HYBRID BASED MODELLING AND DERIVATIVE PRICING IN THE UK ELECTRICITY MARKET

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MARY WANZA MUTINDA

MANCHESTER BUSINESS SCHOOL

TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This paper presents an empirical analysis of a hybrid model for capturing the dynamics of the spot prices of electricity, and contingent claims thereof.

The dynamics of the spot price process are captured as a sum of a deterministic price-demand transform and an Ornstein-Uhlenbeck stochastic component. From the tests on stability of parameters and recovery of the price process for in-sample and out-of-sample data, the model is shown to perform well.

Closed form formulas for forward contracts are also presented using the market price of risk arguments. The parameter for market price of risk, λ , is estimated based on the convergence assumption i.e. the forward prices converge to the spot price experienced for the given future time. The estimated λ however fails the stability test indicating statistically significant changes in the value over time. In particular λ shifts from the expected negative value in seasons of high demand and variability (winter and summer) to indicate a value attached by the market for holding the forward contract; to positive values in low demand and variability seasons (spring and autumn) to indicate an net cost for holding the forward contract given the difficulty in storability of electricity relative to the almost assured production to meet demand.

A discussion is also presented on the model performance as compared with other models defined for the UK market.

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CHAPTER 1

INTRODUCTION AND OVERVIEW OF ELECTRICITY MARKET

1.1 The electricity market

Deregulation of the electricity industry in parts of Europe, Australia and American states from the early 1990's created a competitive environment in the supply of electricity, which has in-turn led to the growth of electricity energy markets trading in the wholesale spot price and power derivative contracts.

The deregulation has mainly taken the form of separating the industry into three distinct sectors: production, transmission and retailing. The production and retailing sectors are open to competition; whereas the transmission is generally a controlled sector to ensure security and quality of the supply, non-discriminatory access to all market participants, and to balance the demand and supply.

Unlike the more mature interest rate and equity markets which display weak mean reversion and relatively infrequent price jumps, the electricity price process exhibits strong mean reversion, seasonality and frequent non-negligible jumps; characteristics shared with the larger class of energy commodities (e.g. oil, natural gas etc). These characteristics can be attributed to the strong influence of the demand-supply curve to the evolution of the energy commodity price.

The mean reversion characteristic can be explained by recovery of the supply-demand balance following a change in demand (mainly driven by weather changes in the case of electricity) or changes in supply (e.g. a generation park outage). The speed of the production to react to changes in demand, or for the cause of change in demand to dissipate away to return the supply and demand to equilibrium remains relatively limited, as evidenced by differences in hourly prices on off-peak and on-peak times. However, the source of the constraints usually dissipates away quickly within days, or even hours (e.g. by bringing another park to transmission or the temperatures reverting back to normal). The speed of reversion back to the mean therefore tends to be rather large.

Seasonality of the energy prices arises from the fact the consumption is highly influenced by repeated activity patterns of the end users. The activities can be dictated by the time of day, day of week, or period of the year. These activity patterns can largely be predicted and supply is then adjusted to reflect these expected patterns.

Generally speaking, lower demand will be satisfied by the cheapest supply source and as demand increases then supply is sourced from more expensive generation plants or energy sources resulting in a general increase in the price levels.

Unexpected capacity constraints, which can be supply or demand driven, coupled with limited storage, is reflected in the jumps (extremely high volatility) of energy prices. Demand for energy (and occasionally supply in the case where extraction or generation of energy is dependent on weather factors e.g. hydroelectricity) is strongly influenced by weather which can occasionally diverge from the expected norm leading to consumer driven constraints. On the other hand, breakdown in a generation plant, with no reduction in demand leads to a constraint in the equilibrium of supply and demand. The lack of (or limitation in) storage means that supply or demand constraints cannot be smoothed by inventories.

The storage limitation also brings into play the convenience yield factor (i.e. the benefit of holding the physical good) into modelling energy price. The convenience yield provides a bridge between the spot and forward prices by using no-arbitrage argument. The varying limitation of storage; and further the non-homogeneity of the measure of convenience yield for energy users, which varies from industrial benefit measured as the opportunity cost of shutting down a production line, down to political and security issues, increases the complexity of deriving models for energy prices.

Though the electricity market shares the above enumerated characteristics with the other energy commodities, the effects on the electricity prices is rather unique or exemplified by the fact that electricity is an extreme case of storage limitation. Once produced, electricity cannot be stored economically. This implies that the intensity and frequency of the price jumps are likely to be more significant in the electricity market than in the other energy markets.

Further, there is essentially no convenience yield measure, hence a break down in the link of spot and forward prices. This implies that no-arbitrage arguments cannot be applied in the usual manner in valuing electricity derivatives since the argument relies on one hand borrowing funds and buying the underlying commodity, and on the other hand purchasing a derivative of the commodity for future transfer; then equating the two values under the arbitrage-free argument.

The non-storability, and further transmission constraints of electricity, causes the electricity market to be extremely local i.e. the prices in a particular market pool will be heavily influenced by the local determinants of supply and demand.

For instance, in comparison with the UK market where only 1% of the electricity is generated using hydro electricity[1], over 50% of electricity in the Nord pool is by the dam-water energy source. Changes in weather patterns, from wetter summers to longer intense winters, have a higher impact on the supply curve in the Nord Pool market as opposed to the UK market.

Another example can be sited in the US market. The rather extreme summer heat and winter cold leads to a discernible predictable pattern of price movements and jumps based on time of year. The UK temperate climate however, makes these patterns less discernable.

Hence, even given the same pricing model, the significance and value of parameters is likely to differ across different market pools.

1.2 About the UK electricity market

Though the UK was one of the first countries to restructure and privatize its electricity industry in 1990, the regulatory controls on how the market prices were arrived at by the UK electricity pool which operated from 1 April 1990 to 26 March 2001, have led to the popular notion that the recorded prices did not adequately reflect the market forces, and could therefore not be used as true market prices.

Under the electricity pool organization, bids were received only from the electricity producers who set the market price (referred to as system marginal price) with no participation from buyers to allow for market supply-demand forces to fully act upon the set price. From 27 March 2001, the New Electricity Trading Arrangement (NETA) came into force. Under this system both the generators and suppliers of electricity trade in the wholesale electricity market. The change in the market organization is particularly important in modelling the UK electricity prices as real market price data is considered to be observed under the NETA arrangement.

Another notable change in the organization of the UK electricity market is the determination of the half-hourly recorded electricity market prices. Under the electricity pool setting the declared market price, referred to as the System Marginal Price, was based on the bid of the most expensive generating unit needed to meet forecasted demand. All successful generators were then paid at this price, irrespective of their indicated lower cost to produce the electricity giving a sort of "uniform-price" auction system.
The use of highest qualifying bid price implicitly makes the source of power generation, which directly affects the cost of production, a significant variable in modelling the electricity prices since the set market price is just the price of generating using a particular electricity generating unit. The types of fuels applied vary on a step-function of marginal costs with increased demand; with nuclear, hydro and coal sources employed first, and with each successive increase in MWh demanded the oil and gas generators are then activated. It therefore follows that the set market price will reflect the price of the last employed fuel generator, intuitively implying a direct correlation of price with fuel type.

In the NETA setting the recorded spot price is the volume – weighted average price for all qualifying contracts[2], defined as contracts in the half-hourly, 2 hours or 4-hours block format specified by the regulators and traded before a set "Gate Closure" period of 1 hour before the requested supply is transmitted. This change to the use of the weighted average pricing reduces the effect of this one – to – one correlation of the spot prices and a particular fuel source.

A further point to note on the UK electricity market is the composition of the electricity generating companies. Though over 100 companies are listed as producers, 90% of the total capacity is accounted for by 13 companies. Even more notable, 60% of total capacity is controlled by just 5 companies. There then exists an oligopoly opportunity – which is not unique to the UK market. Owing to the capital intensive electricity generation process, the market entry cost acts as a natural barrier of entry resulting in few participants in generation. It, therefore, follows that market prices are likely not to directly reflect the marginal cost of the production by fuel type as the full forces of competition are inhibited.

The last two observations on the pricing formula and the composition of the players become significant in the setting of model assumptions on the stability of the deterministic seasonal component and the significance of variables selected in the modelling process.

1.3 Objectives of the study

This study seeks to evaluate the performance of a hybrid model, in particular using demand driven price driver together with a market factor, in modelling of electricity spot prices and valuation of contingent claims. From the general understanding of electricity markets, and by extension the larger class of energy commodities, the influence of the demand-supply constraints in the evolution of the prices is evident. On the other hand, the influence of market behaviour independent of the drivers is discernable from the structure of the market and the players.

In this paper, we define, calibrate and test a hybrid model using empirical data of the UK electricity spot prices and contingent claims thereof. We then offer a discussion of the performance of the model as compared with other models applied to the same market.

The rest of this paper is organised as follows: In Chapter 2 we briefly discuss existing literature on modelling electricity prices. A statistical analysis of the UK electricity data is presented in Chapter 3 and the model is defined and tested in Chapter 4. In Chapter 5 we present solutions to contingent claims and perform a sensitivity analysis and stress test on the result. Finally, in Chapter 6 we present the conclusion and implications of our results, and possible extension of the work.

CHAPTER 2

LITERATURE REVIEW

2.1 Spot price models

From the late 1990's a number of papers have been written suggesting various approaches and models for the electricity prices and valuation of derivatives thereof. The studies have concentrated on the USA, Nord pool and the UK power markets which, based on the early timing of deregulation, have sufficient data to perform empirical studies. Though each region's market may differ in the organisation and level of regulation, the three stylized spot price characteristics are observed in all markets. The main differences noted in the studies have been the value and significance of the parameters calibrated from the empirical testing level which, as indicated earlier on, can be attributed to the composition of types of fuels used in the generation parks and the effect of weather on the demand and supply of electricity among other factors.

There are two main approaches to modelling the electricity market prices: The pure-price models and the hybrid models.

Pure price models take the form of applying stochastic techniques to represent the evolution of the spot prices as observed in the market with no reference to the cause of the movement.

On the other extreme are the fundamental equilibrium models which seek to accurately represent the details of operations in the markets via interaction of generation capabilities (supply) in terms of capacity, fuels, technology employed and marginal costs; demand characteristics based demographic, economic and statistical analysis; and specification of constraints in the system e.g. transmission constraints (capacity and system losses), environmental constraints (pollution quotas, allowed carbon levels and costs), or operational constraints (planned or unplanned outages, reserves etc). These are usually not applied in market models as they do not incorporate market forces and their use is generally limited to use in combination with pure price models for hybrid models.

Hybrid models then combine the two to capture the natural representation of the dynamics of energy prices using fundamental equilibrium models, while employing the pure price models to represent the evolution of the drivers as observed in the market.

We review literature on each of these approaches separately below:

2.1.1 Pure price models

Though literature on electricity modelling remains generally scarce as the electricity markets are rather young, the pure price process is the more commonly applied approach in the papers studying the problem.

Pure-price models have mean-reverting processes as the general choice of base functions to capture the distribution of the electricity spot prices. On this base, various approaches are then been adopted in terms of variable choice, number of factors in the model and distribution of the variables to capture the timing, frequency and distribution of the seasonality and jumps of the price process.

The most commonly applied mean-reverting process is the Ornstein-Uhlenbeck process with the general form,

$$dS_t = \theta(\mu(S,t) - S_t)dt + \sigma(S,t)dW_t,$$

where S_t is the price process, μ the long-term mean and σ the volatility driven by a standard wiener process W_t. Volatility is assumed to be either constant or modelled as a stochastic process.

