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A PROPOSED MODEL MORTALITY TABLE FOR THE RWANDESE POPULATION.

Nthiga, Bryan Njue 074383

A Research Proposal Submitted in partial fulfilment for the award of Bachelor of Business Science Actuarial Science

Strathmore Institute of Mathematical Sciences (SIMS)
Strathmore University
Nairobi, Kenya

Nov, 2016

DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

......Bryan Njue Nthiga...................... [*Name of Candidate*]

.... [*Signature*]

.........2/12/2016.................................. [*Date*]

This Research Proposal has been submitted for examination with my approval as the Supervisor.

.........Dr. Olukem.......................... [*Name of Supervisor*]

.... [*Signature*]

..................................2/12/2016.. [*Date*]

Strathmore Institute of Mathematical Sciences

Strathmore University

ABSTRACT

African countries have had the challenge of lacking country-specific published mortality tables. Up to date, there exists only two countries in the continent that have managed to achieve this milestone and these are Kenya and South Africa. Rwanda is faced with the same problem of lacking published mortality tables reflecting their own unique mortality but are however seeking to develop their own tables. This study takes a keen look on how Rwanda-specific mortality tables can be developed by use of the Brass-logit model. Additionally, pricing for life insurance is carried out using the developed mortality rates so as to come up with suitable premium rates that are applicable to the Rwandan population.

Key-terms: Mortality tables, Rwanda, Brass-logit model, Life assurance pricing.

ACKNOWLEDGEMENTS

I would like to acknowledge the following for their guidance and support throughout the duration of the project:

The Almighty God for granting me good health and enabling all this possible.

My supervisor Dr. John Olukuru for his constructive, timely feedback and commitment during the project duration.

The school administrator Mr. Raphael Karanja for his continued support to the cause.

My parents and classmates for their moral support over the duration of the project.

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LIST OF ABBREVIATIONS

The following are the lists of abbreviations and their respective definitions as used in this study:

ASSAR - Association des Assureurs de Rwanda

MINECOFIN - Ministry of Finance and Economic Planning [Rwanda]

NISR - National Institute of Statistics of Rwanda

AKI – Association of Kenya insurers

IRA - Insurance Regulatory Authority

1.0 INTRODUCTION

Mortality tables or otherwise known as life tables describe the mortality and survival patterns of a population (Meulen, 2012). Mortality tables are a key summary tool for assessing and comparing mortality conditions prevailing in populations (Lopez, Salomon, Ahmad, Murray, & Mafat, 2000). These tables provide information on various parameters such as number of survivors, death probability and life expectancy of people of a given age and sex among a population. Social Security Administration (2013) then suggests the general format for a mortality table as illustrated on Figure 1 below.

Period Life Table, 2013

	Male			Female		
Exact age	**Death probability [a]**	**Number of lives [b]**	**Life expectancy**	**Death probability [a]**	**Number of lives [b]**	**Life expectancy**
0	0.006519	100,000	76.28	0.005377	100,000	81.05
1	0.000462	99,348	75.78	0.000379	99,462	80.49
2	0.000291	99,302	74.82	0.000221	99,425	79.52
3	0.000209	99,273	73.84	0.000162	99,403	78.54
4	0.000176	99,252	72.85	0.000133	99,387	77.55
5	0.000159	99,235	71.87	0.000119	99,373	76.56

Figure 1: Sample mortality table

Mortality tables are used by a broad range of professionals including demographers, epidemiologists and actuaries among others for analytical purposes (Lopez, Salomon, Ahmad, Murray, & Mafat, 2000). Actuaries in particular use the mortality tables when modelling life assurance products and pensions.

Murray, et al. (2003) suggests that a desirable mortality table model should satisfy at least three characteristics which are:

- The model should be simple and easy to use.
- The model should adequately reflect the true range of age-specific mortality patterns as observed in the real population.

If a model is used to select a mortality table to represent mortality by age of a population, there should be a close fit between the predicted and the actual mortality rates. The fit can be measured by the squared error in the death rates.

They key motive in the construction of any mortality table is to construct a system that provides mortality rates by age and gender, defined by a small number of parameters that capture the level as well as the age pattern of mortality (Murray, et al., 2003). If a model adequately reflects the reality, then the characteristics of the given population can be summarized by the parameters of the model and thus enhancing the study of variations within a population over time (Murray, et al., 2003).

Brief Overview of Rwanda

Rwanda is a landlocked country that lies in the Eastern Africa region. Its neighbours include Uganda to the north, Tanzania to the east, Burundi to the south and Democratic Republic of Congo to the west (Republic of Rwanda, 2014). According to the results of a recent census done in 2012, the population of the country is approximately 10,515,973 (NISR & MINECOFIN, 2012). From the census, there was also an observed increase in general population growth rates following its decrease in the 1990s.

Rwanda is a third world country (Gillespie, 1989). However, the vision of the country is to develop a stable and sound financial sector that is sufficiently deep and broad, capable of efficiently mobilizing and allocating resources to address the development needs of the economy and reduce poverty (Republic of Rwanda, 2013). One of the methods they intend to pursue to realize this dream is by constructing Rwandan-specific mortality tables that would be beneficial to facilitate growth of life assurance and annuity products. The National Bank of Rwanda and MINECOFIN are tasked with construction of the mortality tables while insurance companies and ASSAR on the other hand would be required to introduce annuity products which are lacking to meet private pension needs.

Insurance penetration in Rwanda is also substantially low at around 1% (African insurance organisation , 2015). In addition to this, life insurance is less undertaken as compared to General Insurance. Even in the greater East African region, only Burundi had lower life insurance premiums in comparison to Rwanda. This low uptake of life insurance products is attributed to lack of creative products and even fundamental life products in the Rwandan market. The figure below indicates the Rwandan insurance market position as compared to its neighbours. The figure is obtained from the African insurance organisation conference (2015).

