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Where do these two coincide?

Traditional no-arbitrage framework, consider the BSM model:

- Primitive asset: Underlying stock ($\{S_t\}$)

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How does $\{X_t\}$ affect $\{W_t\}$?

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- Hedging (risk-managing) portfolios: $\{V_t\}$ is a function of $\{X_t\}$

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Academic Research vs Practicality - another word of caution

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- A stylized fact in finance refers to an empirical finding that is sufficiently ubiquitous across all instruments, markets and time periods as to be accepted as truth
- In some way, all of these facts are an attack on the central distributional assumptions of mathematical finance

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- A leverage effect
- Volume/volatility correlation
- Asymmetry in time scales
- Most recent work has been done on the fifth item - volatility clustering

Aggregational Gaussianity (AG)

Another well-known stylized fact of financial time-series is the property of Aggregational Gaussianity:

- AG is the phenomenon in which the empirical distribution of log-returns tends to normality (or Gaussianity) as the frequency of observations decreases (or the time scale δt over which the returns are calculated increases) (Eberlein & Keller (1995), Rydberg (2000))

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- In a financial time-series context, log-returns calculated over shorter time periods are known to be kurtotic and often skewed
- As the time-interval over which the returns are calculated is increased, the distribution of returns better approximate normality

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- This has obvious implications for all derivatives & their risk-management
- In particular, many of the standard tools of financial modeling cease to apply: portfolio aggregation, the standard mean-variance portfolio selection models, the Sharpe-Litner-Mossin capital asset pricing models, Brownian motion, marginally Gaussian distributions, aggregated distributions - stable laws and others

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- Log-returns over longer periods of time are in principle the sum of returns over shorter periods - hence the central limit theorem should give a tendency towards normality
- This fact is well known and observed in many financial time-series

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- As a simple illustrative example of how AG manifests Herlemont (2003) examined the log-returns on the CAC-40 Index on the Paris Bourse for the period 1990 - 2003 inclusive
- Herlemont uses a standard Q-Q plot to assess departures from normality at terms from 10 seconds to 10 minutes (Figure 1), and daily to 6-monthly (Figure 2)

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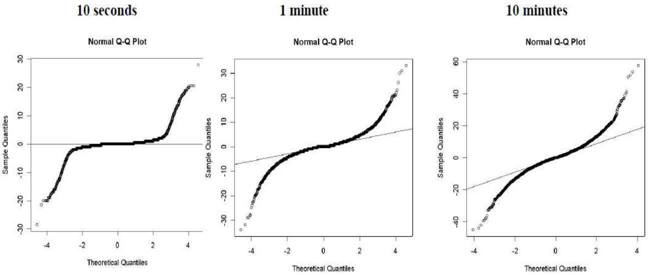

Figure: 1. Q-Q plots from a study by Herlemont (2003) on the CAC-40 (period 1990 - 2003)

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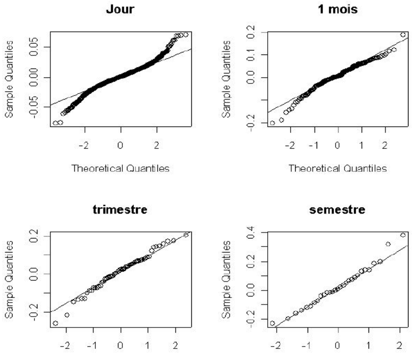

Figure: 2. Q-Q plots from a study by Herlemont (2003)

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- Note: while the use of a Q-Q plot is **qualitative rather than inferential**, when the underlying phenomenon is simply corroborated no further inferential statistics is really required
- When results are more ambiguous, we would need to cast the same investigation into an inferential framework, and one would require a more complex statistical methodology

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- The All-Share index comprises a market-cap weighted index of 162 constituents
- The index is anomalous from the perspective that it is extremely concentrated (5 stocks currently make up 36.33% of the index, and 10 stocks 53.9% of the index)
- It is dominated by large-cap stocks, the majority of which are dual-listed, and resource biased (currently 43% of the top 20 stocks in the All-Share Index are resource companies)