This base model is then typically extended in the power market to capture the non-negligible jumps taking a generic form of,

$$dS_t = \theta(\mu(S,t) - S_t)dt + \sigma(S,t)dW_t + J_t dq_t\ ,$$

where J_t is a discrete random jump Poisson process.

As the return to the long-term mean in the electricity market is usually very fast with price jumps lasting only a few days, sometime just a few hours, most models generally require, and calibrate, a high speed of mean reversion, θ. This has the effect of removing the frequent, daily or hourly variability and characteristics of the (majority) "non-jump" periods i.e. seasonality components. At this level also, the timing, frequency and intensity of the jumps is not captured.

The papers on modelling pure-price electricity spot prices follow up on these shortcomings and suggest various approaches to capture the seasonality component and characteristics of the jumps, while presenting a parsimonious tractable model easy to interpret and apply in pricing.

To capture the salient features of the jumps, more specifically the intensity of the jumps as well as up-and-down jumps, or subsequent up-jumps, Deng (1999) proposes a sequential regime-switching representation model that switches the spot prices between the "high" and "normal" states with the trigger event being unexpected contingencies in transmission of the electricity or generation plant outages. The model does not capture the discernable seasonality component, and further no model fitting to electricity prices is performed to test against observed data.

Lucia and Schwartz (2000) investigate the price behaviour in the Nord pool and focus on the observable seasonality of the price process on changes in time of day, day of week and season of year. The price process is then modelled to have a deterministic seasonal component and a (stochastic) Ornstein–Uhlenbeck process with zero long term mean. The seasonality from high demand to low demand is captured via a Markov regime switching process with the trigger being the deterministic time of year.
One-factor and two-factor models are proposed, calibrated and tested. The two-factor model investigation is motivated by the observation that the spot and forward prices are not perfectly correlated, with the forward prices being less volatile as compared to spot prices. The calibration and testing results indicate substantial improvement in the tested model, giving premise for the use of multi-factor models in modelling spot prices.
Pilipovic (2007) also suggests a two-factor mean reverting model where the parameter for mean $\mu(S,t)$ is also a stochastic process describing the long term dynamics.

Both Lucia and Schwartz (2000) and Pilipovic (2007) models do not incorporate the non-negligible jump process. A possible reason behind this is the additional complexity of defining and calibrating more parameters in the pure-price models when attempting to capture all the stylized features of the electricity price evolution.
Empirical evidence from the Nord pool as with all other electricity markets, however, indicates the jumps are non-negligible in both frequency and intensity and contribute significantly in explaining the spot price process.

Cartea & Figueroa (2005) offer a natural extension to Lucia & Schwartz (2000) model approach by including jumps. The proposed mean-reverting jump diffusion model with a seasonality component is applied to the UK electricity market. The jump process is modelled as homogenous Poisson process where the intensity of the counting process based on the historical average number of jumps. However, the jumps frequency and timing are not conclusively determined due to data constraints.

Geman and Roncoroni (2006), following similar model approach of a mean-reverting model with seasonality and jump components, provide a first attempt at capturing the timing of the price jumps under a pure-price process by seeking to capture periodically – recurrent jumps in data selected from three US electricity markets. The trigger for the occurrence of the jumps is time-dependent based on historical experience of the time of year where imbalances in electricity generated and electricity demanded is most likely to be observed. Culot et al. (2006) however observe that these periodically – recurrent predictable jumps are less discernable in the European pool markets where temperate climate is more prevalent.

Benth and Kallsen (2006) argue that the magnitude and frequency of price jumps are seasonally dependent and propose an additive model of a sum of Ornstein – Uhlenbeck processes with a jump component each reverting back to mean at different speeds dependent on the season. The paper serves to introduce the observation of varying speeds of reversion which appear to be seasonally dependent.

The challenge in adopting pure-price models, as detailed above, revolves around capturing the timing, frequency and intensity of the (non-negligible) jump process which is strongly driven by the interaction by demand and supply curves as well as balancing the number of parameters while capturing all the spot price characteristics. Pure price model tend to define

many parameters to include mean-reversion, seasonal regime switching, jump regime switching, stochastic volatility and two-factor processes. This makes the models rather complex and harder to explain and define economically.
These constraints have motivated the development of hybrid models for capturing the dynamics of electricity spot prices.

2.1.2 Hybrid models

Alternative to the pure-price models where the evolution of the spot prices is fully defined by the observed dynamics of the price, are the hybrid models which explain the price process based on drivers associated with supply and demand.

Hybrid models generally take form of modelling the non-linear relationship of the price with an underlying price driver usually the demand curve or the underlying fuel. Jumps occur naturally in this setup as the observable price jumps and dips are in response to the interaction with the driver. To this end hybrid models offer an extension to the pure price models in defining an intuitive, economically and naturally interpreted, tractable and parsimonious model.
Eydeland & Wolyniec (2003) define the hybrid models as a fusion of fundamental (supply/demand relationship) and pure stochastic models (that capture the evolution of the underlying drivers). The key motivation of investigating these relationships is argued to be the scarcity of stable information in the pure price processes due to low data availability and the fact that the markets may still be evolving and the parameters thus unstable. To model the electricity prices a non-linear transformation of the demand or supply driver is suggested.

Barlow (2002) presents a non-linear Ornstein-Uhlenbeck (NLOU) process for the description of observed electricity spot prices. The demand function is modelled as a mean – reverting process with a non-constant mean given by a deterministic seasonal function. As pointed out in his paper, the model so described is stationary and therefore does not provide a satisfactory explanation of the spot-forward price relationship.

Burger et.al (2004) improves on the stationarity limitation while incorporating the seasonal patterns, price jumps, mean reversion, price - dependent volatilities and long term non-stationarity into their model. They estimate a non linear price-demand curve whose transform describes the price seasonality and jump process. Two other factors are included – Short term and long term measures to capture the price structures over both spot and forward price markets.

Finally, Geman and Cartea (2009) investigate the determination of the seasonality and jump timing and magnitude in the UK market. They argue the use of historical data in determining the timing, frequency and magnitude of jumps, and the calibration of the seasonal component, to be an inadequate and possibly unstable approach as historical data may not be true determinants of future price behaviours.
The basic form of their model follows from the Lucia and Schwartz (2000) framework of a seasonal deterministic and a stochastic component. The seasonal component is however calibrated from the gas prices and the stochastic component a time-varying mean reverting process with a deterministic switching component triggered the forecasted constraints. The trigger is measured as the ratio of forecasted demand to forecasted generation.

In this study we follow closely through the arguments presented by Burger et.al (2004) in defining a hybrid model based on the data defined from the UK market.

2.2 Valuation of Contingent Claims

The stylized characteristics of the electricity prices, and the energy commodity markets on a broader perspective, give rise to unique derivatives products developed to manage the risks that are not found in other (financial) markets. In particular there are the volumetric type options (swing, recall and nominations contracts) giving the holder the right, and obligation to adjust volume of received or delivered commodity. Even the common forward, future and standard option contracts bear unique definitions.

In particular, due to the non-storability of electricity, power delivered at different times of the day, week or month represents different commodities making the pricing of the derivatives more complex. Further still the standard approach of valuing a derivative or contingent claim by constructing a portfolio to replicate the payoff of the contract and using a no-arbitrage argument cannot be directly applied.

This constraint is circumvented by two approaches:

i) modelling the forward prices directly instead of the spot price dynamics or

ii) replacing the convenience yield notion with a market risk premium measure which is measured as the difference between the forecasted future spot price and the forward price (the price a trader is willing to pay today for the future delivery), thereby creating the link between spot-forward market. The contingent claims are then priced by assuming a rational expectation hypothesis i.e. $F_{t,T} = \mathbb{E}[S_T|\mathcal{F}_t]$

The first approach addresses the pricing of options on the forward contracts since the forward/future contract can be used as assets to create replicating portfolios and follow the usual no arbitrage arguments in pricing. Bjerksund et al. (2000) apply this approach of directly modelling the forward price on the Nord pool market, and using no-arbitrage arguments to price European and Asian options on the forward contracts. Benth and Koekebakker (2005) apply concepts from the Heath–Jarrow–Morton theory to model the forward price curve.

Though this approach solves the pricing of contingent claims in the forwards market, the lack of (or difficulty in defining) the convenience yield leads to a breakdown in the theoretical linkage of the forward price to the spot price. This in turn limits the extension of such models in evaluating derivatives on the spot prices.

Only a handful of the papers presenting the spot price models, and discussed above, venture further to present a solution for contingent claims. The limitation, especially for pure price processes stems from the difficulty in estimating the risk premiums in the models.

Lucia and Schwartz (2000) offer empirically supported solutions to contingent claims using the risk-premium approach with one-factor and two-factor models. In particular they observe that two-factor models provide better results than the one-factor models. This is consistent with the general observation in energy markets of multi-factor pricing models. More specific to the electricity market, the forward prices are observed to be less volatile than the spot prices. These two discernable characteristics then intuitively lead to the conclusion of need for two-factor model to capture the short-term and long-term behaviour.

Burger et.al (2004) approach the valuation by implying the risk-neutral mean and volatility of the long term process from the future market and assuming the non-headgeable risk to be zero. In contrast with Lucia and Schwartz (2000) closed form solution, their approach entails use of Monte Carlo simulation.

In our study we apply the rational expectation hypothesis and in contrast to Burger et.al (2004) we seek to define and solve a closed form partial differential equation for solution to the contingent claim.

CHAPTER 3

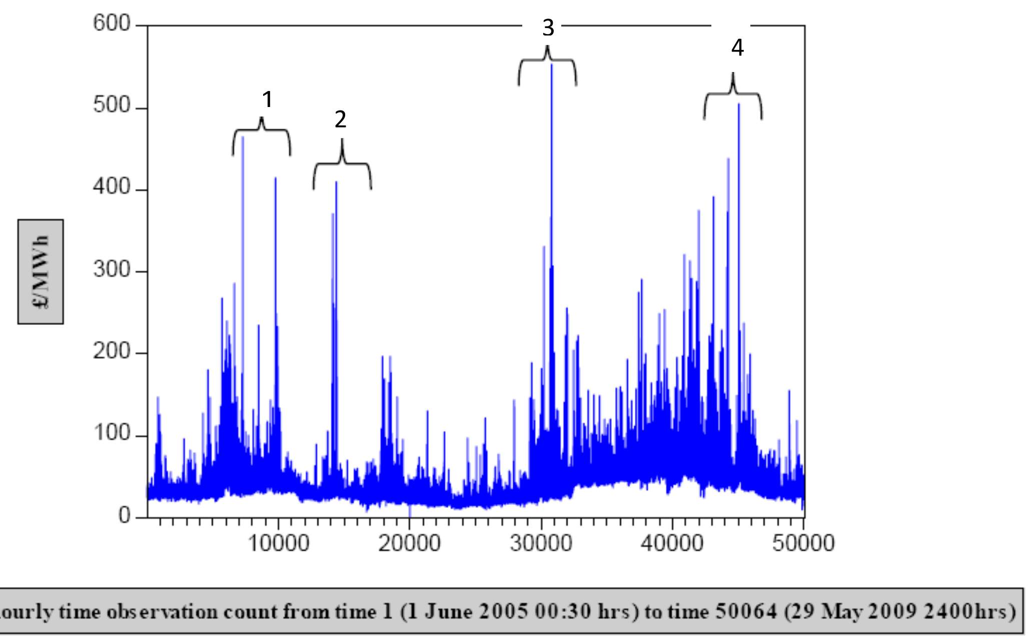
ELECTRICITY DATA ANALYSIS

We seek to define and calibrate a hybrid model that captures the UK electricity spot price evolution. To begin with, we analyze the data to provide a good base description of the significant data features to be considered in the model selection process and the interaction of the price features with the demand/supply curves.

