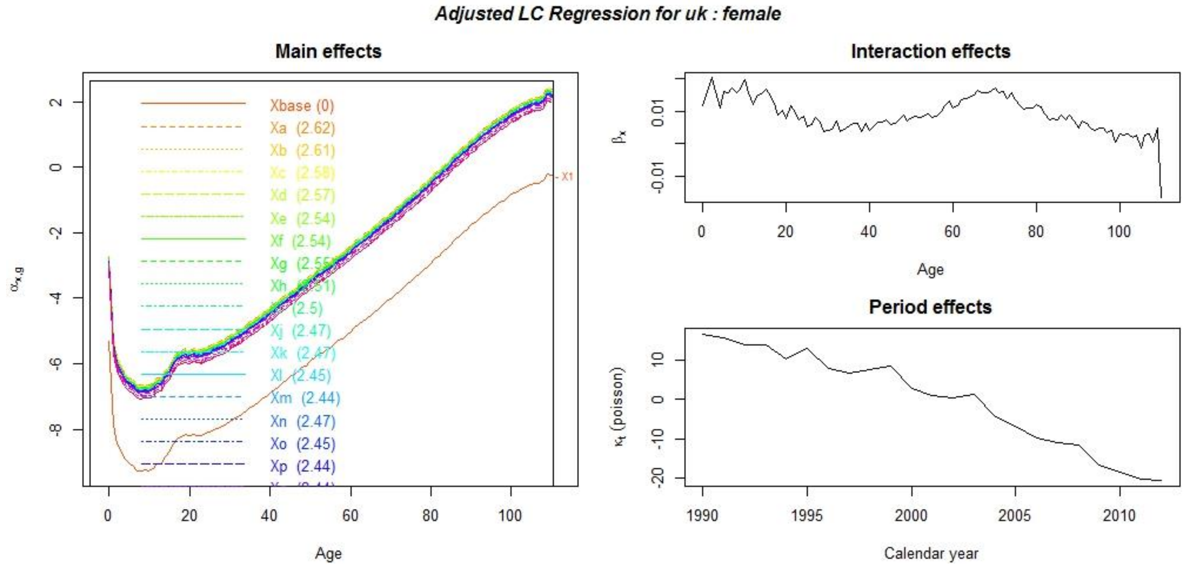


Figure 6(a)

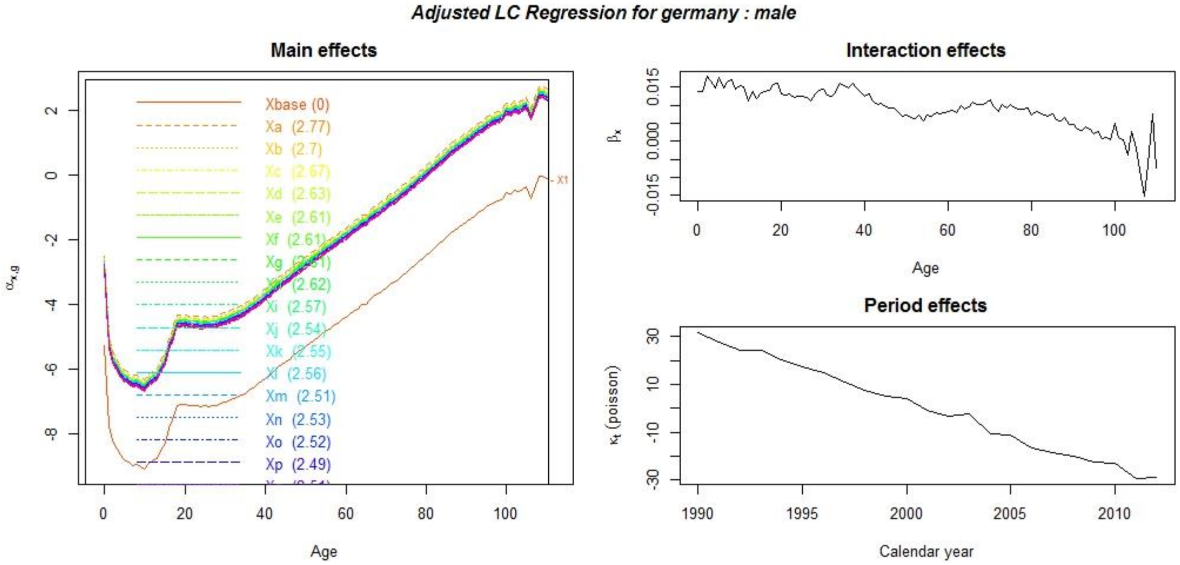
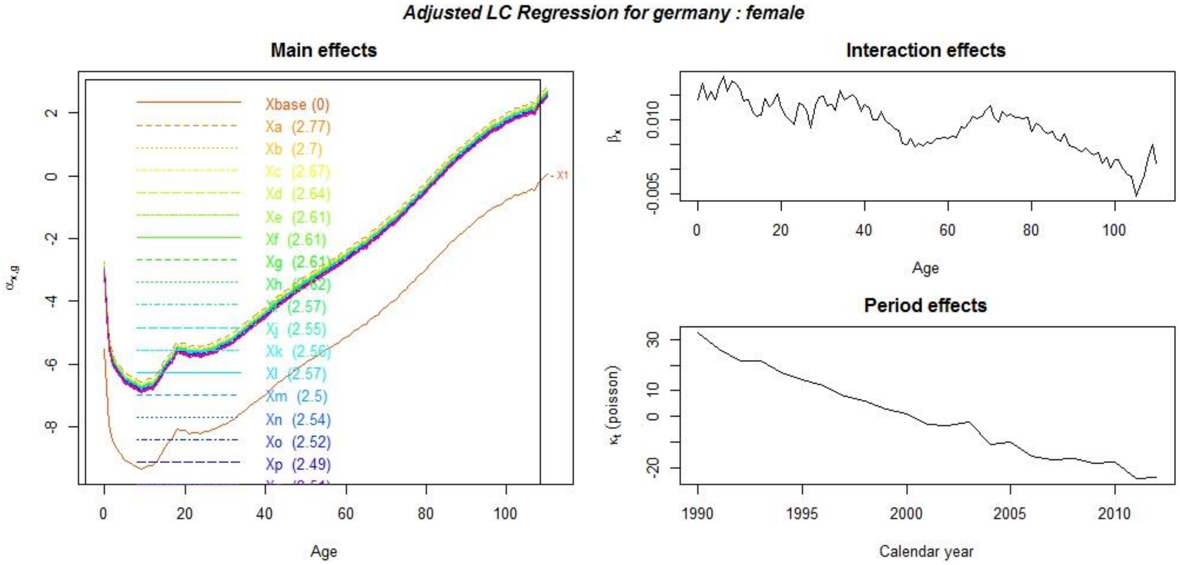