COUNTRY	LIFE INSURANCE PREMIUM US Dollars – Billion	GENERAL INSURANCE PREMIUM US Dollars – Billion	TOTAL PREMIUM US Dollars – Billion	INSURANCE PENETRATION	SHARE OF REGIONS GROSS WRITTEN PREMIUM	EXCHANGE RATE TO: US$
KENYA	**0.569**	**1.002**	**1.572**	**2.93%**	**74%**	**100**
TANZANIA **(2013 data)**	**0.027**	***0.199***	**0.226**	**0.90%**	**11%**	**2100**
UGANDA	**0.028**	**0.165**	**0.193**	**0.85%**	**9%**	**2612**
RWANDA	**0.010**	**0.100**	**0.110**	**0.60%**	**5%**	**745**
BURUNDI (2013 data)	**0.004**	**0.012**	**0.016**	**0.50%**	**1%**	**1555**
TOTAL	**0.638**	**1.478**	2.116		**100%**	

Figure 2: Insurance Overview East Africa

1.1 Problem Statement

Most developing countries lack mortality tables customized to their own population and Rwanda is no exception. Brass (1971) notes that about 80% of the population of world information about mortality is largely insufficient and the relevant authorities who are mainly concerned with mortality tables of developing countries have to attempt to estimate life tables from little data.

Rwanda recognizes the need for having to come up with Rwanda-specific life mortality tables within the period of 2013-2018 (Republic of Rwanda, 2013). This study therefore aims to suggest a method of coming up with Rwanda-specific mortality tables. Users of mortality tables would at times require the graphical representations of various aspects of the mortality tables. This is because it is easier to use graphs when comparing between any two mortality tables. The research will therefore come up with graphs of the survival and death rates from the obtained mortality tables.

Similarly, mortality tables are fundamental in pricing of life insurance contracts. To a life insurer a mortality table is a mathematical tool used to calculate the monetary values of benefits dependent on life contingencies (Campbell, 1940). Therefore, by calculating the expected outflow from a particular insurance contract, an actuary can easily determine the premiums to be paid for the policy contract. Therefore, the absence of published mortality tables in Rwanda is bound to affect the life insurance business in the country in turn affecting the life insurance penetration as observed in figure 2. Having established the importance of mortality tables in pricing, it is thus fundamental to understand how to specifically convert mortalities to expected payouts thus pricing the life insurance contract and thereby coming up with premium rates for the particular insurance product.

1.2 Research Objectives

The study seeks to come up with a Rwandan-specific mortality table using an appropriate mortality model. In addition to coming up with Rwanda-specific mortality tables, the research also aims to come up with a graphical representation of the generated tables for comparison with other mortality tables.

The research also aims to demonstrate how mortality tables are used in pricing life insurance contracts. In turn, a suggested price will be quoted on Rwanda life insurance policies.

1.3 Research Questions

1. What are the mortality rates for the Rwandan population?
2. What are the recommended premium rates to be applied to the Rwandan population?

2.0 LITERATURE REVIEW

In this section we will review the various mortality table models that have been suggested by other authors.

Coale and Demeny mortality table system

Coale and Demeny mortality tables were first published in the year 1966 from a set of 326 mortality tables for both male and female and from actual populations of people (Coale & Demeny, 1966). From these tables, there were four classic mortality age patterns which were corresponding to the geographical location where the particular population came from. These mortality patterns were mainly characterized by their mortality schedule shapes and were conversely named North, South, East and West.

The model has several limitations to it which include:

- There were strict standards of accuracy used in the construction of the model which limited the number of non-European countries represented and therefore the model may not cover mortality patterns existing in the contemporary developing world (Murray, et al., 2003). This is particularly a problem for this study since we are looking to create a mortality table for the Rwandan population and Rwanda is a developing country.
- One of the parameters of the model is discrete and this limits the flexibility of the model. Later models have been developed where both parameters are continuous.

Ledermann Model

The Ledermann model mortality tables were initially published in 1959 but were then revised over the next decade (Ledermann, 1969). The model is based on a factor analysis of some 157 empirical tables (Murray, et al., 2003). An advantage of the model is that the method of selection of the empirical tables was more flexible than the Coale and Demeny tables and there is more inclusion of less developed countries.

However, the main limitation of the model is its complexity that ideally rules out its application in less developed countries. Some of its suggested input values are in practice not easily obtained and this is especially in developing countries where data is at a minimum.

Another limitation is of the model is that the independent variables used in deriving it refer to parameters obtained from combining information on both female and male observations. This forces the user to accept relationships between female and male mortality embodied in the model despite being significant evidence to the contrary (Murray, et al., 2003).

Therefore due to the stated reservations of the model, the study reviews another improved mortality table model.

Brass-logit mortality table system

The Brass logit system of mortality table model was first proposed by Brass in the year 1971 and is among a category of mortality models known as relational models. It features a standard mortality table and has two parameters[1] α and β which relate any mortality table to the standard (Murray, et al., 2003). The general survivorship function is shown by the mortality standard whereas the parameters illustrate the deviations from the standard.