3.1 Overview of the evolution of spot prices

The price data consists of daily half-hourly price per Megawatt Hour as recorded by APX Power-UK, UK's first independent power exchange, and downloaded from DataStream. The data runs from 1 June 2005 to 29 May 2009; the dates so chosen to capture four full-year weather seasons. As a general guide, the winter season falls from December to February, spring from March to May, summer from June to August, and autumn spans September to November.

Figure 1: UK spot price data 1 June 2005 to 29 May 2009



A first initial visual impression of the raw data reveals the rather high stochastic volatility, frequent jumps of high magnitude and, in particular, the mean – reversion feature, supporting the appropriateness of a mean reverting process as a base model.

To gain a better understanding of the evolution of the spot prices, we highlight four visually significant jump periods and seek reasons behind the price jumps.

The first selected cluster corresponds to the November-December 2005 period during which highest recorded price jumps of over 140% for successive half hour periods were experienced. This was caused by a sudden drop of temperatures in mid November 2005, coupled with predictions by the meteorological office of a rather cold winter and an announcement of forecasted gas shortages (which at that time contributed 32% of fuel used in the electricity generation parks). This led to a panic rush of gas purchase creating both supply and demand constraints leading to a huge price jump.

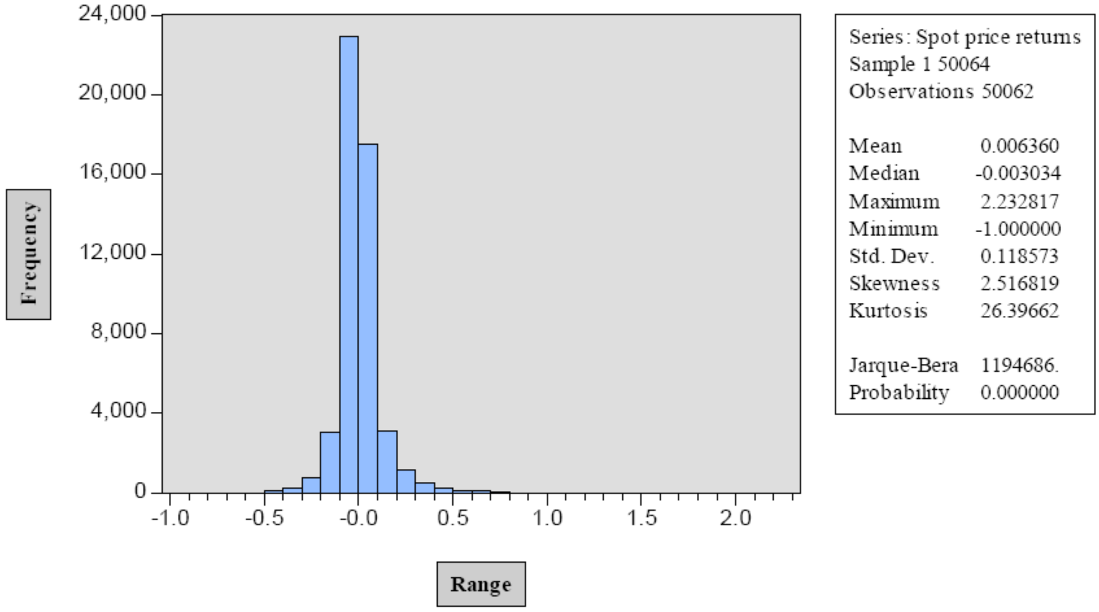
The second selected cluster falls in September 2006, an autumn month which recorded the warmest September temperatures since 1914 as reported by the meteorological department, resulting in a below average demand against provided supply. Similar reasons are behind the third band falling in December 2007-January 2008 which was reported to be the 4th warmest January in England since 1914.

The fourth price jump cluster occurs in December 2008-February 2009, recorded as the coldest winter since 1996.

As evidenced by the review above, and economically intuitive, the price jumps result from constraints which can be (more commonly) demand driven due to significant deviations from expected weather patterns, or supply driven due to shortage of power generation fuels or unplanned outages of power plants. It is also notable that the observation by Culot et al. (2006) of un-patterned (time- independent) jumps in European data (in our particular case – the UK market) holds true as the jumps do not seem to occur at any predictable pattern or points over time of year.

The parametric characteristics of the spot price process are summarized in figure 2 below.

Figure 2: Histogram & descriptive statistics for spot price returns



The clustering of returns around the mean (0.006) as evidenced from the histogram suggests there are significant outlier jumps in the process. Approximately 80% of the changes are +/- 10% around the mean. Jumps of over 10% in absolute percentage over successive half hourly observations account for 20% of the data series.

The positive skewness measure (2.5 against normal distribution of 0) which indicates a bias for upward price jumps, high kurtosis (26 against that of a normal distribution of 3) and Jacque Berra 0% probability of normality of data leads to conclusion that the returns are not normally distributed i.e. the probability of extreme events occurring is much higher than predicted in a Gaussian distribution. This observation leads to conclusion that Brownian motion (or a geometric Brownian motion) is not an adequate model to describe the price evolution.

3.2 Spot price drivers

From the discussion in section 1.2 on determination of the market price in the UK market, it is natural to assume the price drivers from supply point of view to be the cost of running the generation plant (mainly fuel and variable costs) and planned or forced outages, and the drivers from the demand side to be activity pattern of end users and deviation of temperatures from expectation.

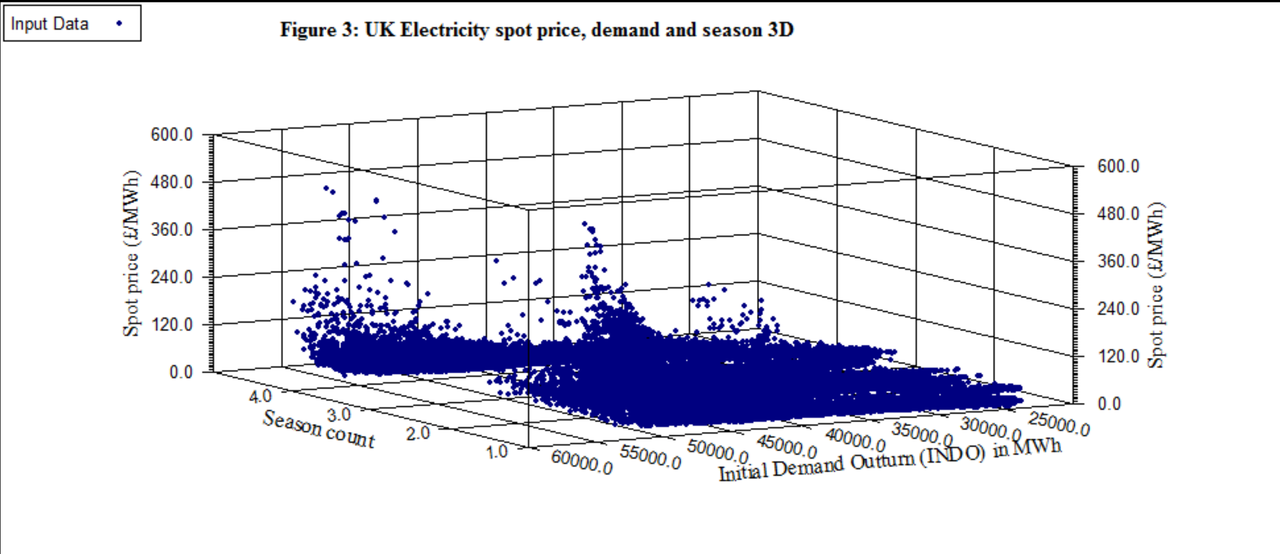
As we seek to define a hybrid model, we need to identify an appropriate driver for the spot price evolution. Possible candidates include (supply side) fuel, outages or variable costs, or (demand side) temperatures or user activity.

The use of a supply side function is complicated by the variety of generation fuels used, unpredictability of outages, and non-uniform variable costs across generation parks based on management, age of the park among other reasons.

The clear correlation between temperatures and demand levels, and singularity in the recording of temperatures makes the use of a demand driver an easier option, while maintaining tractability. As user activity tends to be rather repeated and predictable, with expected variations, it is reasonable to expect that the demand recorded in MWh fully embodies the temperature and activity information.

Figure 3 demonstrates the strong relation between the spot price and demand providing a justification for use of demand as underlying driver in formulation of spot prices.

Figure 3: UK Electricity spot price, demand and season 3D



The nominal numbering of the seasons is 1- summer, 2 - spring, 3 - autumn and 4 - winter. From the plotted graph there is a clear visual correlation of the price and demand distribution with an influence from the season of year.

In areas of high demand there is deterioration in this relationship with the price levels increasing steeply for every additional MWh. A plausible explanation is that in this region the capacity constraints start playing a greater role in affecting the price. This constraint appears to be seasonally influenced implying possible planned outages over seasons to allow for maintenance of plants. In particular the maximum demand in spring, summer and autumn is in the 50,000MWh range at which point the deterioration of demand-price is witnessed. In contrasts, price jumps over the winter period are experienced above the 55,000MWh demand level.

This variation is captured by Burger et.al (2004) by adjusting the demand by relative availability based on planned outages in the summer in Germany. We follow a similar approach by adjusting the demand by a parameter that captures relative capacity availability over the four seasons.

3.3 Deterministic seasonality component

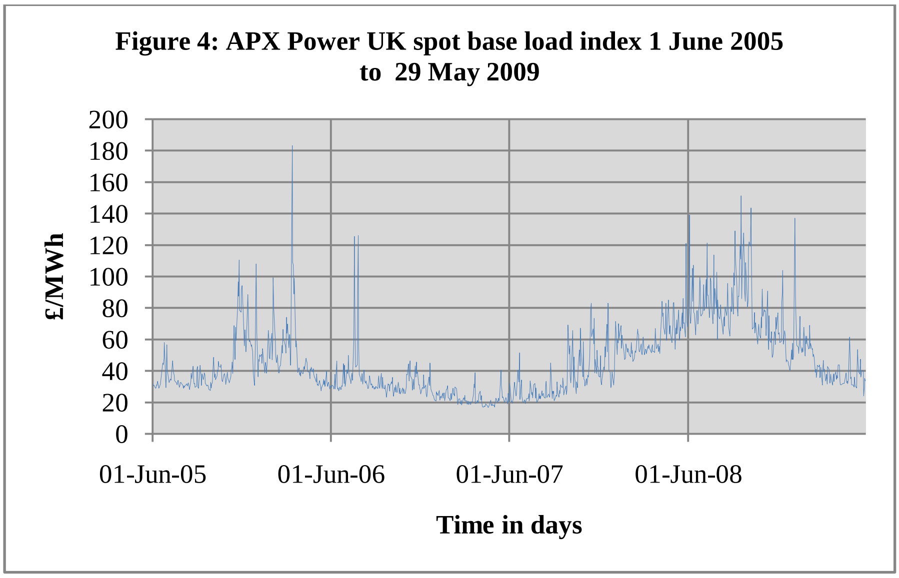
The seasonality component in electricity prices results from observable pattern of variation in the demand for electricity based on economic activity of users and weather conditions over time-of-day, day-of-week and season-of-year. For time-of-day, generally speaking, there is a higher demand for electricity in the early evening hours when electricity is used for cooking, heating and lighting of households. The usage is also expected to be at the lowest levels in the late night to very early morning (10pm to 4am) periods when users have retired for the day.