Figure 6(b)



4.3 Model Deviances

Recall that the extended Lee-Carter model decomposes to the original Lee-Carter model when the level of greenhouse emissions is zero. This result of decomposition is similar to the model decomposition as outlined by (Niu, G., & Melenberg, B., 2014) where the explanatory factor was the level of GDP. Since a randomly stratified mortality dataset was produced with the observed mortality rates as the base mortality experience, then the base deviance can be considered as the deviance associated with the original Lee-Carter model. The total deviance arising from incorporating the levels of greenhouse emission can therefore be used to calculate the fit improvement as a percentage.

Table 2: Deviance comparison by country and gender.

COUNTRY	**MALES**		**FEMALES**	
	Base Deviance	**Total Deviance**	**Base Deviance**	**Total Deviance**
USA	12168.26	11603.25 **(4.64%)**	12027.36	11604.90 **(3.51%)**
UK	2998.32	2848.19 **(5.01%)**	3215.56	3074.12 **(4.40%)**
GERMANY	4103.12	3880.07 **(5.44%)**	4615.79	4396.64 **(4.75%)**

The model deviance outputs indicate that the total deviance when taking into account the level of greenhouse emissions is lower than the deviance from the base mortality experience. This elucidates that the fit of mortality rates while taking into consideration the level of greenhouse emission is improved (Percentage fit improvement is bracketed in table 2). This confirms the premise that addition of exogenous variable to extrapolative models improves the fit of mortality rates (French, D., & O'Hare, C., 2014).

The ranking of fitting improvement is consistent with the ranking of correlation coefficients by country. This means that the proposed model fits mortality rates better in Germany, followed by UK and lastly USA. However, the ranking of fitting improvement by gender is inconsistent with the results of the correlation analysis. Again, this may necessitate the introduction of an additional parameter that captures the sensitivity of the mortality rates to the level of greenhouse emissions in order to further improve the quality of fit.

As such, the model will look like this:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_x \alpha_g + \ln(\varepsilon_{x,t}) \qquad (21)$$

Where γ is the parameter that captures the sensitivity to the level of greenhouse emission and it varies by age and gender.

CHAPTER FIVE: IMPLICATIONS FOR POLICY AND PRACTICE

With climate-change risk gaining considerable attention of experts from various fields, it is important to recognize that the levels of greenhouse emissions accelerates global warming. Consequently, this has a significant impact on the mortality rates of particular countries as evidenced by this study. It follows that mortality models should also account for the levels of greenhouse emissions as this can improve mortality forecasts as well as the accuracy of pricing financial products contingent on death or survival of individuals.

This study recommends the adoption of the proposed model in predicting how future mortality rates will evolve. This can be informative in socio-economic planning for populations. The proposed model should also be taken into consideration when pricing financial products that are contingent on survival or death of individuals so as to ensure that such products are not mispriced.

It is equally important for international agreements on climate change such as the Montreal protocol to be followed to the latter. This will ensure greenhouse emissions are minimized to sustainable levels. The overall effect of this will be reduced heat-related mortality.

CHAPTER SIX: RECOMMENDATION AND CONCLUSION

The inadequacy of models that incorporate greenhouse emissions as a proxy for predicting mortality rates is apparent. Based on mortality data and greenhouse emissions data for Canada, Germany, Japan, UK and USA, this study proposed a mortality model that can be used as a basis for predicting mortality rates while explicitly allowing for greenhouse emissions. From the correlation analysis, it can be concluded that there is a strong positive correlation between the level of greenhouse emissions and the mortality indices particularly in Germany, UK and USA. The extension of the Lee-Carter model that incorporate the level of greenhouse emissions is therefore justified.

The proposed model considers that greenhouse emission affect mortality rates in the same way across all ages. (O'Hare, C. et al., 2015) proved that the impact of temperature-changes on mortality is more significant at older ages than at younger ages. Therefore, further studies should be conducted to develop mortality models with greenhouse emissions having different sensitivities for different ages.

Even so, the extended model is more robust than the original Lee-Carter model since the deviance comparison reveals that the proposed model leads to fitting improvements. As such, the proposed model provides a better fit to mortality rates and would ideally predict future mortality rates more accurately while taking into account the impact of greenhouse emissions. The robustness of the proposed model makes it more effective for use in risk-management decisions such as addressing climate-risk.

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