The Brass model is underlined by the assumption that two distinct age-patterns of mortality can be related to each other by a linear relationship between the logits of their respective survivorship probabilities (Murray, et al., 2003). Therefore, for any two observed survivorship functions l_x and l_x^s where l_x^s is from the standard table and l_x from censored data, it is possible to find α and β whereby:

$$Logit(l_x) = \alpha + \beta Logit(l_x^s) \tag{1}$$

If: $Logit(l_x) = 0.5\ln(\frac{1.0-l_x}{l_x})$ for all ages $x > 0$ (2)

Then for all ages x:

$$0.5\ln\left[\frac{l_0 - l_x}{l_x}\right] = \alpha + 0.5\beta\ln[\frac{l_0 - l_x^s}{l_x^s}] \tag{3}$$

If the above equation holds for every pair of mortality tables, then any mortality table can be generated from a single standard mortality table by changing the values of α and β used (Murray, et al., 2003). In the model α varies the standard mortality level while β varies the slope of the standard. Therefore, if the overall mortality of the sample is

[1] It has two parameters only when a single global standard mortality table is used.

lower than that of the standard table, then α would be a negative value and if there is heavier adult mortality to child mortality then β would be greater than one.

If one then wishes to develop a model mortality table for a population from a standard table, the parameters α and β then have to be estimated for the said population. It is estimated that the parameters α and β in a relational model life table relative to an appropriately-selected standard reproduce exactly the observed values of under-five mortality and observed index of adult mortality[2] (Timaeus, 2013). It is also assumed that the two probabilities[3] apply at the same point in time.

In order to ensure easier fitting of the model, α is expressed as a function of β and the respective observed mortality index of both children and adults.

For:

$$ {}_5q_0: \alpha = logit(l_5) - \beta logit(l_5^s) \tag{4}$$

$$ {}_nq_{15}: \alpha = 0.5\ln\{\frac{\lambda}{(1-\lambda)\exp(\beta c_{15+n}) - \exp(\beta c_{15})}\} \tag{5}$$

Where: $\lambda = {}_{45}q_{15}, c_{15} = logit(l_{15}^s),\ c_{15+n} = logit(l_{15+n}^s)$

This study therefore seeks to use this strategy due to its fairly parsimonious nature and due to the fact that data is available for the study.

According to Murray and others (2000) the advantages of the model include:

- The model has a higher degree of flexibility as compared to the other empirical models[4].
- The model is not dependent on any fixed empirical data and allows the choice of a locally-applicable standard mortality table.
- Using this model, one is able to generate a wide range of reasonably accurate mortality schedules.

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[2] Common examples of observed index of adult mortality are ${}_{45}q_{15}$ or ${}_{35}q_{15}$.

[3] ${}_5q_0$ and (${}_{45}q_{15}$ or ${}_{35}q_{15}$)

[4] The other empirical models include: Coale and Demeny regional model mortality tables and the Ledermann's system of model life tables.

The main disadvantage of the Brass-logit model as (Murray, et al., 2003) notes is that the logit transformation does not entirely linearize the relationship between many survivorship functions.

3.0 METHODOLOGY

This section describes the method undertaken in the research.

3.1 Research Design

This study is seeking to come up with a life mortality table for the Rwandese population. It is a quantitative, descriptive study making use of the Brass-logit model to come up with a Rwanda-specific mortality table and then expressing the mortalities graphically. An additional objective to be met is to be able to adequately price a life insurance policy using the developed mortality tables.

3.2 Data Set Used

The data used in the research is secondary data. An estimate of the Rwandese child mortality (${}_5q_0$) and adult mortality dependent on survival to adulthood (${}_{35}q_{15}$) would be required. This data would be obtained from the World Development Indicators: Mortality (The World Bank, 2016). The data is developed by the UN Interagency Group for Child Mortality Estimation in association with United Nations Population Division's World Population Prospects and was last updated on the 14th of July 2016 (The World Bank, 2016).

3.3 Brass-Logit model

The model used in this research study will be the Brass-Logit model which has two parameters α andβ. The general format of the Brass-Logit model to fit in the mortality of a particular age x is:

$$l(x) = \frac{1}{1 + \exp(2(\alpha + \beta logit(l_x^s)))} \tag{6}$$

In order to fit this model to the respective age groups, α andβ have to be estimated.

Estimating α and β

Due to the fact that the Brass-Logit model is a two parameter model, the parameters can be expressed as a function of each other and one of the two measures of mortality[5]. For under-five mortality, α would then be calculated by:

$$\alpha = logit(l_5) - \beta logit(l_5^s) \tag{7}$$

[5] Under-five and adult mortality.

An important assumption is then made that the adult mortality is a conditional probability of a fifteen year old dying before age 15 + n which is denoted by $_nq_{15}$ and calculated by the formula:

$$1-\frac{l_{15+n}}{l_{15}}=1-\frac{1+\exp(2(\alpha+\beta logit(l_{15}^{s})))}{1+\exp(2\left(\alpha+\beta logit(l_{15+n}^{s})\right))} \quad (8)$$

The value for alpha in the above equation is then substituted with the previously calculated alpha in turn leading to the equation:

$$1-\frac{l_{15+n}}{l_{15}}=1-\frac{1+\exp(2(logit(5)+\beta(logit(l_{15}^{s})-logit(l_{5}^{s}))))}{1+\exp(2(logit(5)+\beta(logit(l_{15+n}^{s})-logit(l_{5}^{s}))))} \quad (9)$$

The value of logit (5) is already known as it is tabulated in the standard table and thus it is possible to solve for β by the use of solver in Excel. This is achieved by reproducing the observed value of $_nq_{15}$ with β. The ideal β should fall within the range of 0.8 to 1.25 (Timaeus, 2013).

3.3.1 Selecting an Appropriate Standard Mortality Table

It is important that the right choice of standard mortality table is made when aiming to create a mortality table for a given population. An ideal standard table is one that aims to describe well a relative balance between child mortality and adult mortality of a country (Moultrie & Timaeus, 2013).

Various standard tables have already been created that match the population characteristics of specific locations and especially in the developed world. Moultrie and Timaeus (2013) suggest that in the absence of adequate data to determine the most suitable family of model tables, one may select the same family as that employed for a neighbouring country with a similar cultural and socio-economic characteristics. This study will then use a standard mortality table from Kenya. The reason for this is because Kenya is located in the same region as Rwanda.