The day-of-week price level seasonality is explained by higher usage over week days with industrial and office consumption together with household consumption, as compared to weekends when factories are closed.

Prevailing weather temperatures explain the season-of-year price level switches. In colder months more electricity is demanded by households in the evening hours for heating whereas in the hotter months the electricity demand peaks at mid of day hours for cooling purposes. Depending on the extremity and duration of the season patterns, the price – level switching over the seasons of the year becomes more or less discernable.

We investigate which of the above characteristics are significant for the model building process by banding the spot price curve to the three conceptualised seasonality effects.

To examine for the season-of-year effect we graph the APX Power UK spot base load Index over the selected study period. The Index is derived as an arithmetic average of the 48 half-hourly recorded prices for the period 2300 hrs to 2300hrs.

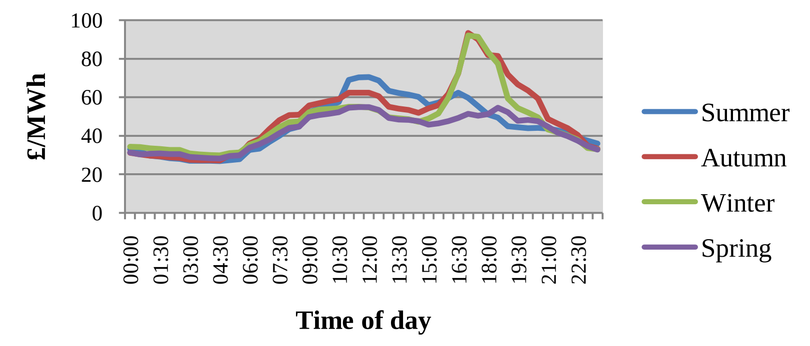


For the selected study period there appears to be no predictable pattern for repeating annual weather seasons. This can be explained by the fact that, though the seasons are clearly distinct, the UK has a rather temperate climate with plentiful rainfall all year round and generally no extreme winters or summers. The season-of-year effect on price level is then rather less discernable.

Due to data limitation, no prices are recorded for the weekend spot prices as markets are closed. We take the assumption that the weekend consumption curve has the same distribution as that of weekday. This appears reasonable as the factory and industrial usage would be expected to be a constant factor with the predictable variations (curves) caused by household activities over the time of day.

To investigate for the time-of-day effect we obtain the mean average of each of the full day 48 time bands for the selected data period. As discussed earlier, though the season-of-year switch is rather not distinct, the timing of the energy use shifts with cold days increasing usage in evening periods and warmer days in mid day hours. We therefore obtain mean averages over seasons to examine for this significance of this effect. The results are presented in figure 5 below:

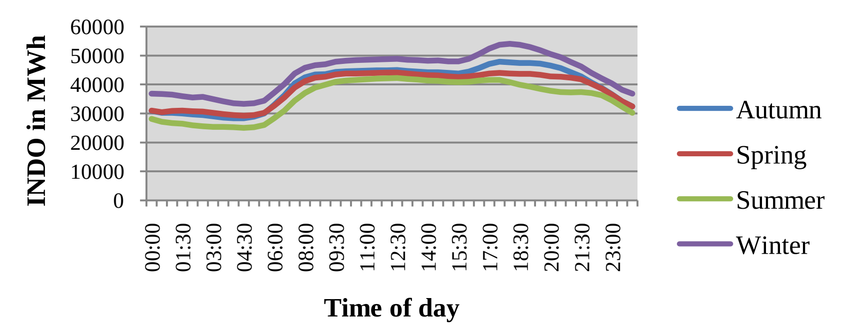
Figure 5: Empirical half hourly season average price patterns for period 1 June 2005 - 29 May 2009



A clear pattern of price movements over the time of day is observable. In particular, peaks for warmer months are recorded in midday hours, whereas colder months record higher average price peaks in the early evening hours. The UK can therefore be described as a winter peaking market with demand and prices higher in the winter. The time-of-day is, therefore, concluded as a significant variable in the description of evolution of electricity spot prices.

As we seek a hybrid model, we investigate if the seasonality information is fully contained in the demand curve. From the graph of averaged demand for each time period presented in figure 6 below the strong correlation of the seasonality price component with demand component is visible.

Figure 6: Half hourly season average Initial Demand Outturn (INDO) for period 1 June 2005 - 29 May 2009



3.4 Residual Stochastic component

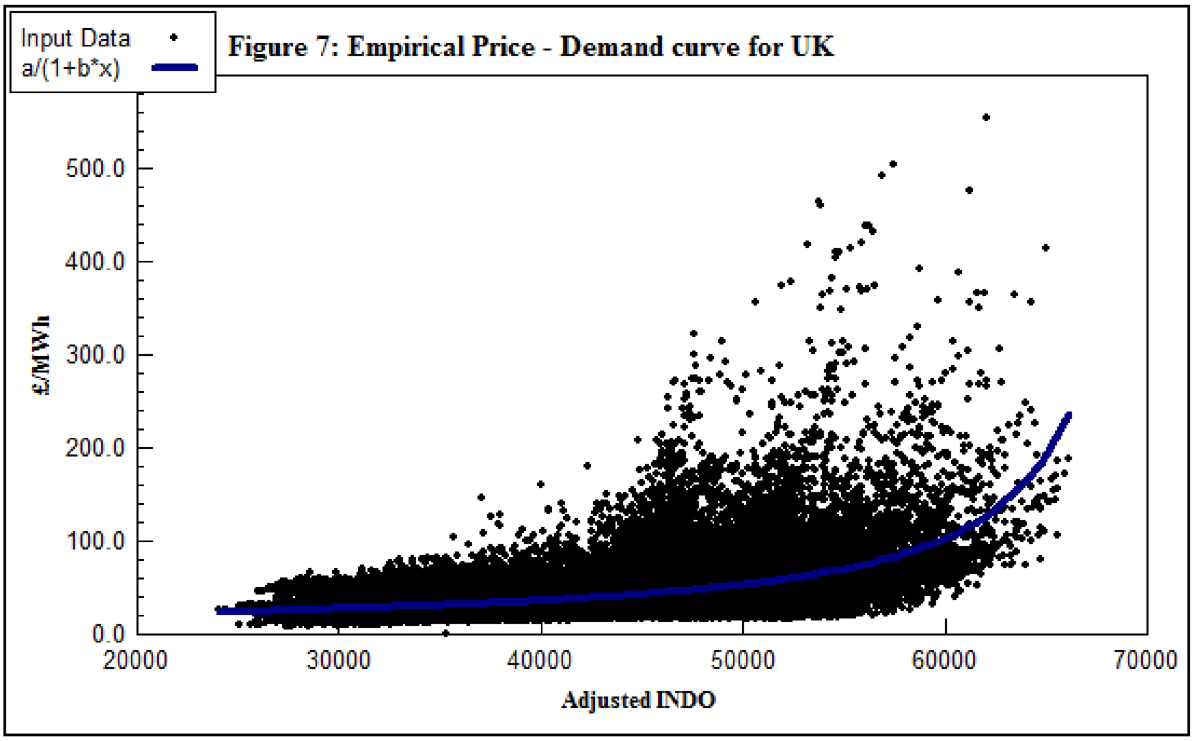
Conceptually, and following general conclusions from previous studies, the spot price model will take the form:

Spot price = Deterministic seasonal effect + Underlying spot price stochastic process

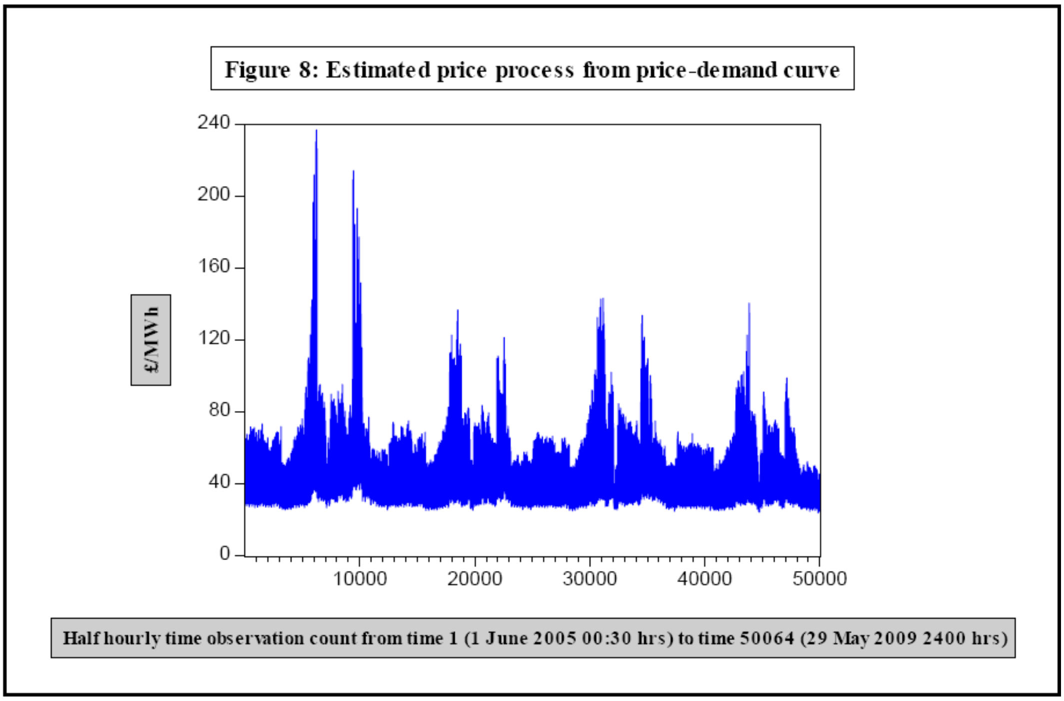
The approach for evaluating the residual stochastic process under a pure price process involves first ‘de-seasonalizing’ the data by deducting the predictable time-of-day seasonal price component. As the deterministic price component is a rather smooth curve, the jumps are wholly captured in this stochastic component. Intuitive Markovian regime switching parameters are the most common method applied to capture the jumps under the pure price process.
Deng (2000) applies this to jump-diffusion models with the switch described as unexpected contingencies in the network. The calibration is however not detailed. Geman and Cartea (2009) apply a Markov regime switching model to capture the timing of the jumps. The regime is decided using the ratio of forecasted demand against forecasted generation capacity. Markov switching is, however, been criticised on fact that it does not allow for the existence of successive upward jumps and, moreover, the models become rather complicated once regime switching and seasonality is combined.

Using hybrid models, jumps are viewed from economic reasons of demand-supply constraints. As the prices are determined by the interaction of the supply and demand curves, a non-linear transformation between the demand and price is taken into account. The residual process can then be explained as ‘market psychology’ e.g. behaviour of speculators and effect of risks such as unplanned outages, or other influences in market prices not related to the technical issues in producing electricity.

