Strathmore Institute of Mathematical Sciences (SIMS)
Special Examination for the Degree of Bachelor of Business Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE: Wednesday 2nd August, 2023 **Time: 8-10am(2 Hours)**

Instructions

- **This examination consists of FIVE questions.**
- **Answer Question ONE (COMPULSORY) and any other TWO questions.**

1. (a) In the context of decision making under uncertainty, briefly but intuitively describe the independence axiom (also referred to as independence of irrelevant alternatives) **(3 Marks)**

   (b) Provide a formal definition of risk aversion **(2 Marks)**

   (c) Consider the utility function of the form

   $$u(w) = -\frac{1}{v}e^{-vw}$$

   Derive the absolute risk-averseion coefficient for this utility function. **(3 Marks)**

   (d) Provide an intuitive explanation of the following concepts as used in asset pricing literature.

   (i) Pricing kernel **(4 Marks)**

   (ii) Mutual fund theorem **(4 Marks)**

   (ii) Complete Markets **(2 Marks)**

   (e) The Arbitrage Pricing Theory (APT) is agnostic about what factors should determine asset returns. However, factor identification has become an empirical issue. Describe three approaches often used in factor identification.**(6 Marks)**

   (f) "*Inference about the risk premia is equivalent to inference about the stochastic discount factor (SDF)*". Briefly explain if you agree with this statement (Hint: Mathematically show the equivalence).**(6 Marks)**

2. (a) An expected-utility-maximizing individual has constant relative risk aversion utility, $U(W) = \frac{W^{\gamma}}{\gamma}$, with relative risk-aversion coefficient of $\gamma = -1$. The

individual currently owns a product that has a probability p of failing, an event that would result in a loss of wealth that has a present value equal to L. With probability $1-p$, the product will not fail and no loss will result. The individual is considering whether to purchase an extended warranty on this product. The warranty costs C and would insure the individual against loss if the product fails. Assuming that the cost of the warranty exceeds the expected loss from the product's failure, determine the individual's level of wealth at which she would be just indifferent between purchasing or not purchasing the warranty. **(6 Marks)**

(b) Assume the utility function $u(w) = 11w - 5w^2$, with $w_0 = \$1$ million, $r_f = 0$, $E[\tilde{r}] = 0.1$, and $\sigma_{\tilde{r}}^2 = 0.2^2$. Find the optimal amount invested in the risky asset assuming that the investor is risk-averse. **(6 Marks)**

(c) The prediction test of the CAPM plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$. Based on these estimates one may estimate the simple linear regression equation

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \epsilon_i, i = 1, \dots, N \tag{1}$$

(i) if the CAPM were true, what should the estimated values for γ_0 and γ_1 be? **(2 Marks)**

(ii) The least squares estimates of equation(1) are summarized below. Using these estimates, test the null hypothesis that the CAPM is true using a 5% significance level. Assume that the average excess return on the market index over the sampled period was 0.0031 **(6 Marks)**

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> summary(sml.fit)

Call: lm(formula = mu.hat ~ betas)
Residuals:
     Min       1Q  Median       3Q     Max
 -0.0252 -0.0047 0.00112 0.00936 0.0142


Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept) 0.002 0.007      0.275   0.788
     Betas  0.001 0.009      0.143   0.888

Residual standard error: 0.0115 on 13 degrees of freedom
Multiple R-Squared: 0.00157
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3. (a) Let $\overline{R} = (\overline{R_1}, \overline{R_2}, \dots \overline{R_n})'$be an $n \times 1$ vector of the expected returns of the n assets. Also let V be the $n \times n$ covariance matrix of the returns on the n assets. V is assumed to be of full rank, it is symmetric and positive definite. Let $\omega = (\omega_1, \omega_2, \dots \omega_n)'$ be an $n \times 1$ vector of portfolio proportions, such that the ω_i is the proportion of total portfolio wealth invested in the ith asset. The portfolio weights need to sum to 1. That is $\omega' e = 1$.