It should be noted that The KE 2001-2003 Tables for Assured Lives that are published will be used as the standard table. The KE 2001-2003 tables contain information on the mortalities of assured lives and it can be used to model the benefits of life assurance products. The reason why insurance lives are preferred on this research is so that they can be used to mirror the potential insured lives in the Rwandan market so as to be able to price a life assurance product using the generated Rwanda-specific mortality tables.

This would aid with the low life assurance penetration in the country. The insurance Act in Kenya was amended in 2011 to adopt the KE 2001-2003 mortality tables that are now used to calculate the liability under life assurance policies (AKI, 2015). However, a limitation of using assured lives table as a standard table is such that in the assured lives table, the mortality from the ages of 0-20 are not available and thus would not be extrapolated even in the generated Rwandan mortality table.

3.4 Deriving the Mortality Table

By solving for β, the value of α can then be defined by substituting it to the equation $\alpha = logit(l_5) - \beta logit(l_5^s)$. Through use of the two parameters and a selected standard mortality table, the values of l_x can then be calculated by use of the formula:

$$l(x) = \frac{1}{1 + \exp(2(\alpha + \beta logit(l_x^s)))} \quad (10)$$

This can be done in an Excel spreadsheet and thus further emphasizing on the simplicity of the model. Once the values of $l(x)$ are calculated, the other mortality table functions like probability of survival and death can then be easily calculated by the respective formulas:

$$probability\ of\ survival\ (year\ x) = \frac{l_x - l_{x+1}}{l_x} \quad (11)$$

$$probability\ of\ death\ (year\ x) = 1 - probability\ of\ survival \quad (12)$$

After coming up with mortality tables, the next step is to express various aspects of the table graphically.

3.5 Pricing Life Insurance Contract Using Mortality Tables

In order to determine the premium rates that are to be charged in Rwanda, a life assurance pricing model is developed. The robustness of this model would first be tested in the Kenyan market where it would be used to price known life assurance products. The desired result of this step is to prove that the price obtained from pricing using a mortality table is around similar to the actual prices of the products in the market. Under this study, an endowment assurance product will be priced. For an endowment assurance, the insurer pays out the sum insured in case the policyholder dies within the policy term or the contract lapses. Payment is made when any of the perilous events occurs first. An endowment policy combines both protection and investments (AKI, 2014). It is due to these qualities of combining protection and investments that an endowment is chosen as a sample life assurance product to be priced since it can have different uses for different consumers.

A model is then created. In order to obtain the annual premiums, the key metrics to be calculated are the expected present value of benefits factor and present value of premiums factor.

3.5.1 Assumptions in the model

- Premiums are payable annually in advance.
- The death and survival benefits are equal at 1,000 Kenya shillings.
- Benefits are payable at the end of year of death
- Term of policy is 15 years
- Age at entry of policyholder ranges from 20-60 years
- Interest rate used is 4%
- Mortalities follow KE 2001-2003 mortality tables
- Initial commission expense of 25% of premium followed by renewal commission expense of 10% of the premium throughout.
- Initial management expense of 15% of premium followed by subsequent management expenses on renewal of 7% of premium throughout the term of the contract.
- Premiums are level.

3.5.2 Pricing model

Pricing was done via Excel. The pricing method to be used is by equating the premiums to the total expenses. The first step is to import the KE 2001-2003 mortality tables to be used for pricing to the spreadsheet. The KE 2001-2003 tables only have the mortality rates q_x. Calculations are then done on q_x to fill the table below for all the ages.

t	x	q_x	p_x	tpx	$k\|qx$
0				1.00	
1	35	0.00200	0.99800	0.99800	0.00200
2	36	0.00219	0.99781	0.99582	0.00219
3	37	0.00242	0.99758	0.99341	0.00241

Table 1: Mortalities pricing table

T symbolizes the time period and is measured discretely where 1 unit represents one year. X represents the age of policyholder. The very first x would be the age of the policyholder on entry into the contract. q_x Column represents the death rates as obtained from the mortality tables. p_x Column represents the probability of survival in the year and it is calculated using the relationship in equation 12. tpx Is the cumulative probability of survival from one year to the next and it begins at a value of 1 when $t = 0$. It is obtained by multiplying the tpx of the previous year to the current year's p_x. $k|qx$ Represents the probability of living for $x + k$ years and then dying the following year. It is calculated by multiplying the tpx of the previous year to the current year's q_x.

After filling out the values in table 1, other columns are then generated to help determine the present values of benefits and premiums as shown in table 2 below.

Time	Age	Discounting factor	PV premiums factor	PV benefits factor
t	x	$v^\wedge t$	$(v^\wedge t) * (tpx)$	$(v^\wedge(k+1)) * (k\|qx)$
0	35	1.00	1.00	
1	36	0.96	0.96	0.001918
2	37	0.92	0.92	0.002024
3	38	0.89	0.88	0.002139

Table 2: Present value calculations

The discounting factor uses the assumed interest rate as listed and it is important since it will be used in the determining the present values of premiums and benefits. The present value of premiums factor is discounted to only t since premiums are paid at the beginning of the year and accounts for the probability of survival since one is required to be alive to pay the next premium. The expected present value of benefits factor is discounted to power $k + 1$ as death benefits are payable in the end of the year. The mean present value (MPV) of premiums and benefits factor are then respectively obtained by summing the values of the respective present values column. Under the MPV of benefits factor, an additional component is added to reflect the survival benefit in case an insured life survived to the end of the contract. This component can be referred to as the pure endowment component which is calculated by the formula: $v^n nP_x$. This is simply discounting the probability of survival to the end of the contract to present time.