We employ Burger et.al (2004) approach by first adjusting the data by relative availability factor and then fitting in a non-linear curve to the data.
Graphically the fitted model is presented in figure 7 below:



The fitted process from the non-linear price-demand relationship is then presented in figure 8 below



Visibly the characteristics of seasonality, mean-reversion and jumps are captured in the transform. We are then able to extract the residual process and analyse the characteristics for modelling.

The residual price process and descriptive statistics are presented in figure 9 and 10 below respectively:

Figure 9: Residual price series

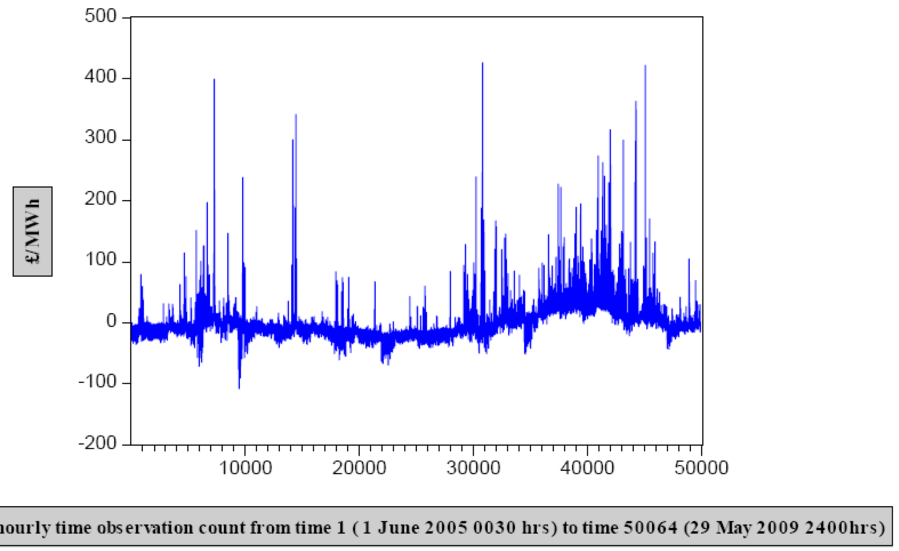
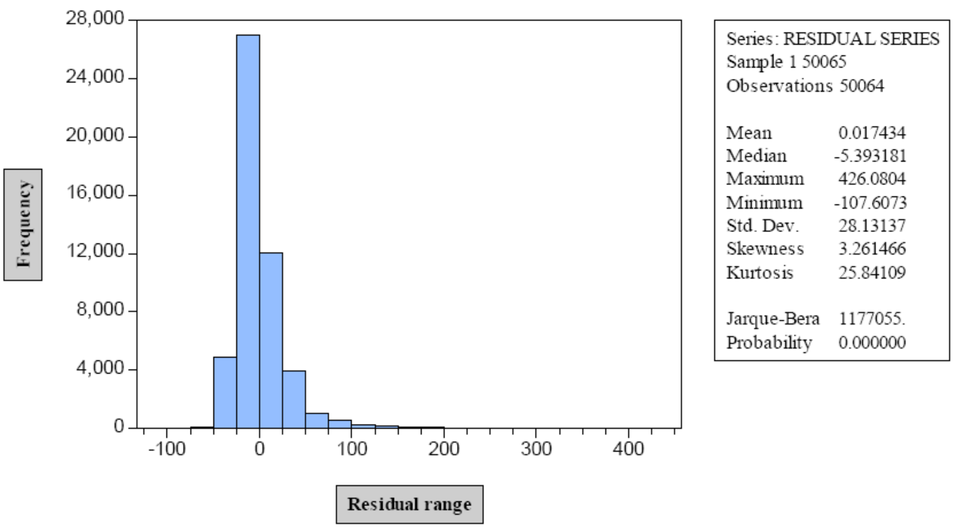


Figure 10: Histogram and Descriptive statistics for residual price process



From the graph and descriptive statistics the residual process generally oscillates about the mean zero value with significant occasional jumps. This can be explained by unplanned outage, market psychology or transform function limitation since the transform explains only 23% of the variability and the price-demand relationship generally breaks down at higher demand levels. Though the normality has marginally improved, the data is still leptokurtic implying more observations on the tails than normal.

3.5 Summary results from UK data analysis

From the above analysis of selected UK electricity price data a reasonable price evolution process should be able to capture the observed properties of time-of-day seasonality, mean reversion, leptokurtosis and price jumps.

By employed the non-linear price demand relationship, 23% of variability is accounted for and the estimated price paths embody all four properties as presented in figure 8.

However the residual process remains significant and non-normal. This can be attributed to the influence of non-technical information in the price more-so for significant jumps where the price-demand curve relationship deteriorates. The residual process has a long run mean of about zero and exhibits all properties with exception of seasonality (i.e. strong mean reversion, jumps and non-normality).

To capture the price evolution and distribution characteristics we propose a model of form

Spot price = price-demand curve transform + Residual zero mean-reverting process

The full description of the model is detailed in chapter 4.

CHAPTER 4

THE SPOT PRICE HYBRID MODEL

4.1 Theoretical framework

The fundamental equation of spot price evolution is presented as:

$$S_t = f(t, L_t/v_t) + X_t$$

$$dX_t = -\beta X_t dt + \sigma dW_t \qquad \text{.......... 4.1}$$

Where:

- L_t is the electricity demand and v_t a deterministic value of expected relative availability of the generation plants such that L_t/v_t gives the demand adjusted for forecasted availability of the plants. In our analysis we assume $v_t = 1$ for winter season i.e. highest capacity available is in the winter period.

  The function $f(.)$ dependent on time and the adjusted demand describes the non-linear relationship between price and demand estimated from empirical data.

  Therefore the first term $f(t, L_t/v_t)$ captures price characteristics dependent on demand i.e. the expected price at a given demand value. This way, the seasonality and jumps are intrinsically captured from observed demand.

- X_t is the residual stochastic process that captures the short-term market influences on the price superimposed on the deterministic function. This can be thought to be the market reaction to news of possible or realised unexpected constraints. By observation of the process (figure 9) we model this to follow a (stationary mean-reverting) Ornstein-Uhlenbeck $(0, \beta, \sigma)$ process i.e. with long term mean of zero, speed of mean reversion $\beta > 0$, the initial value defined as $X(0) = x_0$ and dZ increments to a standard Brownian motion process Z_t which represents the source of uncertainty. By defining the process in this way we miss out the leptokurtic nature of the process and include in our assumptions the normal distribution of the residuals for the fitted stochastic process. Though this is clearly not supported empirically, we argue the omission to be insignificant for our hedging purposes and further assess the effect of violation of this assumption in chapter 5.

4.2 Underlying Assumptions

Assumption 1

The electricity demand is readily observable and is deterministic

We assume that most market participants can directly observe the demand beforehand and therefore have good estimates for the whole market demand. This implies that the electricity demand L_t is assumed to be persistent so that the best estimate for demand for the next half hour is the present half-hour observation. This has that there is no source of uncertainty in the $f(.)$ function.

This appears reasonable as demand levels are heavily influenced by weather patterns which tends to have a persistence attribute i.e. the best guess of weather (demand) today is that yesterday. This assumption does ignore the unexpected/ unplanned demand spikes/ supply shortages which we assume to be overall negligible.

Assumption 2

The dynamics of the residual stochastic component spot price can be modelled by mean reverting diffusion process with constant volatility

The variations of the residual spot prices are explained as market reaction to temporary imbalance in supply and demand in excess of generation and consumption fundamentals and modelled using a mean reverting Ornstein-Uhlenbeck process with zero equilibrium price level, a constant variance per unit time and a continuous sample path. By extension of the O-U properties then, the process is Gaussian and the error term from fitting the models is assumed to be normally distributed. This appears not contradict the data analysis that indicates a non-normal distribution of this residual process.

Empirical analysis and previous studies also indicate a stochastic volatility which is seasonally dependent with higher volatility in high demand seasons. For simplicity we have assumed the volatility to be constant.

Further still studies have indicated that two factor models which include an analysis of the long-term mean or equilibrium rate offer a better pricing model. We choose to define a one-factor model with zero long term mean in part due to data limitations on forward prices, and also weighing benefit of a complex sophisticated model vis-a-vis an easy to use and tractable model.

The effects of these settings: Gaussian error term, constant volatility and one-factor model, will be evaluated in chapter 5 to discuss their effects on overall results.

Assumption 3

The price-demand curve and mean-reverting residual term are stochastically independent.

We assume the function $f(.)$ and the residual price process X_t are independent and the parameters can be estimated independently. This appears reasonable by construction as $f(.)$ captures the empirically estimated price-demand curve driver whereas X_t captures the market psychology, unplanned outages or other unexpected events.

4.3 Parameter Calibration

Applying the assumption of stochastic independence we are able to estimate the two components separately. For each component we follow the three step approach of identification, estimation and diagnostic checking presented below.

4.3.1 Deterministic non-linear transform

We take two steps in identification: first we scale the demand over each season by level of availability. Assuming availability in winter time to be 1, then the adjustment parameter across the seasons is 0.9 for autumn, 0.87 for spring and 0.8 for summer. The summary results ranked in order of best fit to data for various non-linear curves are in table 1 below:

Table 1: Price-Demand Curve fitting summary results

Rank	Model	Std Error	Residual Sum	Residual Sum of Squares	Adjusted R^2
1	Tenth order polynomial	28.0469783	-0.000810835	39373341.26	0.236636136
2	Ninth order polynomial	28.0473385	-1.74575E-05	39375139.17	0.236616529
3	Eighth order polynomial	28.0518292	-1.65536E-05	39388535.83	0.236372058
4	Seventh order polynomial	28.0521993	-6.7353E-07	39390362.24	0.236351906
5	Sixth order polynomial	28.0538977	1.49502E-06	39395918.98	0.236259437
6	Fifth order inverse logarithm	28.0677236	0.021970599	39435547.72	0.235506455
7	Fifth order logarithm	28.0688962	0.007377117	39438842.64	0.23544258
8	a*x^5+b*x^4+c*x^3+d*x^2+e*x+f	28.0742746	4.73095E-09	39453958.22	0.235149551
9	a*x^4+b*x^3+c*x^2+d*x+e	28.0761997	3.03535E-08	39460157.61	0.235044652
10	Fifth order inverse polynomial	28.1008552	-9.7943E-08	39528703.23	0.233700552
11	Fourth order logarithm	28.1045004	-0.001922489	39539748.94	0.233501733
12	Fourth order inverse logarithm	28.1211914	-0.000125595	39586727.71	0.232591026
13	*a/(1+b*x)*	*28.1316599*	*872.7983539*	*39618580.65*	*0.232019565*
14	Cubic spline	28.1621688	-1.26392E-07	39702974.11	0.230352904
15	a*x^3+b*x^2+c*x+d	28.1621688	-7.63996E-08	39702974.11	0.230352904

We choose the 13th model due to its ease of use, moreover the explained variability of 23% (as captured by the adjusted R^2 does not vary significantly from the top ranked).The estimated parameters are: a = 15.7045 and b = -0.000014129 hence the deterministic component is fully defined as:

$$f(t, L_t/v_t) = \frac{15.7045}{1-(0.00001412\ 9*\frac{L_t}{v_t})} \quad \text{........... 4.2}$$

Testing of the fitted data is done in section 4.4 below.

