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**A DYNAMIC FINANCIAL OPTIMIZATION APPROACH TO
STRUCTURING MORTGAGE BACKED SECURITIES IN KENYA**

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Bachelor of Business Science in Actuarial Science at Strathmore University**

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This Research Project has been submitted for examination with my approval as the Supervisor.

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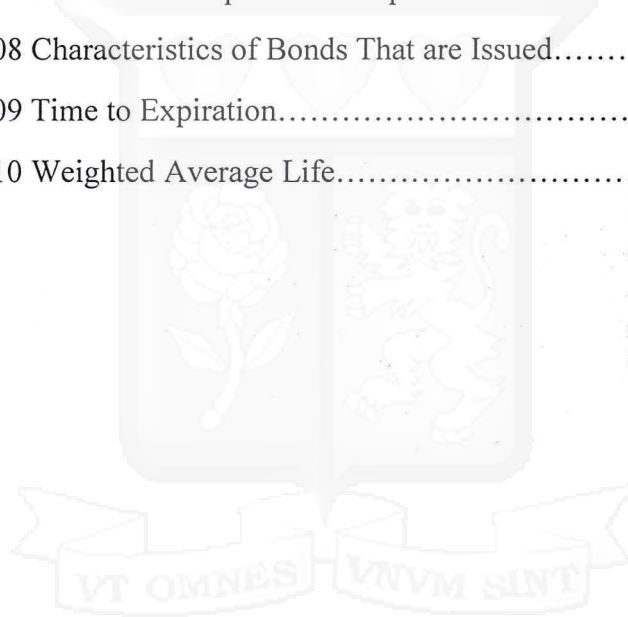
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LIST OF ABBREVIATIONS

CMO	Collateralised Mortgage Obligation
DP	Dynamic Programming
MBS	Mortgage Backed Security
REIC	Real Estate Investment Conduit
WAL	Weighted Average Life



1. Introduction

1.1. Background

A mortgage is a loan secured by the collateral of some specified real estate property that obliges the borrower to make a predetermined series of payments. It gives the lender (mortgagee) the right to foreclose on the loan and seize the property to ensure that the debt is paid off if the borrower (mortgagor) defaults (Pachamanoova & Fabozzi, 2010a). On the other hand, a mortgage-backed security is a debt instrument backed by a pool of mortgage loans. Many mortgages are gathered in a pool, and this pool is then sliced into small pieces for sale to investors as packages, according to Duru (2009).

These securities have developed since the 19th century. This development was possible because the lack of profits confined credit to the margins of American capitalism (Hyman, 2012). Also, usury laws made the lending of cash unprofitable due to high transaction costs. As a result, there was minimal consumer credit investment.

In the 1920s, financial intermediaries emerged to buy debt from shops enabling them to extend credit and boost sales. These intermediaries could, in turn, borrow from financial institutions large sums and pay back with its profits. This facilitated consumer debt, leading to the American economic boom of the 1920s. However, in the face of the depression of 1930, the debt was constrained as banks were not willing to lend in the uncertain environment. As a result, State intervened by introducing the Federal Housing Agency, a financial intermediary that was guaranteed by the government, to buy debt from issuers of credit and sell it to investors. The aim was to rebuild the economy after the crisis. This saw large institutional investors such as insurance companies invest in the securities, with no fear of economic downturns. Given the stability of this consumer debt industry in the 1970s, the government introduced mortgage-backed securities to split the bulk of debt into packages to be sold to many investors. Soon afterwards, different forms of these securities began to emerge, appealing to more investors as well as increasing in complexity (Hyman, 2012).

The success of this industry led to the concept of securitization being extended to other forms of debt, and hence asset-backed securities emerged. As a result of these

mortgage-backed securities, formerly red-lined neighbourhoods that had been denied credit for decades suddenly became awash with loan offers as lenders scoured for the remaining most marginal borrowers (Hyman, 2012).

The MBS market was one of the fastest growing towards the end of the twentieth century (Boudoukh, Richardson, Stanton, & Whitelaw, 1998). This is supported by the fact that in 1993, the outstanding face value of the securities was \$1.5 trillion whereas, in 1980, it stood at \$100 million. Further studies of this development found that not only is this industry increasing in popularity, but also the importance of these financial instruments is becoming more and more significant. As of June 2000, the MBS market amounted to almost \$2 trillion Bandic (2004).

The increasing size of this industry was matched by increasing home ownership in the United States of America (Hyman, 2012).