Annual premium value (P) is then obtained as shown in equation 13.

$$P = \frac{Sum\ assured \times MPV\ of\ benefits\ factor}{\left((1 - renewal\ expense) \times MPV\ premiums\right) - Initial\ expenses} \tag{13}$$

4.0 DATA ANALYSIS AND DISCUSSION

Under this section, the secondary data obtained from the United Nations World Development Indicators Database will be analyzed before and after it is run through the model in order to justify the later results from the model. Additionally, premium rates obtained from the pricing model will be explained.

4.1 Preliminary Data Analysis

Under this stage a broad comparison is to be made between the statistics obtained about the Rwandan population and those from the chosen standard country, Kenya. In order to come up with Rwanda-specific mortality tables using the Brass-Logit model, the under-five mortality and adult mortality would be needed. These respective statistics are considered as the mortality indicators for a particular country. For the case of Rwanda, the mortality indicators are summarized in figure 3 below: The figures represent the number of deaths for a particular age group per a thousand people.

		Rwanda		
Under-5 mortality			**Adult Mortality**	
Male	Female		Male	Female
45	38		296	178

Figure 3: Rwanda mortality indicators

From an initial glance, there is a notable difference between the mortalities of the males and their counterpart females of the same age group. In both the under-five and adult mortality categories, there tends to be more male deaths than female ones. However, this is expected since in all developed countries and most developing ones, women tend to outlive men (Perls & Fretts, 1998).

The mortality indicators for the Kenya are then obtained next and are shown in figure 4 below: Similarly, the lives are expressed per a thousand people.

		Kenya			
Under-5 mortality			**Adult Mortality**		
Male	Female		Male	Female	l15
53	45		296	251	937

Figure 4: Kenya mortality indicators

It is important to note that an additional indicator is included in the Kenyan mortality indicator which is $l15$. This simply implies the lives alive at age 15 and this statistic will be crucial in future calculations.

Comparing between figure 3 and 4, several similarities can be seen. There exists a trend in both countries such that the males have higher mortality rates than females. Also, comparing within the under-five mortality, it is clear that the Kenyan population has higher mortality rates than the Rwandan one but the numerical value of the difference remains fairly constant between the males and the females. On adult mortality, the mortality rates are same for the males but differ when it comes to females as the Kenyan female population has a higher mortality.

4.2 Model Implementation

In order to generate the Rwanda-specific complete mortality tables, the Brass-Logit model is implemented. First, the logit values of the mortality indicators of the various countries are obtained using equation 2. After the logit values have been found, the next step is to look for the value of β by use of a solver function with the aid of equation 9. This would in turn give a value for α. This step is repeated for both the males and females to obtain different values for each. The results from the solver runs are as illustrated in the figures below.

Female	
Alpha	2.17816
Beta	2.483675
nq15 (calc)	0.822001
nq15 (observ.)	0.822

Figure 5: Female α and β

Male	
Alpha	1.341958
Beta	1.990618
nq15 (calc)	0.704001
nq15 (observ.)	0.704

Figure 6: Male α and β

After determining the values for α and β , the next step is to now generate the entire mortality table. This would be done with the aid of equation 10.

4.3 Findings

The direct output from the model is in form of projected lives lx. It should be noted that lx obtained from the model are in decimals since one can only obtain the logit function of a decimal. The decimal value represents lx in 100,000's. The line chart below illustrates the projected lives in comparison to the Kenyan lives and also the brass general standard table as obtained from (Murray, Ahmad, Lopez, & Salomon, 2000).

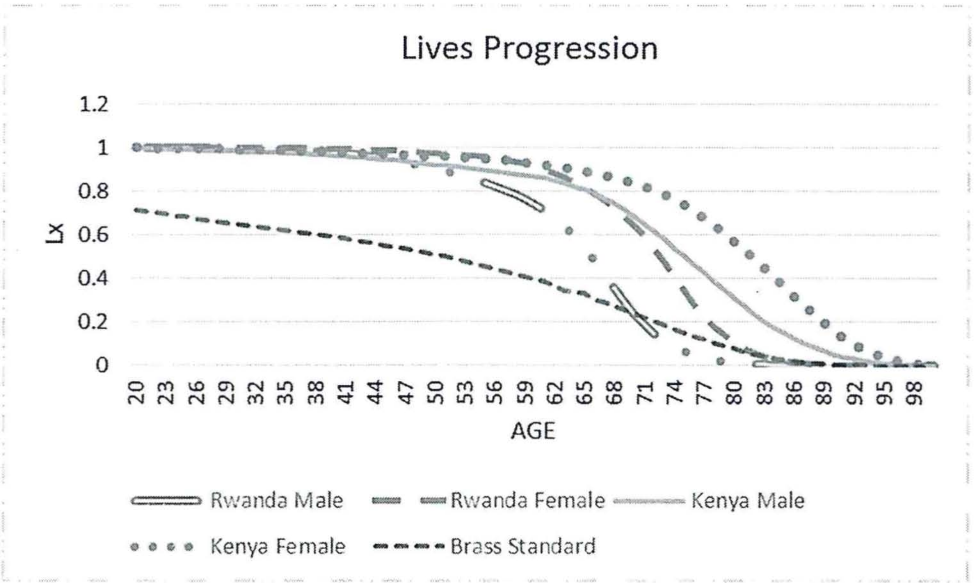

Figure 7: Lives progression

The lives progression table displays how lives decrease over the ages as they tend to 0. From these l_x statistics, the mortality rates q_x are obtained by equation 11 and thus generating a mortality table as displayed on figure 1. These generated tables can be found in the appendix.