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- The various frequencies of returns comprises five distinct "terms" - daily, weekly, monthly, quarterly and annually
- These observations for the total 16-year period are displayed in a Q-Q plot below (Figure 3)

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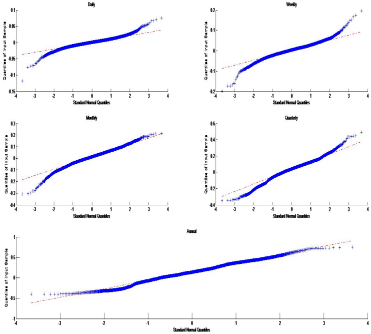

Figure: 3. Q-Q plots from the JSE All-Share Index for the period 1996 - May 2012 (**Data is overlapping)**

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- How can we assess whether the qualitative patterns manifest in Q-Q architecture are sufficient and necessary to infer AG?
- For this objective, we need to move beyond Q-Q plots and into an inferential framework

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  - The derivation of return intervals over rolling periods introduces serial-dependency - highly-correlated return values are not independent and have a vastly deflated variance

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- We utilize the well-known Sobol quasi-random uniform sequencing algorithm
- This sampling regime and sampling effort adequately prevents overlapping returns due to sampling-error

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- We conduct two goodness-of-fit tests for normality - the Shapiro-Wilk (SW) test and Anderson-Darling (AD) test
- Both tests are two-sided, and an alpha level of significance of 5% is adopted for both
- In Table 1, we report on the percentage of trails that are rejected by the null-hypothesis (of no difference to normality) for each sampling term
- Our a priori expectations are twofold:
  - The percentage of rejections should decrease as sampling term increases if AG is a clear feature of returns on the JSE
  - Terms from 2-4 weeks and longer should be overtly Gaussian

Results

Table 1. Percentage of trials that fail the goodness-of-fit tests for normality

Term	Shapiro-Wilk	Anderson-Darling
Daily	95.2%	92.3%
Weekly	89.1%	85.0%
Monthly	68.9%	63.6%
Quarterly	86.8%	93.3%
Annually	24.7%	38.9%

Each trial comprised a sample of 5% of the terminal return values being drawn from the population with the aid of a Sobol-sequence. 1000 trials are conducted.

Results

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  - The proportion of observations conforming to normality does not simply decrease as term is increased
  - There is an anomalous characteristic of this proportion increasing at terms of 3-months
  - All terms (from daily through to annually, inclusive) have a very high incidence of the underlying return data being generated by non-normal processes

Results

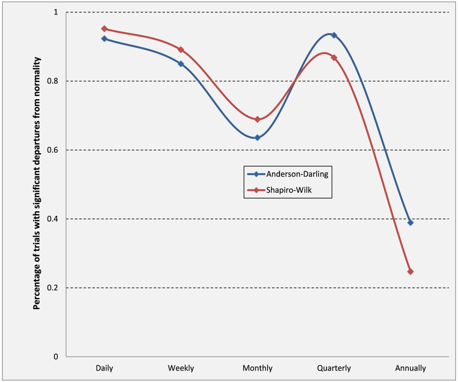

Figure: 4. A graphical illustration of the proportion of trials failing normality tests (via SW and AD) as term is increased

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- If the true process driving the returns is at least AG, then the tail-risk will decrease over long time periods and risk managers and option traders can rest easily
- Option implied volatility surfaces seem to confirm this as a general market view
- In the absence of AG, the observed term-structure and flattening of the volatility surface and the consequential mark-to-market process represents a serious risk

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- This means that volatility (the square-root of variance) has a translational meaning under each measure and historical (statistical) volatility may be compared with implied volatility
- Without some form of AG, this comparability breaks down
- It is still unclear how the statistical measure communicates with the implied risk-neutral measure

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