4.3.2 *Residual stochastic process*

For identification of the short term Ornstein-Uhlenbeck (O-U) process, we begin by reviewing the correlogram results. The series has a very persistent autocorrelation function that dies away very slowly and only the first partial autocorrelation coefficient appears significant. The Unit root tests supports that the data is stationary.

These observations support the use of a first order autoregressive process, AR (1), which is considered the benchmark model for mean-reverting processes for a discrete time version of O-U process.

$$X_t = c + aX_{t-1} + \xi_t, \qquad \text{..........} \qquad 4.2$$

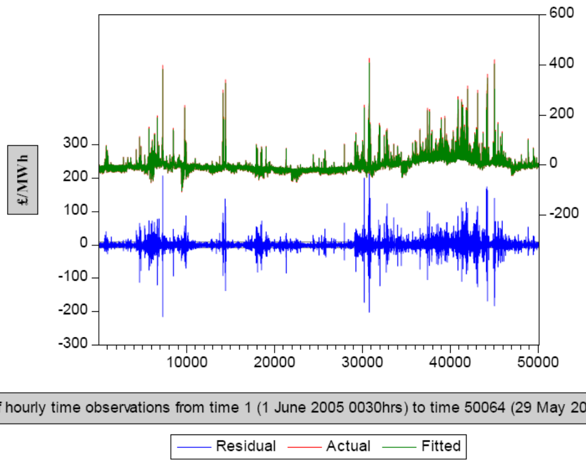
where the error term ξ_t is assumed to be white noise with constant variance σ_ξ^2

Summary results for the estimated parameters are presented in table 2 below:

Table 2: Estimated parameter for residual stochastic O-U process				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C -constant	0.021901	0.865454	0.025305	0.9798
a	0.958661	0.001272	753.8	0.0000
β	0.020119606	R-squared	0.919031	
σ	8.028014592	Adj. R-squared	0.91903	
		S.E. of regression	8.004944	

And the graphical presentation of the estimation and the residual is presented in figure 11 below.

Figure 11: Stochastic component (Ornstein - Uhlenbeck process) fit results



Half hourly time observations from time 1 (1 June 2005 0030hrs) to time 50064 (29 May 2009 2400hrs)

The residuals are not normally distributed implying inadequacy of the model to capture all variability. This stems from limitations of the linear Gaussian AR (1) model that assumes error structure to be Gaussian and independent over time and thus cannot accommodate residual price spikes in the data due to market psychology or other non-electricity generation factors. However, this inadequacy is considered reasonable for our purposes since we are not aiming for accuracy in price forecasting, but adequacy of the model for risk management, in particular hedging.

Testing of the fitted data is done in section 4.4 below

4.4 model testing

The assessment of model suitability for hedging purposes can be tested on two levels:

i) The ability of the model to recover the distribution of the original price process. This can be done by moments matching of the first 4 moments of the empirical and estimated price returns.

ii) The robustness of the model to replicate the price evolution. To test this, we perform parameter stability and out-of-sample tests.

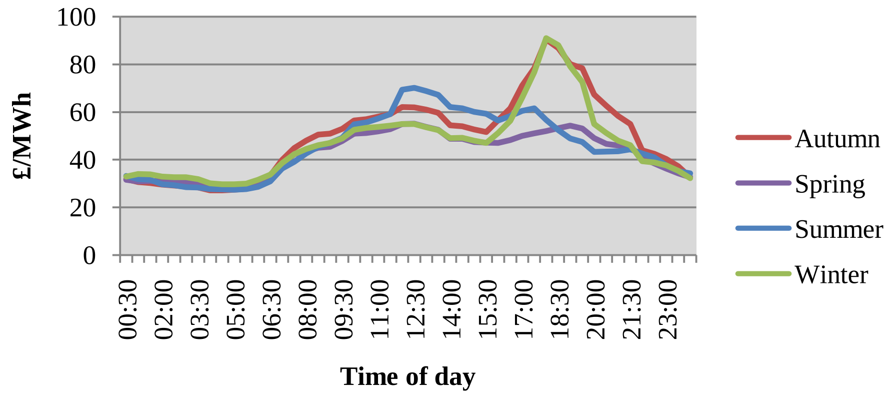
   The parameter stability test is of particular importance as this is a key criticism of the jump-diffusion pure price models where significantly different parameters are calibrated across different time bands, a problem attributed to lack of adequate data for the rather young markets.

   For the out-of-sample tests, in addition to reviewing the closeness of fit (through the R^2 statistic) we also review the success ratio, which measures the percentage of right sign prediction (i.e. how closely the pattern of increase / decrease in the estimated prices matches that of the empirical data). This statistic is of more importance in our modelling and risk management purposes as it can be loosely applied as the indicator of the timing of price jumps which are the focal point for risk management.

4.4.1 *Distribution analysis*

For a first visual test to examine the preservation of the price evolution we plot out the arithmetic average of the estimated prices and compare this with the empirical data plotted in figure 5.

Figure 12: Half hourly seasonal average price patterns from estimated price process for period 1 July 2005 - 29 May 2009



The evolution of the price closely mirrors the observed patterns from empirical data and from a visual level, the model reproduces average paths well.

To obtain a statistical measure of the goodness of fit we review the distribution characteristics of the estimated price process presented in figure 13 below.

Figure 13: Histogram & descriptive statistics for estimated spot price returns

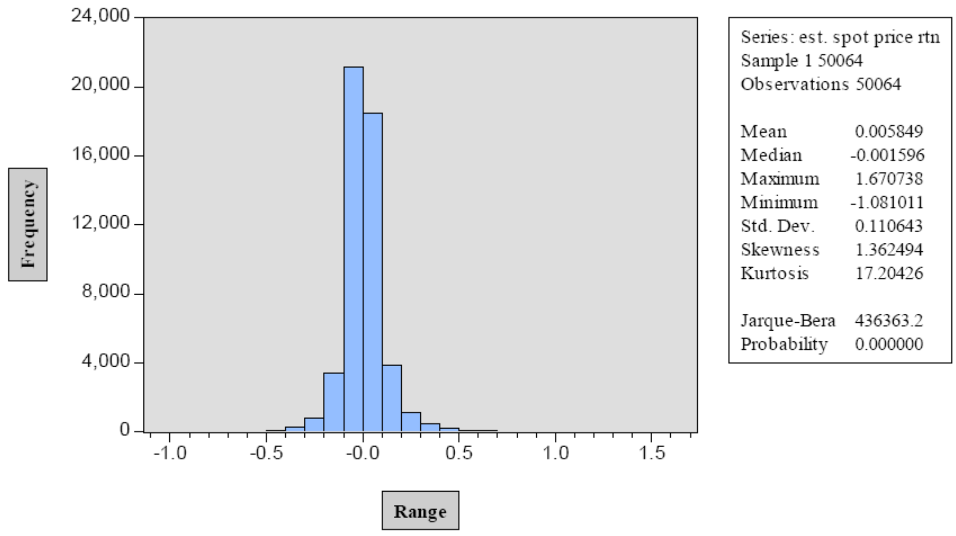


Table 3 presents the estimated price return as compared to the actual price return distribution.

Table 3: Moment matching		
	Empirical	**Estimated**
Mean	0.006360	0.005849
Standard Deviation	0.118573	0.110643
Skewness	2.516819	1.362494
Kurtosis	26.39662	17.20426

The model produces roughly similar value of mean and variance as with the empirical data. The skewness and kurtosis measures however decrease significantly as compared with empirical values. This can be attributed to the assumptions of normality in using an AR (1) process in estimating our mean reversion, hence the AR (1) process with gaussian error term underestimates the extreme events arising in the system due to unplanned outages or market psychology and instead assumes a normally distributed disturbance term.

4.4.2 Parameter stability test

We apply the Chow test and split the data into two sub-periods each covering 2 full years with first set running from 1 June 2005 to 31 May 2007 and second subset from 1 June 2007 to 29 May 2009.

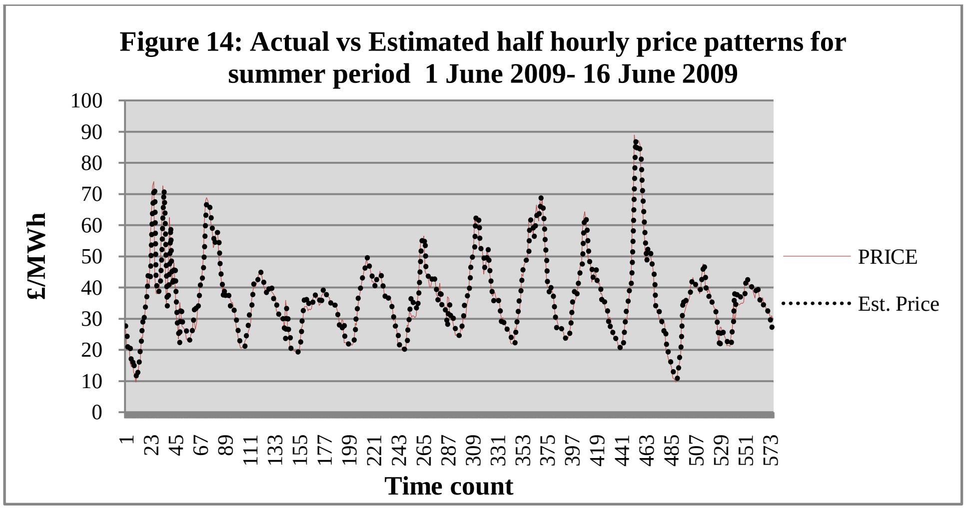
The results are summarised in table 4 below:

Table 4: Parameter stability test (Chow test) from results the test statistic is less than critical value, we therefore accept the null hypothesis that the parameters are stable over time

RSS $_{\text{full sample}}$	**RSS** $_{\text{sub-period 1}}$	**RSS** $_{\text{sub-period 2}}$	**T** $_{\text{total obs.}}$	**K** $_{\text{parameters calibrated}}$	**Chow test statistic**	**Critical value F(k, T-2k)**
3,222,483.77	1,002,544.45	2,219,990.56	50,064	4	-0.1990	2.370

4.4.3 Out of sample testing

We test the robustness of the model by applying the parameters and model assumptions to 576 observations from 1 June 2009 to 16 June 2009 outside of the sample period. The resulting estimation is presented graphically in figure 14 below:



From figure 14, the estimate price has a rather good fit to the empirical data.

The (economic loss function) success ratio of 64.69% is rather favourable indicating approximately 2/3rds of the time the model predicts the right direction of price movement (irrespective of whether it is a small or large change).

4.5 Summary results from hybrid model fitting

From the model testing results above, the defined hybrid model provides a good fit to historical data, has stable parameters and is able to reproduce the price paths and distribution.

The formula derived to estimate the deterministic price process is presented in equation 4.2 whereas the stochastic component is estimated as an AR (1) process with assumed normal error terms.

The residual term from the fitting is however not normal as presented in figure 11 indicating inadequacy of the model in fully explaining spot price evolution. This is also reflected in table 3 (moment matching) where the 3rd and 4th moments differ significantly from empirical data. The model therefore underestimates the frequency of extreme events. However, given the good visual fit, recovery of the 1st two moments and 2/3rd success ratio we believe the model is adequate for our contingent claim pricing purposes.

To complete the modelling requirements, we extend and test the model in risk evaluation and contingent claim pricing in chapter 5.