Similarly, the premium rates of the 15-year endowment assurance contract was obtained for the various ages using the KE2001-2003 male tables. The full premium rates table for all the ages can be found at appendix 4. For the Kenyan mortality, only the male premium rates have been calculated since the comparing industry prices were all quoted to the masculine gender. The Kenyan premium rates in this research are compared to the industry prices to check for robustness of the model and thus there is no need to come up with female premium rates since data could not be obtained of the particular 15-year female endowment contract.

4.4 Discussion

With the aid of a brass-logit model, a Rwandan specific mortality table has been able to be developed. It should be noted that the developed rates are already smoothed and thus there is no need for graduation. The key reason for the smoothed curve is because the standard table used, in this case the KE 2001-2003 is already a smoothed table. Smoothened rates are vital since they help in computation of financial quantities like premiums for life insurance (Seal, 1941). This is also desired so that there are no abrupt changes in the prices of same insurance policies across adjacent ages.

The developed progression of lives are displayed in figure 7 in comparison to both the Kenyan lives and the standard logit table. Due to this, several comparisons can be made with relation to both the Kenyan tables and the standard table. Generally, it is observed that the male lives decrease faster than their female counterparts in both Kenya and Rwanda consistently for all the ages. This confirms the findings of the preliminary analysis where male lives were deemed to have heavier mortality rates.

In comparing between Rwanda and Kenya lives progression, they all tend to begin at the same level and on par with each other. However, as the ages increase, it becomes evident that Rwandan lives tend to decrease faster than their Kenyan counterparts in both the males and the females. For the males, the lives progression gap starts showing at around age 50 whereas for the females this gap starts forming at around age 60.

Unlike the Kenyan and Rwandan tables, the Brass standard table starts at a lower number of lives. The reason for this is that the Brass standard tables begin at age 0 and thus by age 20, several lives would have decreased. The standard table that was used in this research the KE 2001-2003 only contains assured lives and there is no data for lives aged less than 20 since they do not partake in insurance. This would in turn inform the assumption that under the standard tables, the first life dies between age 20 and 21. It is due to this that lx begins at 1 on age 20. This assumption would in turn affect the graduated tables to feature the same characteristics of lx being 1 at age 20.

A similarity that can be found among all the graphs is that there tends to be a starting period where the lives decrease at a slow rate, then there is another phase where the lives tend to decrease at a faster rate while approaching 0. This can be graphically observed from figure 7. For the Brass standard, this change occurs at around age 56 while for the Rwandan population, this change occurs at ages 53 and 65 for the males and females respectively. As for the Kenyan population, this change occurs at ages 65 and 75 for the males and females respectively.

In order to prove the validity of mortality tables in pricing the endowment insurance, a suitable endowment insurance is obtained from the market in order to compare its premium prices to those calculated by the mortality tables in Excel. The select endowment insurance used is the 15-year Endowment Plan 101 by Britam Insurance. It is to be noted that the product itself is not offered in isolation but is used to be able to price more complex endowment insurance offered like the "Elimu Bora Education Plan" (Mureithi, 2016). A justification for choosing Britam Insurance to price a Kenyan product is that they are the market leaders in life assurances in the Kenyan market with a market share of 38.95% (IRA, 2016). Due to this significant market share, Britam should be enjoying economies of scale (Buzzell, Gale, & Sultan, 1975) and therefore their prices are more likely to feature less profit loadings which would be adequate for the research. The table below shows the comparison between the calculated premium rates and the premium rates used by Britam Life Assurance. The data for the Britam Premium rates was obtained from (Mureithi, 2016).

Age	Britam Premium Rates per mille (males)	Calculated Premium Rates (males)
30	58.94	60.63
35	61.01	61.45
40	62.39	62.07

Table 3Premium Rates Comparison

It is clear that the calculated premium rates are not exactly the same as those used by Britam. However, the values are very highly correlated as they have a correlation figure of 0.993. This indicates that the premium rates very much exhibit nearly the same characteristics and thus the model can be used as a reliable model in pricing.

This model is then used to price suggested price quotes for the same product using the Rwandan mortality as developed by the Brass-logit model. The assumptions used remain unchanged. The resulting suggested premium rates can be obtained in appendix 5 of this research.

5.0 CONCLUSION, RECOMMENDATIONS AND LIMITATIONS

5.1 Conclusion

The research aims to come up mortality tables for the Rwandan population by using the Brass-logit model. In order to do this, an appropriate standard table is chosen. The standard table chosen is the published Kenyan mortality table KE2001-2003. Rwandan mortality tables are then generated using this model.

In addition to this, another model is created to price a life assurance product. The robustness of the model is tested by using it to price a life contract in the Kenyan market using the known KE2001-2003 mortality tables. The product chosen is the 15-year endowment product. The prices generated are compared to those in the market and they appear consistent thus proving that the model is accurate. This same model is then used to price a 15-year endowment product using the developed Rwanda mortality tables. The Rwandan premium rates are obtained for both genders and across multiple ages and are thus presented as a suggestion of the level of premiums that Rwandans can pay for the same contract.

5.2 Recommendations

A recommendation that can be made from this study is that The National Bank and Rwanda and MINECOFIN can use the Brass-logit model to be able to come up with Rwandan mortality tables as demonstrated in this study. Alternatively, they could also seek to develop their own tables from their own insurance data like how Kenya developed theirs.

Also, there is need for further research to be done by other African countries to develop their own unique mortality tables. Even those African countries that have already developed their own tables (Kenya and South Africa) need to periodically adjust their tables to reflect the changing mortality situations that change over the years. This would lead to more accurate life insurance products which could in turn lead to an increase in insurance penetration in the continent.

Another recommendation to be made from this study is on the prices of endowment contracts to the population of Rwanda. The methodology used to price the product in this study can be adopted in the country as well and be able to gauge the actual prices of the contracts.