In Africa, homeownership poses a challenge to those of low income, especially in major cities like Nairobi in Kenya in which Kibera slum is located, the largest slum in Africa (Desgroppes & Taupin, 2011). As per the last census, more than 34% of Kenya's total population lived in urban areas, and of this, more than 71% was confined to informal settlements. At the time, Kenya's informal settlement growth rate was 5%, the highest in the world and was set to double in 30 years' time (Mutisya & Yarime, 2011). One major cause of homelessness in Kenya is the inability to access adequate financing to purchase or to put up housing structures. Households have difficulty accessing debt financing from financial institutions (Mutisya & Yarime, 2011).

This current Kenyan situation can be related to early 19th Century American economy described above, where consumers lacked access to debt financing. However, as evidence from America has shown, consumer debt can gain popularity through the use of mortgage-backed securities.

1.2. Motivation for the study

Given this success in the American economy, this study seeks to determine the most suitable structure for Mortgage-Backed Securities in a Kenyan context, given the need for increased consumer debt financing to increase home ownership in the economy.

1.3. Problem Statement

Ideally, there should be a match between the demand for housing in an economy and its supply. This would ensure stability in house prices as well as affordability (Munene, 2010).

In sub-Saharan Africa, slum populations grow at 4.5 percent per annum (Marx, Stoker, & Suri, 2013). At this rate, slum populations would double every fifteen years. It is expected that Nairobi's population will grow to over 8 million by the year 2025 given the rapid urban population growth rate. Unlike cities in developed countries, this increase is not accompanied by a corresponding improvement in socio-economic and environmental development (Mutisya & Yarime, 2011).

Low-income earners have difficulty accessing credit, given their low financial capacity to repay. This is evidenced in the 2017 Budget Speech made by the Kenyan Cabinet Secretary for Finance who stated that the per capita income in Kenya is estimated to be 152,671 (Rotich, 2017). In addition to this, a Reuters report shows that the documented average monthly wage in Kenya is Ksh.6,498 Macharia (2013). It is therefore clear that low-income earners in Kenya cannot afford to obtain mortgage financing, despite Kenya recently being declared a mid-income economy. A stark difference in the country's income distribution was admitted by the President of Kenya in his 2017 State of the Nation Address, who stated that "In simple terms, 50% of all the money collected as revenues in Kenya goes into the pockets of less than 2% of the country's total population." (Kenyatta, 2017).

The estimated current urban housing needs are 150,000 units per year for the urban areas and 300,000 units per year for the rural. The current production of new housing in urban areas is only 20,000-30,000 units annually, giving a shortfall of over 120,000 units per annum (Munene, 2010).

In as much as low-income earners in Kenya cannot afford to finance their housing, there is a fast growing demand for houses in urban areas Rotich (2017). Should this demand not be matched by supply, home prices will soar beyond the reach of even the middle-income citizens. It is for this reason that this study sought to find a sustainable solution to this ballooning problem.

1.4. Research Objective

1. To determine the optimal structure for investible MBS in Kenya.
2. To carry out sensitivity analysis of the MBS structure defined.

1.5. Research Question

1. What is the optimal MBS structure to adopt in the Kenyan context?
2. How does cost of issuing MBS vary with changing parameter values?



2. Literature Review

2.1. Securitization

Securitization is a financial operation which allows a financial institution to transform unmarketable financial assets, such as mortgage assets or lease contracts into marketable securities (Mansini, Speranza, & others, 2002). This is done by selling pools of loans or debt to a special purpose vehicle, which in turn finances the purchase by selling securities in the capital markets. There are various forms of securitization which include sales, asset-backed commercial papers, conduits, structured investment vehicles, collateralized debt obligations and collateralized loan obligations. However, for investors to be willing to buy this debt, their repayment must satisfy conditions set out by (Gorton & Metrick, 2012). The concern of this paper is on how special purpose vehicles float their debt on the capital market. This could take many forms including but not limited to pass-through securities, collateralized mortgage obligations (CMO) and real estate mortgage investment conduits (Jaffee & Rosen, 1990).

Pass through securities were the first to develop in the 1930s as the US Government tried to revive the consumer credit market. Given the success of these securities, Collateralized Mortgage Obligations were developed and introduced in the 1970s, appealing to different investor classes. Further success led to the development of asset-backed securities in the 1990s and real estate investment conduits emerged in 2002, to resolve problems concerning taxation that faced the mortgage backed securities.

A pass-through security receives pro-rata payments in relation to how the mortgages are amortized. Even the principal prepayments are distributed on a pro-rata basis to all tranches (Pachamanova & Fabozzi, 2010a).