CHAPTER 5

PRICING CONTINGENT CLAIMS ON ELECTRICITY SPOT PRICES

5.1 Theoretical framework

The non-storability of electricity renders the electricity market highly incomplete and breaks down the spot – forward relationship.

This limitation can be circumvented by directly modelling contingent claims on the forward/future electricity prices which provides a complete market scenario where risk neutral pricing arguments can be applied. However, various over-the-counter products depend on spot price evolution and the use of forward price curves offers no linkage back to the spot price process. This creates the need to define solutions for contingent claims based on the spot price process which we present below.

As discussed earlier, to price contingent claims using the spot price process, we need to identify the market price of risk.

To begin with, we re-state the stochastic process followed by the spot prices of form:

$$S_t = f(t, L_t^*) + X_t \qquad \text{........... 5.1}$$

where L_t^* refers to the demand adjusted for availability. It therefore follows that

$$X_t = S_t - f(t, L_t^*) \qquad \text{........... 5.2}$$

And from our prior assumptions X_t follows a (stationary mean reverting) Ornstein-Uhlenbeck process with zero long run mean and a β speed of mean reversion i.e.

$$dX_t = -\beta X_t dt + \sigma dW_t \qquad \text{........... 5.3}$$

Assuming that the function $f(.)$ is deterministic and satisfies appropriate regularity conditions we can then re-write 5.1 and 5.3 as

$$d(S_t - f(t, L_t^*)) = -\beta \, (S_t - f(t, L_t^*))dt + \sigma dW_t \qquad \text{........... 5.4}$$

which has the intuitive interpretation when spot price S_t deviates from the deterministic price derived from demand levels $f(t, L_t^*)$, it is pulled back to it as a rate proportional to the deviation from this level.

The solution for this O-U process is explicitly given as:

$$S_t - f(t, L_t^*) = e^{-\beta t} \ (S_0 - f(0, L_0^*)) + \sigma e^{-\beta t} \int_0^t e^{\beta s} \, dW_s \qquad \text{.......... 5.5}$$

Since $S_0 - f(0, L_0^*) = X_0$, then we can re-write the expression as

$$S_t = f(t, L_t^*) + X_0 e^{-\beta t} + \sigma \int_0^t e^{\beta(s-t)} \, dW_s \qquad \text{.......... 5.6}$$

From 5.5 we can then see that the spot price conditional distribution is Gaussian satisfying

$$S_t \sim N \ \left(f(t, L_t^*) + X_0 e^{-\beta t} \ , \ \frac{\sigma^2}{2\beta} \left(1 - e^{-2\beta t}\right) \right) \qquad \text{.......... 5.7}$$

5.1.1 Market price of risk, λ

The price at time t of a forward contract expiring at future time T (denoted as $F_{t,T}$) is given as the expected value of the spot price at time T under a (risk neutral) Q-martingale measure, conditional on the information available up to time t i.e.

$$F_{t,T} = \mathbb{E}_t^Q \ [S_t | \mathcal{F}_t] \qquad \text{.......... 5.8}$$

In a complete market this Q-martingale measure is unique, ensuring only one arbitrage-free price of the forward.

However, in the electricity (incomplete) market the measure is not unique. We, therefore, define and calibrate the market price of risk which gives a measure to account for the risk premium observed in the market.

The market price of risk is the defined as the difference between the return in the original ‘risky’ probability measure $\mathcal{P}$ and the return under the ‘risk-neutral’ martingale measure $\mathcal{Q}$.

In our analysis we take the assumption that is λ a deterministic constant hence a predictable process.

By Girsanov's theorem there exists a probability measure Q equivalent to the original 'risky' probability measure $\mathcal{P}$ such that

$$W_t^{\lambda} = W_t + \int_0^t \lambda(s)\, ds = W_t + \lambda_t \qquad \text{.......... 5.9}$$

is a standard Brownian motions under Q.

Then by Ito's calculus, and using the relation $X_t = S_t - f(t, L_t^*)$ we can re-write 5.4

$$dX_t = \beta\,(-\,X_t\,)dt + \sigma d(W_t^{\lambda} - \lambda t)$$

$$= \beta \left(\frac{-\lambda\sigma}{\beta} - X_t\right) dt + \sigma d(W_t^{\lambda}) \qquad \text{..........5.10}$$

Following similar path with derivation of 5.4 – 5.7 the explicit solution for 5.9 is given as

$$S_t = f(t, L_t^*) + X_0 e^{-\beta t} + \alpha(1 - e^{-\beta t}) + \sigma \int_0^t e^{\beta(s-t)}\, dW_s^{\lambda} \qquad \text{.......... 5.11}$$

where $\alpha = \frac{-\lambda\sigma}{\beta}$

then the conditional distribution under Q is given as

$$S_t \sim N\ \left(\ f(t, L_t^*) + X_0 e^{-\beta t} + \alpha(1 - e^{-\beta t})\ ,\ \frac{\sigma^2}{2\beta}\left(1 - e^{-2\beta t}\right)\right) \qquad \text{.......... 5.12}$$

It therefore follows, if we estimate λ we are able to know the price dynamics of the stochastic component X_t and can therefore price claims on the spot price.

To calibrate the market price of risk we imply it from the option prices in a similar technique as that of obtaining implied volatility from the Black-Scholes model. Hence we seek to find λ^* such that the mean square error of the observed and model contingent claims is minimized.

5.1.2 Forward price

Following from equation 5.8 the value of a forward contract at time zero on the spot price with maturity time T can then be expressed using the defined hybrid model as:

$$F_{t,T} = \mathbb{E}_t^Q\,[S_0|\mathcal{F}_0] = \mathbb{E}_t^Q\,[S_0] = f(T, L_T^*) + X_0 e^{-\beta T} + \alpha(1 - e^{-\beta T}) \qquad \text{.......... 5.13}$$

5.2 Underlying Assumption

In addition to the three assumptions on the underlying hybrid model defined in section 4.2, two additional assumptions are included for purposes of pricing contingent claims

Assumption 4

Perfect "Frictionless" Market

We assume for the given market participants:

- Borrowing and short selling of securities is permitted and full proceeds are received
- Borrowing and lending rates are equal
- There are no transaction costs or tax rates
- Security trading takes place continuously
- The risk free rate of interest, *r* is known and constant and is same for all maturities. In this setting therefore, forward and future prices are equal

Assumption 5

Convergence assumption

We assume that at the expiration the forward/future price converges to the spot price of electricity at the time T i.e.

$$F_{T,T} = S_T \qquad \text{.......... 5.13}$$

This allows us to apply the 'modified' risk-neutral environment to include a measure for market price of risk.

5.3 Forward contract valuation using hybrid model

Below we present analysis for a simple forward contract contingent claim on the spot price.

Due to data limitations, in particular unavailability of forward price data, we do not calibrate the market price of risk from forward prices. Instead we take approach of estimating λ under premise that the convergence assumption holds.

We therefore seek to derive a forward price from the hybrid model and assess the various market risk (λ) measures for goodness of fit based on our convergence assumption that the forward price converges to the spot price at expiry time.

To proceed we first classify the 48 half hourly market price observations to off-peak, extended off-peak and peak hours to match relevant APX product definitions.

Peak is defined as duration of 12 hours commencing 0700-1900hrs. Extended peak covers 16 hours from 0700 – 2300hrs hence an extra four hours from 1900-2300hrs. Off-peak periods are defined as the remaining 8 hours from 2300-0600hrs when there is low activity and general minimal use of electricity.

5.3.1 Day – ahead forward prices

The inputs to the formula for day ahead contingent claim are:

Equation: $F_{0,1} = f(1, L_1^*) + X_0 e^{-\beta} + \alpha(1 - e^{-\beta})$

$\beta = 0.020119606 \qquad \sigma = 8.028014592 \qquad \alpha = \frac{-\lambda\sigma}{\beta}$

Market price: $\lambda = 0, -0.01, -0.012, -0.015, -0.02, -0.05$

We estimate the forward prices curve for the full period under investigation (1 June 2005 to 29 May 2009). The results are presented in figure 15 below.

Figure 15: Day Ahead forward price curve for λ=0 for period 2 June 2008 to 29 May 2009

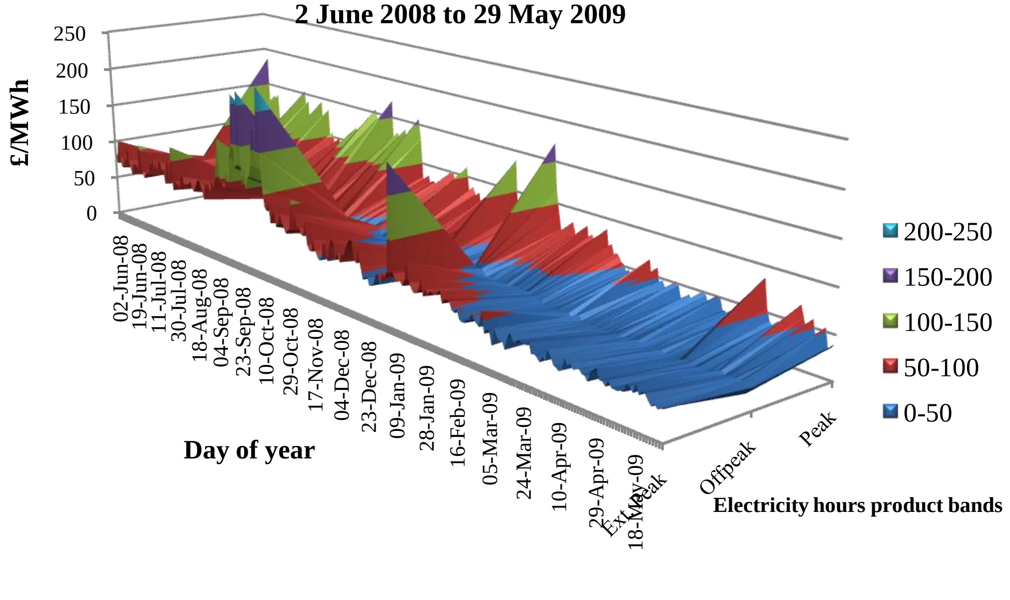


Figure 15 exhibits some desirable properties observed from previously presented models and studied in the electricity forward markets.

To begin with the "trough" look for the inter-day price seasonality is captured with off peak prices being lowest at the bottom of the trough while peak bands have the highest forward price, similar to visual result presented by Figueroa and Cartea (2007). Further, as observed and noted by Pilipovic (2007), the forward prices clearly reflect summer and winter peaks with highest price volatility bands falling in December-January winter period and May-June summer periods.

We conducted two tests on the estimated forward prices. First, using the solver tool in excel, we sought to find the market price of risk λ that minimizes the Root Mean Square Error (RMSE) measure. Next, we sought to determine the stability of the derived λ parameter through seasons.

Estimated market price of risk

The estimation of λ is based on the null hypothesis that the convergence assumption holds. Under this assumption the consumer is indifferent between entering into a forward price contract for delivery in future time and purchasing and 'holding' the electricity commodity. Hence at the future time we have $F_{T,T} = S_T$. The three set time bands: peak, off peak and extended peak are evaluated as three different products and the results are presented in table 5 below:

Table 5: Estimated values of day-ahead λ based on convergence assumption

Day Ahead product	**Summer**	**Autumn**	**Winter**	**Spring**	**Overall**
Peak hours	(0.025)	0.039	(0.039)	0.092	(0.018)
Off Peak hours	(0.007)	0.016	(0.007)	(0.006)	(0.005)
Extended Peak hours	0.020	0.013	0.010	(0.008)	0.005

As the market price of risk can be interpreted as the extra premium paid as insurance for delivery over and above the risk neutral prices and therefore deducted to arrive at risk neutral prices, it is ideally negative.

Concentrating on the main off peak and peak hour products, the results presented in table 5 indicate overall negative λ for both products. However in between the seasons the market price varies from negative to positive factors, results which are consistent with Bessembinder and Lemmon (2002) equilibrium model implying that the forward price is a downward biased estimator of future spot prices if the expected demand and variability of electricity is low which falls in the spring and autumn.

The negative λ describes the state in which there is increased investor desire to hedge their positions due to increased probability of price spikes hence huge losses. The expected forward price then bids up to compensate for this increased risk. The market price of risk is greatest in the winter where the highest demand and variability is recorded.

The positive λ on the other hand reflects the difficulty to inventory electricity hence the cost of holding a forward contract in seasons of low demand and low variability weighs in on the benefit of guaranteed prices as the probability of price spikes is low.

The results of the Chow test indicate that the λ parameters are unstable over time. The results are summarized in table 6 below:

Table 6: Day – ahead Market price of risk λ stability test (Chow test). From results the test statistic is greater than critical value, we therefore reject the null hypothesis that the estimated market price of risk is stable over time

Day Ahead product	RSS whole	RSS period 1 (521 obs.)	RSS period 2 (518 obs.)	Test statistic	Critical value (5%) F(1, 1037)
Peak hours	392,416.3	157,823.7	143,509	313.7554	3.84
Off Peak hours	11,811.79	6,613.43	2,952.338	243.7202	3.84
Extended Peak hours	245,419.4	83,806.32	128,566.3	161.5206	3.84

This implies that our assumption of a deterministic constant λ does not hold. The finding is consistent with Weron (2008) paper which concludes that the market price of risk is local over given time periods.

5.3.2 Year ahead forward prices

In this case, the inputs to the formula for year ahead contingent claim (covering 4 seasons) are:

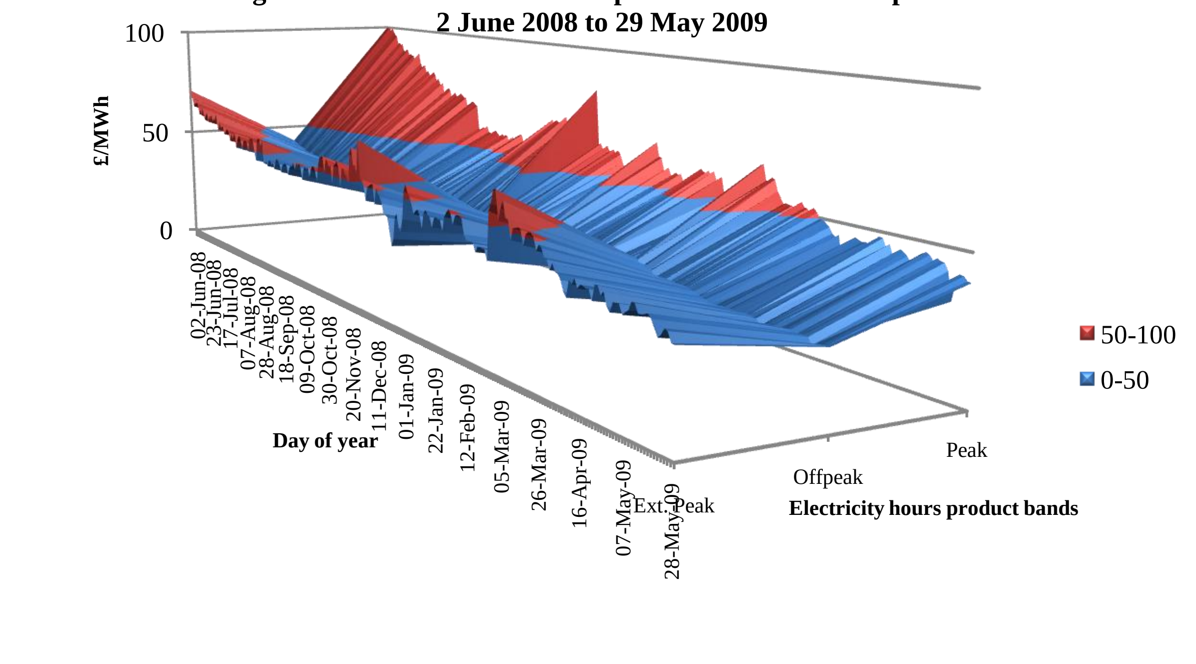
Equation: $F_{0,1} = f(t, L_t^*) + X_0 e^{-\beta t} + \alpha(1 - e^{-\beta t})$

$\beta = 0.020119606$ $\sigma = 8.028014592$ $\alpha = \frac{-\lambda\sigma}{\beta}$ $t = 0,1,2 \ldots\ldots,257$

Market price: $\lambda = 0, -0.01, -0.012, -0.015, -0.02, -0.05$

The results are presented in figure 16:

Figure 16: Year Ahead forward price curve for λ=0 for period 2 June 2008 to 29 May 2009



The year ahead forward prices exhibit lower volatility than day-ahead forward contracts for the same period. This is similar to Bessembinder and Lemmon (2002) equilibrium model results and Pilipovic (2007) observation that the forward market in the long term exhibits less variability compared to the spot market.

Table 7: Estimated values of year ahead λ based on convergence assumption

Day Ahead product	**Summer**	**Autumn**	**Winter**	**Spring**	**Overall**
Peak hours	(0.008)	0.024	(0.035)	0.017	(0.004)
Off Peak hours	(0.003)	0.0002	(0.004)	0.002	0
Extended Peak hours	0.0013	0.012	0.012	0.015	0.002

The estimated λ's generally follow similar pattern to that of day ahead and again fail on the stability test inferring that the market price of risk is rather local.

5.4 Critical review of model assumptions and estimates

The underlying assumptions and estimates that have a significant effect on the final results include:

A - The spot price evolution can be modelled as a sum of deterministic price-demand transform function and stochastic mean-reverting diffusion process

The two components of the hybrid model that describes the evolution of the spot prices are:

i) A deterministic price-demand transform from a persistent (pre-observable) demand level i.e. demand persists over consequent half-hour observations.

ii) A stochastic mean-reverting diffusion residual process with normally distributed error terms.

Under this setting the model is greatly simplified to a one factor model ignoring any source of uncertainty from the demand process. Empirical evidence from work by Lucia and Schwartz (2000), Burger et.al (2004) and Pilipovic (2007) indicates that two-factor models perform better than the simplified one factor models. In particular Burger et.al (2004) offers a two factor hybrid model with the demand modelled as a stochastic process.

Further the assumption of a normalized error term, which from our workings is contradicted, the probability of occurrence of huge price spikes is greatly underestimated. Our model therefore, underestimates the magnitude and frequency of price jumps.

From a market player’s perspective, therefore, the hybrid model intrinsically implies that prices are largely determined by industry participants using economic reasoning of demand levels and supply constraints, rather than outside speculators.

Other models on the UK electricity market have provided better approximation of the spot price evolution with respect to matching of moments of the empirical and estimated prices, but with a general effect of a more complicated model. In particular the model presented by Geman and Cartea (2009) better matches the Skewness and Kurtosis measures in comparison to our hybrid model.

Bearing in mind that our aim is not to provide accurate forecasts of spot prices, but rather a model that captures the qualitative aspects of electricity spot prices including mean reversion and jumps for risk management, and further given the model success ratio of 64.69% we are therefore able to meet our objectives under these assumptions and model simplifications.

B –Constant deterministic interest rates, volatility and market price of risk

These may not be realistic assumptions and estimates given the nature of the electricity market and the larger financial market.

As noted by Lucia and Schwartz (2000), Eydeland and Wolyniec (2003) among others; in the energy markets and in particular the electricity market, we do not have convergence in the strict sense. We therefore apply our market price of risk, λ, to capture this statistically significant basis between the actual future spot price and forward price at expiration.

To the extent that the estimated λ is not accurate, so are our evaluated risk neutral forward prices. In particular, from empirical chow test, the market price of risk is evaluated as not stable implying it takes on statistically significant different values and directions over time. Previous studies have also indicated seasonally dependent non-constant volatility. This is also visually evident from figure 1.

Further, we have estimated the volatility and market price of risk parameters from historical data. This may not provide a good estimate given that the high volatility of electricity markets and the fact that these markets are considered still too young to be judged as stable.

An alternative approach using stochastic seasonally- dependent processes for these two parameters may be more appropriate. We however believe that the method applied gives a sufficiently reasonable result for our purposes.

5.5 Limitations of study

The study is limited to access of empirical data of recorded forward prices for the UK forward market. The estimated forward prices are therefore not compared against actual prices. However, using the convergence assumption, the distribution characteristics of estimated forward prices are observed to follow previous studied characteristics.

CHAPTER 6

CONCLUSIONS

In this paper we propose and calibrate a hybrid model for the evolution of spot prices that recovers the main characteristics of electricity spot price dynamics namely seasonality, mean-reversion and jumps.

The hybrid model is presented as a sum of two components: a deterministic price-demand transform function and a residual mean reverting stochastic function. By definition of the transform, the stylized characteristics are intrinsically captured and the stochastic component then captures the residual price movements largely due to market psychology and effects of anticipated and known news in the market.

The results obtained from calibration and estimation of the spot prices perform well in both in-sample and out-of-sample test. In comparison with other models presented for the same market namely: Cartea and Figueroa (2005) mean-reverting jump diffusion model and Geman and Cartea (2009) hybrid model with forward looking capacity constraints, the model does well in matching the first two moments – mean and variance but the skewness and kurtosis of the estimated model are much lower than empirically observed ones due to assumption of normally distributed error terms in estimation of our stochastic component. In contrast, Geman and Cartea (2009) model performs well for all four moments rather successfully whereas Cartea and Figueroa (2005) offer no such comparison.

However the simplicity of the model coupled with a good success ratio and recovery of price paths makes it a good alternative to more accurate but complicated models for the purposes of valuation of contingent claims.

The evaluated day ahead and year ahead forward prices exhibit desired characteristics of forward prices with lower volatility and recovering the seasonality and jumps in the process.

The calibrated market price of risk exhibits two significant features – unstable and change of sign. From the chow test, the market price of risk is significantly different over time and this seems largely seasonally dependent. The premium is highest in months of high demand (winter and summer months) where price jumps have a higher probability of occurring, and is positive in low demand - low volatility months and due to non-storability of electricity, or high expense in storing the energy form for conversion to electricity; it becomes a negative benefit to hold a forward contract. This result is in line with observations by Bessembinder and Lemmon (2002).

This study can be extended in a number of ways to better reflect market observed spot and forward prices while preserving the simplicity and tractability. To begin with, the availability and use of actual forward prices could greatly improve the calibration and testing of the market price of risk parameter. Further still, relaxation of assumptions to evaluate more realistic experiences could also be considered. In particular extension to a two-factor model, use of stochastic season-dependent volatility and a season-deterministic market price of risk.

Footnotes

1. Source: Digest of UK Energy Statistics (DUKES) http://www.berr.gov.uk/energy/statistics/publications/dukes/page45537.html
2. Source: Elexon Market Index Definition Statement http://www.elexon.co.uk/

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