5.3 Limitations of the Study

One of the limitations of the project is brought about by the choice of standard table. The standard table used is the KE 2001-2003 which is obtained from insured lives. Therefore, there are no values of mortalities between the ages of 0-20 years. This gap is transpired to the generated tables as the generated tables will also lack mortality estimates between the ages of 0-20. Additionally, the standard table used might not accurately reflect the current situation of mortalities since circumstances might have changed from the period of 2001-2003 to 2016.

Another limitation is the lack of more data sets on price quotes of similar 15-year endowments as used in the project. This limits the level of analysis that would be undertaken to determine the appropriateness of using mortality tables in pricing of life assurance

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APPENDIX

Appendix 1: Rwandan Female Mortality Table as generated by Brass-Logit

Female										
Age	l_x	q_x		*Age*	l_x	q_x		*Age*	l_x	q_x
20	1	2.28E-06		*47*	0.982967	0.002321		*74*	0.408566	0.159508
21	0.999998	1.37E-05		*48*	0.980685	0.002547		*75*	0.343396	0.178244
22	0.999984	3.97E-05		*49*	0.978187	0.0028		*76*	0.282188	0.195031
23	0.999944	7.6E-05		*50*	0.975449	0.003097		*77*	0.227153	0.209886
24	0.999868	9.39E-05		*51*	0.972428	0.003417		*78*	0.179476	0.223146
25	0.999774	5.11E-05		*52*	0.969104	0.003823		*79*	0.139427	0.235381
26	0.999723	8.28E-05		*53*	0.965399	0.004297		*80*	0.106608	0.246944
27	0.999641	0.000117		*54*	0.961251	0.00485		*81*	0.080282	0.25837
28	0.999523	0.000156		*55*	0.956588	0.005993		*82*	0.05954	0.269966
29	0.999367	0.000189		*56*	0.950856	0.007139		*83*	0.043466	0.282299
30	0.999178	0.000219		*57*	0.944067	0.008585		*84*	0.031196	0.295573
31	0.998959	0.000254		*58*	0.935963	0.010365		*85*	0.021975	0.310041
32	0.998706	0.000298		*59*	0.926261	0.012989		*86*	0.015162	0.326227
33	0.998408	0.000357		*60*	0.91423	0.015681		*87*	0.010216	0.344142
34	0.998052	0.000428		*61*	0.899895	0.018718		*88*	0.0067	0.364337
35	0.997624	0.000506		*62*	0.88305	0.021487		*89*	0.004259	0.386925
36	0.997119	0.000607		*63*	0.864076	0.025112		*90*	0.002611	0.412327
37	0.996514	0.000708		*64*	0.842377	0.029298		*91*	0.001534	0.436579
38	0.995808	0.000837		*65*	0.817698	0.034283		*92*	0.000865	0.46379
39	0.994975	0.000965		*66*	0.789665	0.03966		*93*	0.000464	0.493849
40	0.994015	0.001112		*67*	0.758347	0.045685		*94*	0.000235	0.526801

41	0.99291	0.001269		***68***	0.723702	0.053576		***95***	0.000111	0.562453
42	0.99165	0.001432		***69***	0.684929	0.06368		***96***	4.86E-05	0.600874
43	0.99023	0.001583		***70***	0.641312	0.075954		***97***	1.94E-05	0.641212
44	0.988663	0.001752		***71***	0.592602	0.09373		***98***	6.96E-06	0.682443
45	0.986931	0.001921		***72***	0.537057	0.116642		***99***	2.21E-06	0.725671
46	0.985035	0.0021		***73***	0.474414	0.138798		***100***	6.06E-07	1

Table 4: Female Rwandan mortality table

Appendix 2: Rwandan Male Mortality Table as generated by Brass-Logit model

Male								
Age	l_x	q_x	*Age*	l_x	q_x	*Age*	l_x	q_x
20	1	8.89E-05	***47***	0.934992	0.009992	***74***	0.086581	0.243816
21	0.999911	0.000241	***48***	0.92565	0.010974	***75***	0.065471	0.251381
22	0.99967	0.000361	***49***	0.915491	0.011914	***76***	0.049013	0.25863
23	0.999309	0.000455	***50***	0.904584	0.012774	***77***	0.036337	0.266548
24	0.998855	0.000528	***51***	0.89303	0.013821	***78***	0.026651	0.27685
25	0.998328	0.000586	***52***	0.880687	0.014817	***79***	0.019273	0.288317
26	0.997742	0.000636	***53***	0.867638	0.01595	***80***	0.013716	0.294435
27	0.997108	0.000682	***54***	0.8538	0.017144	***81***	0.009678	0.300524
28	0.996427	0.000731	***55***	0.839162	0.018727	***82***	0.006769	0.310762
29	0.995699	0.000788	***56***	0.823447	0.020818	***83***	0.004666	0.317018
30	0.994915	0.000859	***57***	0.806304	0.023446	***84***	0.003187	0.327422
31	0.99406	0.000952	***58***	0.787399	0.026778	***85***	0.002143	0.337979
32	0.993114	0.001072	***59***	0.766315	0.031943	***86***	0.001419	0.345293
33	0.992049	0.001229	***60***	0.741836	0.038831	***87***	0.000929	0.360009
34	0.990829	0.001432	***61***	0.71303	0.047665	***88***	0.000595	0.376486
35	0.98941	0.001687	***62***	0.679043	0.058752	***89***	0.000371	0.399479
36	0.987741	0.001999	***63***	0.639148	0.072327	***90***	0.000223	0.423746
37	0.985766	0.002376	***64***	0.592921	0.088477	***91***	0.000128	0.449259
38	0.983424	0.00282	***65***	0.540461	0.107095	***92***	7.06E-05	0.480784
39	0.980651	0.003339	***66***	0.48258	0.127794	***93***	3.67E-05	0.517783
40	0.977377	0.003934	***67***	0.42091	0.149918	***94***	1.77E-05	0.556911
41	0.973531	0.004609	***68***	0.357808	0.172571	***95***	7.84E-06	0.597906