A collateralized mortgage obligation, on the other hand, rearranges the cash flows to make them more predictable. The basic idea behind a CMO is to restructure the cash-flows from an underlying mortgage collateral into a set of investible bonds with different maturities. These two or more investible tranches receive sequential rather than pro-rata principal pay down. Interest payments are made on all tranches (Cornuejols & Tutuncu, 2006). Collateralized mortgage obligations appeal to more investors than pass through securities because of the different durations of each investible tranche, that match the various liability durations of the different investors

(Pachamanoa & Fabozzi, 2010b). Also, their multiple security classes reduce the uncertainty of cash flows for any particular maturity class. This makes them dominate pass-through securities (Jaffee & Rosen, 1990).

An advantage of improving the predictability of the cash flows is that we can structure tranches of different credit quality from the same mortgage pool. With the payments of a vast pool of mortgages dedicated to the fast-pay tranche, it can be structured to receive an AAA credit rating even if there is a significant default risk on the part of the mortgage pool. This high credit rating lowers the interest rate that must be paid on this slice of the CMO. There is equivalently lower credit quality and higher interest rates for slow pay tranches, as there is less principal left to be repaid (Cornuejols & Tutuncu, 2006).

Real Estate Mortgage Investment Conduits emerged because of tax constraints faced on CMOs in the United States economy. This led to a broader range of structure including but not limited to coupon stripping and floating rate securities (Jaffee & Rosen, 1990). Because of the domination of collateralized mortgage obligations over pass-through securities, and given that there are no tax laws hindering securitization of mortgages in Kenya yet, we focus on determining the optimal tranche structure of mortgage-backed securities. We intend to offer suggestions to solve this problem by the use of optimisation techniques.

Issuers make money by issuing CMOs because they can pay interest on the tranches that is lower than the interest payments being made by mortgage holders in the pool. The mortgage holders pay 10 or 30-year interest rates on the entire outstanding principal, while some tranches only pay 2, 4, 6 and 8-year interest rates plus an appropriate spread (Cornuejols & Tutuncu, 2006). We optimise our structure by minimising the cost associated with the issuance of tranches.

2.2. Optimisation

Optimisation is a branch of applied mathematics that involves the minimization or maximisation of a given objective function of several decision variables that satisfy functional constraints (Cornuejols & Tutuncu, 2006).

Many factors affect whether an optimisation problem can be solved efficiently. The number of decision variables and the total number of constraints are good predictors of how difficult it will be to solve a given optimisation problem. Other factors are related to the properties of the functions that define the problem. Problems with linear objective functions and linear constraints are easier than problems with convex objective functions and convex feasible sets.

An optimisation problem is defined as finding $x^* \in \mathbb{R}^n$ that solves $\min_x f(x)$ given $\text{s. t. } x \in S$ a function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ and a set $S \subset \mathbb{R}^n$ (Cornuejols & Tutuncu, 2006).

The function f is the objective function, and S is the feasible region.

Optimisation models play an increasingly important role in financial decisions. They are used in asset allocation, risk management, option pricing and model calibration (Cornuejols & Tutuncu, 2006).

There are many optimisation techniques. These include linear, quadratic, integer, dynamic, stochastic, conic and robust programming optimisation techniques that are encountered in financial models.

Linear programming is the problem of optimising a linear objective function subject to the linear equality and inequality constraints. When either the objective or the limitations are not linear, it becomes a non-linear programming model. The best-known methods for solving linear programming models are the simplex method and the interior-point methods

Quadratic programming is applied where the objective function is a quadratic function of the variables. These can be solved in polynomial time using interior point methods.

A conic optimisation problem arises when the non-negativity constraints are replaced by general conic inclusion constraints. The problem can further be categorized into second-order cone optimisation and semi-definite optimisation.

Integer programs are optimisation problems that require some or all of the variables to take integer values. This restriction on the variables often makes the problems very hard to solve.

Dynamic programming (DP) refers to a computational method involving recurrence relations. This technique was developed by Richard Bellman in the early 1950s. It arose from studying programming problems in which changes over time were necessary, thus the name dynamic programming. However, the technique can also be applied when the time is not a relevant factor in the problem. The idea is to divide the problem into stages to perform the optimisation recursively. Incorporating stochastic elements into the recursion is possible.

Stochastic programming is an approach used when the data uncertainty is random and can be explained by some probability distribution. Robust optimisation is used when one wants a solution that behaves well in all possible realisations of the uncertain data. These two alternatives approaches are not problem classes, but rather modelling techniques for addressing data uncertainty.

