Strathmore
UNIVERSITY

SCHOOL OF COMPUTING & ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN CYBER NETWORK & SECURITY
END OF SEMESTER EXAMINATION

ICS 1104: DISCRETE MATHEMATICS

DATE: 26th July 2024 Time: 2 hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE: (30 MARKS)

a) Define the following terms as used in logic theory

i) A proposition (1 marks)

ii) A simple statement (1 marks)

iii) A truth table (1 marks)

b) Consider the set $A = \{1,2,3,4\}$.

i. How many relations are on the set A.? (2 marks)

ii. Find the Power set of A. (2 marks)

c) Seven Women and nine Men are on a faculty in the mathematics department at a school. How many ways are there to select a committee of five members of the department if at least one woman must be in the committee? (4 marks)

d) Prove by induction that $7 + 10 + 13 + 16 + \ldots\ldots\ldots\ldots + (3n+4) = \frac{n}{2}(3n+11)$ (6 marks)

e) Let $A = \{4,5,6\}$, $B = \{a,b,c\}$.let the R be a relation defined by $R = \{(4,b),(5,a),(6,c)\}$.

Find the domain, range, inverse of R and complement of R (4 marks)

f) i. Find the first five terms of the sequence defined by $a_n = na_{n-1} + n^2 a_{n-2}$, $a_0 = 1, a_1 = 1$

(3 marks)

ii. Solve the recurrence relations with Solve the recurrence relations with

$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4, a_2 = 8$

(6 marks)

QUESTION TWO: (20 MARKS)

a) Clearly state the difference between a relation and a function (2 marks)

b) Let $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find M_{S0R} (4 marks)

c) Let $A = \{1,2,3\}$, $B = \{a,b,c\}$ and $C = \{x,y,z\}$. Consider the following relation R and S.
$R = \{(1,a),(2,b),(2,c),(3,b)\}$ and $S = \{(a,y),(b,z),(c,x),(c,y)\}$

i. Find the composition relation RoS (3 marks)

ii. Find the elements of R^c , S^c and $(RoS)^c$ (3 marks)

d) Construct a relation on the set $\{a,b,c,d\}$ with minimum number of elements that is

i. Reflexive, symmetric and transitive. (2 marks)

ii Neither reflexive, symmetric, ant symmetric, nor transitive. (2 marks)

e) Let $A = \{1,2,3,4,6,8,12\}$ and $R = \{(a,b) : a \le b\}$ Is

i. List the elements of R (2 marks)

ii. Is R a partial ordering? Justify your answer

(2 marks)

QUESTION THREE: (20 MARKS)

a) Let x be a any real number, use direct proof to show that $x^2 - 6x + 15 \neq 0$ (4 marks)

b) Find the first five terms of the sequence $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $a_0 = 2$, $a_1 = 5$ $a_2 = 15$

(5 marks)

c) Show that the sequence $a_n = 3 \times 2^n - 2 \times 3^n$ is a solution to the recurrence relations $a_n = 5a_{n-1} - 6a_{n-2}$. (6 marks)

d) Solve the recurrence relations with $a_n = 6a_{n-1} - 8a_{n-2}$ with $a_0 = 4, a_1 = 10$ (5 marks)

QUESTION FOUR (20 MARKS)

a) let $A = \{1,2,3,4,5\}$.Determine the truth value of each of the following statements

i. $(\exists x \in A)(x + 3 = 10)$

ii. $(\exists x \in A)(x + 3 < 5)$

iii $(\forall x \in A)(x + 3 < 10)$

iv $(\forall x \in A)(x + 3 \leq 7)$ (4 marks)

b) Show that $\neg[p \vee (\neg p \wedge q)]$ *and* $\neg p \wedge \neg q$ are logically equivalent. (4 marks)

c) i) Define permutations and combinations of r objects from n objects (2 marks)

ii) The manager of a supermarket has been supplied with 6 products, but there are 4 available display slots for the products. Determine the number of ways the manager can arrange the products given that repetition is not allowed. (3 marks)

d) Find the binomial expansion for $(1+8x)^{1/2}$ up to and including x^3.By substituting 0.01 for x in $(1+8x)^{1/2}$ and its expansion, find $\sqrt{3}$ correct to 4 decimal place. (7 marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms as used in set theory

i. A proper subset (1 marks)

ii. Absolute complement of a set A (1 marks)

iii. Infinite set (1 marks)

b) Use set laws to simplify $(A \cap B) \cup (A - B)$ (5 marks)

c) One hundred Students were asked whether they had subscribed to any of three mobile telephone networks, Airtel, Safaricom and Orange. The results were:35 had Airtel line,38 had Safaricom line ,31 had Orange line ,18 had Airtel and Safaricom,9 had Airtel and Orange ,4 had Orange and Safaricom and 23 were not connected to any of the lines;

i. Determine the number of students who have all the three lines (4 marks)
ii. Draw a Venn diagram to represent the survey (3 marks)
iii. How many students had Airtel and Safaricom but not Orange (1 marks)
iv. How many students had Safaricom or Orange but not Airtel (2 marks)
v. How many students had at most two lines (2 marks)