Strathmore UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

MASTER OF SCIENCE IN BIOMATHEMATICS

END OF SEMESTER EXAMINATION

BMA 8101: INTRODUCTION TO MATHEMATICAL COMPUTING

DATE: 13TH DECEMBER 2024 TIME: 3 HOURS

INSTRUCTIONS: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (30 MARKS)

(a) Describe the concept of computational complexity and give three examples of different computational complexities. **[3 marks]**

(b) Outline any two challenges of mathematical computing. **[2 marks]**

(c) Describe the following as used in **MATLAB** and give an example of each.

(i) Switch group. **[2 marks]**

(ii) Conditional operators. **[2 marks]**

(d) The general formula for the rotation of angle ω per residue of any polypeotite helix is given by

$$3.\cos\omega = 1 - 4\cos\left(\frac{\psi+\phi}{2}\right)^2,$$

where ψ and ϕ are the directional angles of the dihedral angle. Write a **MATLAB** script to calculate the rotation angle ω for ψ and ϕ varying for every 5°between $0° \leq \psi, \phi \leq 360°$.

[5 marks]

(e) Consider the following **MATLAB** script and identify the errors and explain how they can be remedied.

```
inits='x(0)=1,y(0)=2,z(0)=3';
[x,z,y]=dsolve('Dx=x+2y-z','Dy=x+z','Dz=4*x-4*y+5z',inits);
t=linspace(0,.5,25);
xx=eval(vectorize(x));
y=eval(vectorize(y));
zz=eval(vectorize(z));
plot(t, xx, tt, yy, t, zz)
```

[3 marks]

(f) Outline the procedure of the least squares fitting and how it is a standard minimization problem. Describe how this is used in model calibration in presence of data. **[4 marks]**

(g) Briefly explain the concept and framework of the single step methods used in the solution of initial value problems and how the single step methods is improved by using the Heun's method. **[4 marks]**

(h) Consider the predator-prey model given by

$$\begin{aligned}\frac{dr}{dt} &= \alpha r - \beta r x,\\ \frac{dx}{dt} &= \delta r x - \gamma x,\end{aligned}$$

where $r(t)$ is the population of rabbits and $x(t)$ is the population of foxes. Given that $r(0) = 5$ and $x(0) = 3$ and the parameters $\alpha = 1.1$, $\beta = 0.4$, $\delta = 0.1$ and $\gamma = 0.1$. Outline how this system of ordinary differential equations can be solve using **MATLAB** for $r(t)$ and $x(t)$ for $0 \leq t \leq 70$ where t is the time duration in days.

[5 marks]

QUESTION TWO (15 MARKS)

(a) Outline the procedure of non-linear least-squares approximation using the data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, including the iterative scheme of Gauss-Newton method that is used in the solution.

[7 marks]

(b) During the monkey pox outbreak in Kenya, the following data was recorded at the Ministry of Health.

Table 1: *Ministry of Health data on Monkey pox cases*

Day	1	2	6
Number of infections	3	5	21

Find the best-fit curve $x = a_0 \exp(a_1 t)$ using the procedure outlined in (a) above. And use it to determine number of cases at day 11.

[8 marks]

QUESTION THREE (15 MARKS)

(a) In a bid to calculate the effect of Greenhouse gases on the Earth's atmosphere, scientists derived a model of the form

$$C\frac{dT}{dt} = (1 - \alpha(T))\, Q - \epsilon\sigma T^4,$$

where $\alpha = 0.3$, $Q = 342 Wm^{-2}$, $C = 91.6 \times 10^6 Wm^{-2}$ and $\epsilon = 0.75$ and $\alpha(T) = 0.5 - 0.2 \tanh\left(\frac{T-265}{10}\right)$. Write a **MATLAB** script that solves for the temperature T and also include how the plot of T versus the time t can be done. Consider time t=365 days. **[5 marks]**

(b) A mathematical model for flu for a county having 1 million people has four population compartments namely, susceptibles, exposed, infected and recovered. Assuming that demographics was consider and there was no acquired immunity. Write a **MATLAB** script that solves the mathematical model considering a period of one year with the initial conditions given as $S = 0.65N, E = 150000, I = 400, R = 0.001N$, choose arbitrary parameters.

[6 marks]

(c) Considering (a), the daily number of the infected persons is recorded in a *.csv* file, using this information, outline a procedure we can use to estimate the parameters using available data of infected persons.

[4 marks]

QUESTION FOUR (15 MARKS)

(a) Define the following terms

(i) Error tolerance as used in **MATLAB** . **[2 marks]**

(ii) A stiff ordinary differential equation. **[1 marks]**

(b) Find the line $y = a_o + a_1x + a_2x^2$ that best fits the data points $(3, 1.9)$,$(4, 4.1)$,$(6, 9.3)$ and $(9, 14.4)$. **[4 marks]**

(c) Consider the following system of ordinary differential equations

$$\begin{aligned} 14x + 3y - 4z + 6s + 1.1t &= 11.41 \\ x + 47z + 0.5s + 2.478t &= 0.41 \\ 1.94x + 34y + 4.11z + 1.345s + 0.1t &= 48.45 \\ 12.3y - 3y + 29.23z + 0.032s + 7.5t &= 0 \\ 3.67x + 32.7y + 3.89z + 12.9s + t &= 2.8 \end{aligned}$$

with the initial conditions given as $x(0) = 4.5, y(0) = 1$,$z(0) = 0$,$s(0) = 3$ and $t(0) = 2$. Using the *dsolve* function, write a **MATLAB** script to obtain the analytical solutions.

[4 marks]

(d) Outline the procedural steps of Monte-Carlo methods.

[4 marks]