Dynamic programming models are based on Bellman's principle of optimality, namely that for all overall optimality in a sequential decision process, all the remaining decisions after reaching a particular state must be optimal on that state. Therefore, if a sub-optimal decision is made at any one stage, the entire decision process becomes sub-optimal (Cornuejols & Tutuncu, 2006).

This principle allows us to formulate recursive relationships between the optimal strategies of successive decision stage and these relations form the backbone of DP algorithms.

Common elements of DP models include decision stages, a set of possible states in each stage, transitions from states in one stage to states in the next, value functions that measure the optimal objective values that can be achieved starting from each state, and finally the recursive relationships between value functions of different states.

There are two approaches to dynamic programming; the backward recursion and the forward recursion approaches. These two methods both result in the optimal path to take in decision making. However, the additional information generated by each approach taken differs.

There are situations where one prefers to have one set of information over the other. The backward recursion produces optimal paths from each node in the tree to the final

stage nodes. The forward recursion produces optimal path from the initial stage node to all nodes in the tree.

If the actual transition state happens to be different from the one intended by an optimal decision, it would be important to know what to do when in a state that is not on the optimal path. In that case, the paths generated by the backward method would have the answer. For this reason, we choose to use backward recursion in our MBS structuring program

2.3. Theoretical Framework

Based on the theory of dynamic programming developed by Richard Bellman, we are interested in a class of mathematical problems which arise in connection with situations which require that a bounded or unbounded sequence of operations be performed for the purpose of achieving the desired result. Particularly important are the cases where each operation gives rise to a stochastic event, the result of which is applied to the determination of subsequent operations (Bellman, 1952).

In many cases, the problem of determining an optimal sequence of operations may be reduced to that of determining an optimal first operation. The general class of functional equations generated by problems of this nature have the form:

$$f(p) = \begin{cases} \min. \\ \max. \\ k \end{cases} (T_k(f)) \quad (2.1)$$

Where T_k is an operator. In many cases of interest, the operator has the form:

$$T_k(f) = g_k(p) + h_k(p)f(S_k p) \quad (2.2)$$

In which S_k is a point transformation.

N is a deterministic total number of stages which at which we make decisions. Let q_x be the probability that on reaching stage k , we are unable to make a decision and t_x be the time consumed in one decision.

(Bellman, 1952) sets out theorems to prove the existence and the uniqueness of solutions that optimise different problems that may be framed in a dynamic programming context. These theorems are:

2.1.1. Theorem 1.

Consider the equation

$$f(p) = \max_{1 \leq h \leq n} (g_h(p) + h_h(p)f(S_h(p))) \quad (2.3)$$

Where we assume that

if $p \in R$, a region of n – dimensional space, then $S_k p \in R$

$$|g_k(p)| \leq c_1 \text{ for } p \in R$$

$$|h_k(p)| \leq c_2 \leq 1 \text{ for } p \in R$$

$$g_k(p), h_k(p) \geq 0 \text{ for } p \in R$$

Under these assumptions, there is unique bounded solution to **Error! Reference source not found.**

2.1.2. Theorem 2

Consider the Equation:

$$f(x) = \max_R [a(x_1, x_2, \dots, x_N) + f(b(x_1, x_2, \dots, x_N))] \quad (2.4)$$

Where $R = R(x)$ is defined by $x_k \geq 0, \sum_{k=1}^N x_k = x$

If

$a(x_1, x_2, \dots, x_N)$ is continuous over $R(x)$ for $0 \leq x \leq x_0$, non – negative and $a(0,0, \dots, 0) = 0$,

$b(x_1, x_2, \dots, x_N)$ is continuous and non – negative over R

$$b(x_1, x_2, \dots, x_N) \leq cx, 0 < c < 1, \text{ in } R(x)$$

$$\sum_{l=0}^{\infty} h(c^l x_0) < \infty, \text{ where } h(x) = \max_R a(x_1, x_2, \dots, x_N)$$

There is a unique solution to **Error! Reference source not found.** for which $f(0) = 0$, for $0 \leq x \leq x_0$

2.1.3. Theorem 3.

Consider the equation

$$f(p) = \min \left\{ \begin{array}{l} 1 + \sum_{k=0}^{N-1} p_k f(x_k) \\ 1 + f(S_t p) \end{array} \right\} \quad (2.5)$$

$$f(x_0) = 0$$

Where $l = 1, 2, \dots, M$ and

$$p = \begin{pmatrix} p_0 \\ p_1 \\ \vdots \\ p_N \end{pmatrix} \quad S_t p = \begin{pmatrix} p_{0,t} \\ p_{1,t} \\ \vdots \\ p_{N,t} \end{pmatrix} \quad x_k = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

The 1 occurring in the k th place. Each p and $S_t p$ is a probability vector, $p_k \geq 0$, $\sum_{k=0}^N p_k = 1$ and $f(p)$ is a scalar function of p .

