STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

MASTER OF SCIENCE IN BIOMATHEMATICS

END OF SEMESTER EXAMINATION

BMA 8203: STOCHASTIC MODELLING

Date: 11th December, 2023 Time: 3 Hours

INSTRUCTIONS

Answer QUESTION ONE and ANY OTHER TWO questions

Question One (20 Marks)

(a) Define Brownian Motion and explain its historical significance in the field of physics (3 marks)

(b) Consider the SDE:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t),$$

where $-\infty < \mu < \infty$, $\sigma > 0$. Using Coefficient Matching Method, find the solution of $X(t)$). (6 marks)

(c) Write R code to simulate a Wiener process using the cumsum function to accumulate random normal increments at discrete time steps. (7 marks)

(d) Show that the process $X(t) = B^2(t)$ is not a martingale, where $\{B(t),\, t \geq 0\}$ is a standard brownian process. (4 marks)

Question Two (20 Marks)

(a) How is Ito calculus used in the field of epidemiology, and what role does it play in modeling the spread of infectious diseases? Provide an example of how stochastic modeling with Ito calculus can help epidemiologists make predictions and evaluate disease control strategies. (8 marks)

(b) Find the mean, variance and covariance of the process defined by: $Z(t) = B(t) - tB(t)$, where $\{B(t), t \geq 0\}$ is a standard brownian process. (9 marks)

(c) X is a Poisson distributed random variable with parameter λ. Calculate $\mathbb{E}\left[3^X\right]$ (3 marks)

Question Three (20 Marks)

(a) Find the mean, the variance anf hence the asymptotic distribution of the O-V process:

$$V(t) = Ve^{-\beta t} + \frac{\sigma e^{-\beta t}}{\sqrt{2\beta}} B\left(e^{-\beta t} - 1\right)$$

(8 marks)

(b) Provide R code for a stochastic Gillespie SEIR model. [Please note that you will need to define the specific parameters and initial conditions for your model. (12 marks)

Question Four (20 Marks)

Consider the following Markov chain one-step transition matrix, where some elements are replaced by $*$:

$$P = \begin{pmatrix} 1/2 & 1/2 & ? & ? \\ 1/3 & 1/4 & ? & 1/4 \\ 1/4 & ? & 1/4 & 1/4 \\ ? & ? & 1/3 & 2/3 \end{pmatrix}$$

(a) Explain why the missing elements of P can be determined from the available information, and find them. (4 marks)

(b) Draw the transition diagram and use it to explain why this Markov chain possesses an equilibrium distribution. (4 marks)

(c) Without performing any calculations, describe as best you can, the eigen-values of the matrix P. (4 marks)

(d) Find the equilibrium distribution. (8 marks)