42	0.969044	0.005361		***69***	0.29606	0.194631		***96***	3.15E-06	0.640395
43	0.963849	0.006185		***70***	0.238438	0.207632		***97***	1.13E-06	0.683956
44	0.957888	0.007076		***71***	0.188931	0.21758		***98***	3.58E-07	0.727955
45	0.95111	0.008019		***72***	0.147823	0.229888		***99***	9.74E-08	0.771753
46	0.943484	0.009		***73***	0.11384	0.239456		***100***	2.22E-08	1

Table 5: Male Rwandan mortality table

Appendix 3: The Brass General Standard Table

Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value
0	1.000													
1	0.850	-0.867	21	0.707	-0.440	41	0.583	-0.167	61	0.383	0.2394	81	0.065	1.330
2	0.807	-0.715	22	0.700	-0.425	42	0.576	-0.153	62	0.368	0.2701	82	0.054	1.428
3	0.788	-0.655	23	0.694	-0.410	43	0.569	-0.138	63	0.345	0.3204	83	0.044	1.535
4	0.776	-0.622	24	0.688	-0.396	44	0.561	-0.123	64	0.338	0.3364	84	0.036	1.649
5	0.769	-0.602	25	0.683	-0.383	45	0.553	-0.107	65	0.322	0.3721	85	0.028	1.772
6	0.764	-0.588	26	0.676	-0.369	46	0.545	-0.091	66	0.306	0.4097	86	0.022	1.905
7	0.760	-0.577	27	0.670	-0.355	47	0.537	-0.075	67	0.289	0.4494	87	0.016	2.049
8	0.756	-0.567	28	0.664	-0.341	48	0.529	-0.057	68	0.272	0.4912	88	0.012	2.205
9	0.753	-0.558	29	0.658	-0.328	49	0.520	-0.040	69	0.255	0.5353	89	0.009	2.374
10	0.750	-0.550	30	0.652	-0.315	50	0.511	-0.021	70	0.238	0.5818	90	0.006	2.557
11	0.748	-0.543	31	0.647	-0.302	51	0.501	-0.002	71	0.221	0.6311	91	0.004	2.756
12	0.745	-0.537	32	0.641	-0.289	52	0.491	0.018	72	0.203	0.6832	92	0.003	2.973
13	0.743	-0.530	33	0.635	-0.276	53	0.481	0.038	73	0.186	0.7385	93	0.002	3.208
14	0.740	-0.522	34	0.628	-0.263	54	0.470	0.060	74	0.169	0.7971	94	0.001	3.464
15	0.736	-0.513	35	0.622	-0.250	55	0.459	0.082	75	0.152	0.8593	95	0.001	3.742
16	0.733	-0.504	36	0.616	-0.236	56	0.447	0.106	76	0.136	0.9255	96	0.000	4.046
17	0.729	-0.494	37	0.610	-0.223	57	0.435	0.130	77	0.120	0.9960	97	0.000	4.376
18	0.724	-0.482	38	0.603	-0.209	58	0.423	0.155	78	0.105	1.0712	98	0.000	4.735
19	0.719	-0.469	39	0.597	-0.196	59	0.410	0.182	79	0.091	1.1516	99	0.000	5.127
20	0.713	-0.455	40	0.590	-0.182	60	0.397	0.210	80	0.078	1.2375			

Source: (Murray, Ahmad, Lopez, & Salomon, 2000).

Appendix 4: Calculated Premium Rates for 15-year Endowment Using Kenyan Mortality

Age	Calculated Premium Rates (Males)
21	60.5
22	60.45
23	60.41
24	60.39
25	60.38
26	60.39
27	60.42
28	60.47
29	60.54
30	60.63
31	60.73
32	60.85
33	60.99
34	61.13
35	61.45
36	61.53
37	61.61
38	61.77
39	61.92
40	62.07
41	62.22
42	62.35
43	62.48
44	62.6
45	62.72
46	62.84
47	62.98
48	63.14
49	63.34
50	63.6
51	63.95
52	64.39
53	64.97
54	65.69
55	66.61
56	67.75
57	69.13
58	70.77
59	72.72
60	74.97

Table 6: Kenyan calculated Premium Rates

Appendix 5: Calculated Premium Rates for 15-year Endowment Using Rwandan Mortality Generated with Brass-Logit Model.

Age	Suggested Premium Rates for Rwanda (Males) per mille	Suggested Premium Rates for Rwanda (Females) per mille
21	59.66	59.35
22	59.72	59.36
23	59.77	59.38
24	59.83	59.4
25	59.89	59.42
26	59.97	59.45
27	60.06	59.49
28	60.17	59.52
29	60.31	59.57
30	60.48	59.62
31	60.67	59.67
32	60.91	59.74
33	61.18	59.81
34	61.5	59.89
35	61.87	59.98
36	62.27	60.07
37	62.72	60.17
38	63.22	60.28
39	63.75	60.4
40	64.33	60.53
41	64.94	60.67
42	65.58	60.82
43	66.26	60.99
44	66.98	61.12
45	67.74	61.43
46	68.57	61.73
47	69.49	62.09
48	70.55	62.52
49	71.8	63.06
50	73.34	63.71
51	75.25	64.51
52	77.63	65.47
53	80.6	66.62
54	84.29	68.01
55	88.85	69.67
56	94.41	71.61
57	101.09	73.89
58	109.03	76.6
59	118.42	79.82
60	129.31	83.58

Table 7: Suggested Rwanda Premium Rates