If for each l it is true that

$$\sum_{k=1}^N p_{kt} \leq c_1 \sum_{k=1}^N p_k, \quad 0 < c_1 < 1$$

There is a unique bounded positive solution to **Error! Reference source not found.**

2.1.4. Simplification

The simplifications and solutions to these general equations are provided by (Bellman, 1952). Of interest to us is the solution of $f(x) = \max_{0 \leq y \leq x} [g(y) + h(x - y) + f(ay + b(x - y))]$ where $0 < a, b < 1$ may be reduced to that of

$$\max [g(x) + f(ax), h(x) + f(bx)] \quad (2.6)$$

In $0 < x < x_0$, if g and h are monotonically increasing functions such that $g(0) = h(0) = 0$, $g'', h'' \geq 0$ in $[0, x_0]$

If $g'', h'' \leq 0$, the situation is much more complicated and no such simple result simpler equation above holds in general (Bellman, 1952).

This is the simplified function is what we will use to structure our securities.

2.4. Empirical Framework

In 2007, a dynamic model was used to structure Colombian securities by (Medina, Riaño, & Villarreal, 2007). They chose to use the dynamic programming because the sequential nature of the decision making process and it helps determine the optimal strategies that will take into account both recent and future information as well as the space of possible decisions to make.

They implemented the model in a Java package for Markov Decision Process (jMDP) framework. They chose jMDP because it is an object oriented framework that allows the user to build a computational representation of these objects. By exploiting the abstraction capabilities of JAVA, the user does not have to worry about the algorithms needed to solve the problem and can rather concentrate in representing the model by extending the classes provided by the framework and implementing abstract functions for the costs, probabilities, etc.

The implementation of a real life instance problem, by extending a dynamic programming model from the jMDP framework, proved the capacities and suitability of this context to address Markov Decision Problems. The development of this project in a rigorous way turned out to be efficient and achieved the objectives (Medina et al., 2007).

2.5. Discussion of previous works

The economic significance of securitization broadly discussed in the literature. Securitization has been studied particularly in respect to the final step of securities issuance. Simulation models are known for a portfolio of mortgage-backed securities while a wide range of literature has investigated the evaluation and pricing of

mortgage-backed securities as well as prepayment of the underlying assets (Mansini et al., 2002).

There is literature on generic structures for mortgage-backed securities as well as on how they developed. These structures range from pass through Mortgage Backed Securities to Stripped Mortgage Backed Securities which comprise of principle only and interest only securities. There are also collateralized mortgage obligations which include sequential pay structures and planned amortisation class bonds and support bonds (Pachamano & Fabozzi, 2010b).

There is also literature on the problem of selecting the financial assets to transform into securities. In this case, the problem is solved as a multi-dimensional knapsack problem (MDKP), as shown by (Mansini et al., 2002).

A dynamic programming model was developed for structuring mortgage-backed securities and implemented using original data of the Seventh issuance of a Colombian securitizing firm by (Medina et al., 2007). They found that by using the model to allocate the securities optimally found a 6.57% reduction in the cost of issuance of the securities, which translated to approximately USD\$ 11,000,000.

The reason for the difference was shown when they confirmed their hypothesis that a less rigid and predetermined structure, that is, a more flexible structure that is aware of the characteristics of the pool of mortgages being securitized would be less expensive and cost efficient (Medina et al., 2007). Given that there are no regulatory constraints on the size, structure or number of tranches in Kenya, we use this model to determine the optimal structure.

2.6. Research Gap

There have been few studies concerning mortgage-backed securities in Kenya. Mutuku (2006) examined factors influencing the development of Secondary Mortgage Market in Kenya with a particular emphasis on Mortgage Backed Securities and found that there is a potential market for trading with Mortgage Backed Bonds (Munene, 2010).

A feasibility study of the introduction of Mortgage Backed Securities in Kenya was carried out by Munene (2010) who found it feasible for them to be introduced. There

are regulations in Kenya, introduced in the year 2007, regulating the issuance of asset-backed securities. The law allows for credit enhancement through issuing subordinated tranches. It, however, does not specify the size, the structure or the number of tranches. Furthermore, there is no literature on the optimal structure of collateralized mortgage obligations in Kenya. Hence this research paper intends to fill this gap.

2.7. Link between our research and discussion of works

As Medina et al. (2007) use dynamic programming to determine the optimal structure for mortgage backed securities in Colombia and found that the use of this method made significant savings in cost, we intend to use the model to structure securities for the Kenyan market.

However, unlike Medina et al. (2007), we will use excel in combination with VBA as our statistical software, where VBA will be used to write excel user defined functions and macros. We choose this because we intend to visualise the table construction algorithms, and use less time than would be required in building a jMDP framework.

2.8. Conceptual Framework

Based on Bellman's principle of optimality, that to achieve general optimality in a specific model that follows a process of sequential decision making, all the remaining decisions after reaching a particular state must be optimal in that state (Bellman, 1952). Therefore, if a solution strategy follows a suboptimal decision in any intermediate step of the process, the problem as a whole will not be optimal (Medina et al., 2007).

Given that we are taking the perspective of the CMO issuer, the costs we incur are those related to the coupon and principal outgo to the tranches of investors. We seek to minimise the present value of these payments made to the bondholders identifying the optimal structure.

Our objective function is to find the optimal partition of the time horizon of the issuance (duration of the tranches) so as to minimise the cost of publication of the MBS, by minimising the present value of the payments made to the bondholders.

$$v(k, t) = \min_{j = k - 1, \dots, t - 1} \{v(k - 1, j) + T_{j+1, t}\} \quad (2.7)$$

$v(k, t)$ is the least cost solution of structuring a CMO with k tranches. That is, the minimum present value of total payments to bondholders in years 1 through t when the CMO has k tranches up to year t .

T_{jt} is the present value of payments on tranche (j, t) starting at the beginning of year j and ending at the end of year t .

States: Years t of the time horizon of the issuance where $t = 1, 2, 3, \dots, T$.

Stages: Number of tranches desired up to the year T .

Actions/Decisions: duration of each tranche. Say we are in state $i = t$ (initial year of the tranche), by choosing state j (final year of the tranche we would have defined by the n th tranche (i, j) and would have taken the action j (go to year j).

Transition function is the function that returns the state reached once action α is taken. In this particular model, taking action α is the same as going to state α , then the transition function $f_n(\alpha) = \alpha$.

Immediate cost: This cost can be found in an already constructed matrix M that has as its elements the present value of the future payments to the bondholders T_{jt} of each possible tranche.

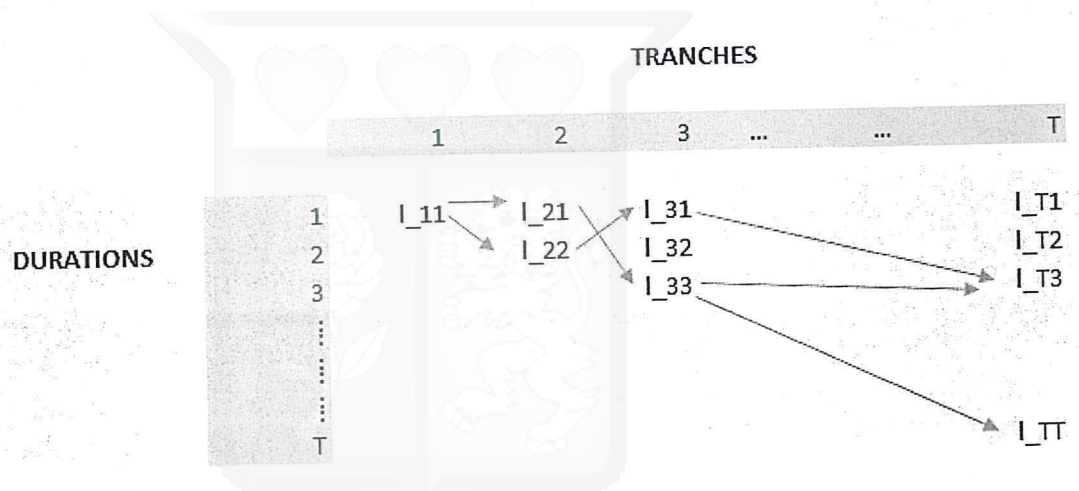
The construction of this matrix is done by programming an algorithm presented by (Cornuejols & Tutuncu, 2006) that takes into account the financial variables that affect the value of issuing each possible tranche.

We start with a given value function for the last stage, $V_N(t)$, and proceed backwards to find all other optimal value functions, $V_{N-1}(t)$, $V_{N-2}(t)$, etc, for stages, $N-1$, $N-2$, etc. The optimal policy is the sequence of actions that at each stage minimize the Bellman equation.

Once these elements have been identified, a dynamic programming model has been fully defined, and it can be solved.

The advantage of dynamic programming becomes clear as k increases. As k becomes bigger, there is no need to compute the minimum of thousands of possible combinations of k tranches. Instead, we use the optimal structure $v(k - 1, j)$ already computed in the previous stage. So the only enumeration is over the size of the last tranche (Cornuejols & Tutuncu, 2006).

Diagrammatically, optimisation can be presented as the following reiterative process, where the arrows point in the direction of optimal decisions. Following each decision path indicated leads to the minimum possible cost. Two arrows from the same point indicate identical costs.



Source: Author (2017)

3. Methodology

3.1. Research design

We will carry out an experimental study. We intend to vary our variable inputs to achieve an optimal solution, noting that the aim of our study is to determine the optimal structure for a mortgage-backed security in Kenya. Optimal in this case means the number of tranches, their pool sizes, the coupon and duration that minimises the cost to the issuing company.

3.2. Nature of study

The study will be quantitative in nature, using a dynamic programming model to achieve the required quantitative result of optimality.

3.3. Population of study

This study focuses on mortgages in the Kenyan economy.

3.4. Data

We will use secondary cross-sectional data, relating to the last period a report was made by the Central Bank of Kenya and the World Bank. The cross-sectional studies carried out by the world bank and those made by the Central Bank of Kenya both give similar and consistent data for the mortgage market in Kenya, indicating its validity. The data is retrieved from the websites of both the Central Bank of Kenya and the World Bank and is, therefore, convenient and cheap to obtain.

It was found that 40% of commercial banks each hold mortgage loans of over Ksh.1 billion by (Munene, 2010). There are 42 listed commercial banks according to the Central Bank of Kenya (Central Bank of Kenya, 2017). In 2015, the Central Bank of Kenya documented that the value of mortgage loan assets outstanding increased from Ksh.164 billion in December 2014 to Ksh.203.3 billion in December 2015 due to increased appetite for home ownership as opposed to rentals. Also, the outstanding value of non-performing mortgages increased from 10.8 billion in 2014 to 11.7 billion in 2015. The average loan size stood at Ksh.8.3 million, and the average loan maturity was 9.6 years with a minimum of 5 years and a maximum of 20 years. The interest rate

charged in 2015 averaged at 17.1%, and 89.3% of mortgages were on variable interest rate basis. (Central Bank of Kenya, 2016). The average duration of a mortgage loan in Kenya is fifteen years according to an economic report by (World Bank Group, 2017). Given these statistics, we have enough data to generate suitable durations for mortgage-backed securities in Kenya.

3.5. Methodological approach

In this section, we present a dynamic recursion for solving the problem. First, we demonstrate that the collateral can service the payments on the issued CMO tranches under several scenarios. These scenarios are well defined and standardised and cover conditional prepayment models as well as the two extreme cases of full immediate prepayment and no prepayment at all (Cornuejols & Tutuncu, 2006). A study by (Munene, 2010) on the feasibility of the Kenyan mortgage market found the development of mortgage-backed securities feasible. We will, therefore, assume that this first requirement has been satisfied. Second, we determine the tranche sizes, subject to standard buffers required for each tranche set out by (Cornuejols & Tutuncu, 2006). Third, we determine the least cost solution of for issuing tranches using a dynamic programming approach, by determining the appropriate durations for each bond.

3.5.1. Determining the required tranche sizes

The convention in mortgage markets is to price bonds on their weighted average life (WAL) (Cornuejols & Tutuncu, 2006).

$$WAL = \frac{\sum_{t=1}^T tP_t}{\sum_{t=1}^T P_t} \quad (3.1)$$

Where P_t is the principal payment in the period t ($t = 1, \dots, T$).

A bond with a WAL of x years will be priced at the x -year treasury rate plus a spread. Typically the WAL of CMOs are high but by splitting the collateral into several tranches, some with low WAL and some with large WAL, lower rates are obtained on the fast-pay slices while higher rates are gotten for the slower paying slices. The issuer ends up with a lower rate on average on the CMO than on the collateral. The objective

is to maximise the profit from the issuance by choosing the size and duration of each tranche (Cornuejols & Tutuncu, 2006).

The required tranche sizes are determined such that they achieve high-quality rating, by having the capacity to sustain higher than expected default rates without compromising payments to the tranche holders. For this reason, credit ratings are assigned based on how much money backs the current tranche (Buffer). That is, how much outstanding principal is left after the current tranche is retired, as a percentage of the total amount of principal.

$$\text{Buffer} = \frac{(\sum_{k=t+1}^T P_k)}{(\sum_{k=1}^T P_k)} \quad (3.2)$$

Where P_k is the principal payment at time t .

Early tranches receive higher credit ratings since they have greater buffers. According to (Cornuejols & Tutuncu, 2006), a tranche with AAA rating must have a buffer equal to six times the expected default rate. This is referred to as the loss multiple. They give further loss multiples are as follows:

Credit Rating	AAA	AA	A	BBB	BB	B	C
Loss Multiple	6	5	4	3	2	1.5	0

Table 3.1. Source: (Cornuejols & Tutuncu, 2006)

The required buffer may also be computed from the following formula:

$$\text{Required Buffer} = \text{WAL} * \text{Expected Default Rate} * \text{Loss Multiple} \quad (3.3)$$

Buffer is the amount of principal collateralizing the tranche.

WAL is the weighted average life of the tranche

Loss multiple is a multiple of expected default rate used to determine credit rating.

Each tranche is priced based on credit spread to the current Treasury rate for a risk-free bond of that approximate duration. Spreads on corporate bonds with similar credit ratings would provide reasonable figures. Our calculation is simplified by assuming bonds sold at par.

Assuming a level payment on a par bond (principal + interest) each year, then if the outstanding principal is Q , coupon rate is r , and amortization occurs over k years, the scheduled amortization in the first year is

$$\frac{Qr}{(1+r)^k - 1} \quad (3.4)$$

The next amortisations include interest on Q

We will assume a prepayment model that follows 100% annual PSA industry standard benchmark, the rate of prepayments is defined as 1% of outstanding principal in the first year, 3% in the second year, 5% in the third year and 6% in all other years, all defined at year end. Our assumption of the annualised PSA industry standard benchmark is based on its simplicity. Also, Kenya does not have a standardised prepayment model of its own yet.

We denote PP_t the prepayment in year t , I_t as the interest payment in year t , A_t as the amortization payment in year t and P_t as the principal pay-down in year t .

$$P_t = A_t + PP_t \quad (3.5)$$

3.5.2. Determining the least cost solution (Optimal bond duration)

Define T_{jt} to be the present value of the payments on tranche (j, t) , given that we have coupon rate c_{jt} and a term structure of spot rates r_t , T_{jt} is computed as follows. In each year k , the payment C_k is made for tranche (j, t) . T_{jt} is equal to the present value of C_k summed over all possible values of k .

Let $t = 1, \dots, T$ index the years. The states of the dynamic program will be the years t and the stages will be the number k of tranches up to year t . Given the matrix T_{jt} , we are ready to solve the dynamic programming recursion.

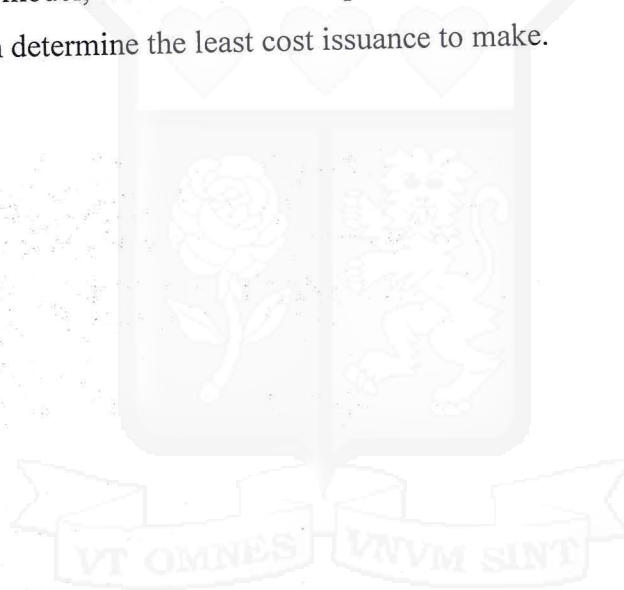
$v(k, t)$ is the least cost solution of structuring a CMO with k tranches. That is, the minimum present value of total payments to bondholders in years 1 through t when the CMO has k tranches up to year t .

T_{jt} is the present value of payments on tranche (j, t) starting at the beginning of year j and ending at the end of year t .

$$v(k, t) = \min_{j = k - 1, \dots, t - 1} \{v(k - 1, j) + T_{j+1, t}\} \quad (2.7)$$

Note that $v(1, t)$ is simply T_{1t} .

With this model, we determine the optimal durations for each of the bonds issued and hence can determine the least cost issuance to make.



4. