STRATHMORE BUSINESS SCHOOL

BACHELOR OF SUPPLY CHAIN AND MANAGEMENT

END OF SEMESTER EXAMINATION

SCM 4205: ADVANCED OPERATION RESEARCH

DATE: 23RD JULY, 2024 Time: 3:30 P.M-5:30 P.M

Instructions

1. This examination consists of **FIVE** questions.
2. Answer Question **ONE (COMPULSORY**) and any other **TWO** questions.
3. Do not write on the question paper.

QUESTION ONE (30 marks)

1a) Explain five components of a linear programming problem (5 marks)

1b) A company manufactures two products, A and B each of which must be processed through process 1 and 2. The table below summarizes labour-hour requirements per unit for each product during each process. Also presented are labour? Hour capacities for each process and respective profit margins for the two products.

Activity	Products		Weekly labour capacity
	A	B	
Process 1	3hr per unit	2hr per unit	120hrs
Process 2	4hr per unit	6hr per unit	260hrs
Profit margins	$5	$6	

i) Formulate a Linear programming problem to determine the number of units to produce each product so as to maximize profit. (4marks)

ii) Determine the maximum profit using simplex method (6marks)

1c) i) Discuss two advantages and application of duality (2 marks)

ii) Write the dual of the following primal problem (3 marks)

$$\max z = x_1 - x_2 + 3x_3$$

s.t

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

1d) Given non-linear programming equality problem, use Lagrange's multiplier technique to solve (10 marks)

$$\text{optimize } z = 6x_1^2 + 5x_2^2$$
$$s.t \quad x_1 + 5x_2 = 7$$
$$x_1, x_2 \geq 0$$

QUESTION TWO (20 marks)

2a i) State four rules for converting any primal problem to its dual (4 marks)

ii) Given the primal problem, convert into dual problem and solve the dual problem by simplex method

$$\max z = 3x_1 + x_2$$
$$s.t \quad x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$
$$x_1, x_2, \geq 0$$

(16 marks)

QUESTION THREE (20 marks)

3a) (i) Explain the difference between linear programing and non- linear programming (4 marks)

iii) Define a general non-linear programing problem (4 marks)

3b) A research firm has come up with a mathematical data for two products X1 and X2 which afirm manufacture. It has been found that this is a non-linear programming problem having linear constraints and the objective function is of quadratic form. The data gathered is as follows:

$$\text{Maximize } z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$
$$s.t \quad 2x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

Determine the maximum contribution and the number of units that can be expected from these products. Using Kuhn-Tucker method. (12 marks)

QUESTION FOUR (20 marks)

4a) (i) Define integer linear programing model (2 marks)

ii) State the general integer programming problem (3 marks)

4b) Five projects are being evaluated over 3-year planning period. The following table gives the expected returns for each project and the associated yearly expenditure

Project	Expenditure			Returns (in millions)
	1	2	3	
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available funds in millions	60	60	60	

i) Formulate the integer programming model (6 marks)

ii) Suppose now that we have the additional condition that if project 3 is selected, project 5 cannot be selected. How should the IP problem modified (1 mark)

iii) If project 4 is selected, then project 2 must also selected. How should the IP problem modified (2 marks)

4c) Use the graphical techniques to the integer linear programming problem below

$$\max z = 6x_1 + 7x_2$$
$$s.t \quad 4x_1 + 5x_2 \leq 20$$
$$10x_1 + 7x_2 \leq 35$$
$$x_1, x_2, \geq 0$$

(6 marks)

QUESTION FIVE (20 marks)

5a) Consider the Markov chain with three states S={1,2,3}, that has the following transition matrix

$$\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

i) Draw the state transition diagram for this chain (4 marks)

ii) If $p(x_1 = 1) = p(x_2 = 2) = \frac{1}{4}$,find $p(x_1 = 3, x_2 = 2, x_3 = 1)$ (4 marks)

iii) Find $f_{11}^{(2)}$ and $f_{21}^{(2)}$ (4 marks)

5b) here are a row of 6 coins of values {5, 1, 2, 10, 6, 2}; the objective is to pick up the maximum amount of money subject to the constraint that no two coins adjacent in the above list can be picked up. Develop a dynamic programming solution for this optimization problem (8 marks)