(i) Write down the expression for the expected return on the portfolio **(2 Marks)**

(ii) Write down the expression for the variance of the portfolio return **(2 Marks)**

(ii) Set up the investor's portfolio choice problem and solve for the optimal portfolio weights **(8 Marks)**

(b) Suppose an expected utility maximizing individual investes her beginning of period wealth, W_0, in a particular portfolio of assets. Let $\tilde{R}$ be the random return on this portfolio, so that the individual's end-of-period wealth is $\tilde{W} = W_0\tilde{R}_p$. Note that $\tilde{R}_p$ is defined as gross return. Denote this individual's end-of-period utility by $U(\tilde{W})$ and for simplicity express $U(\tilde{W}) = U(W_0\tilde{R}_p)$ as just $U(\tilde{R}_p)$. Provide a taylor series expansion of $U(\tilde{R}_p)$ around the mean of $\tilde{R}_p$, ($\mathrm{E}[\tilde{R}_p]$) and show that for any probability distribution of the portfolio return $\tilde{R}_p$, quadratic utility leads to expected utility that depends on the mean and variance of $\tilde{R}_p$ **(8 Marks)**

4. Suppose a typical investor solves the following problem

$$\begin{gathered}\max_{\alpha} U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})] \\ \text{subject to } c_t = e_t - \alpha p_t \text{ and } c_{t+1} = e_{t+1} + \alpha x_{t+1}\end{gathered} \tag{2}$$

where c_t and c_{t+1} denotes consumption at date t and $t+1$ respectively. The other parameters in the model are; e_t (investor's endowment at period t and $t+1$ respectively), p_t (asset price at time t), α (the number of units of the asset purchased by the investor), and x_{t+1} (payoff of the asset at period $t+1$), β represents the time preference parameter of the investor ($0 \leq \beta \leq 1$). The general functional form $U(.)$ represents the investor's utility function. An often convenient utility function used in many applications is the power utility $u(c_t) = \dfrac{1}{1-\gamma}c^{1-\gamma}$

(i) Show that the price of an asset at any given time can be represented as follows: **(3 Marks)**

$$p_t = E_t[\frac{\beta u'(c_{t+1})}{u'(c_t)}x_{t+1}]$$

(ii) Provide an intuitive interpretation of the term $\dfrac{\beta u'(c_{t+1})}{u'(c_t)}$ and briefly discuss how this term influences the asset price p_t **(3 Marks)**

(iii) Show that a simple manipulation of the price equation in (i) can yield the following equation (ignoring time subscripts)

$$1 = E(mR)$$

where R represents the gross return of the asset, while $m = \dfrac{\beta u'(c_{t+1})}{u'(c_t)}$ **(2 Marks)**

(iv) Assuning the power utility function provided and the manipulations in parts (i)-(iii) above, one can show that the risk-free rate can be expressed as

$$R^f = \frac{1}{\beta}(\frac{c_{t+1}}{c_t})^{\gamma}$$

Intuitively discuss three factors that influence interest rates in an economy. **(6 Marks)**

(v) Show that the basic pricing equation ($1 = E(mR)$) can be rewritten as follows;

$$E(R) = R_f - \frac{Cov(u'(c_{t+1}), R)}{E(u'(c_{t+1}))}$$

Interpret this equation. **(6 Marks)**

5. (a) Briefly explain the main advantage of the Arrow-Debreu pricing over the capital asset pricing model **(3 Marks)**

   (b) You are given the following information: Show how you can combine stock and

	Stock	Bond	up.A-D	down.A-D
upstate	1260	1050	1	0
Downstate	840	1050	0	1
Current Price	1050	1000		

   bonds in such a way that you obtain an upstate and downstate pure security **(7 Marks)**

   (c) There are three possible states in the future, states 1,2,and 3, and the consumption at each state is denoted by $c_1, c_2,$ andc_3. There are one million homogeneous consumers with expected utility function

$$q_1 u(c_1) + q_2 u(c_2) + q^3 u(c_3)$$

   , where $(q_1, q_2, q_3) = (0.5.0.3, 0.2)$, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 0.5$, and with endowment $(y_1, y_2, y_3) = (100, 81, 64)$ Treating the state 1 good as numeraire, find the general equilibrium prices for the state 2 good and for state 3 good. **(10 Marks)**

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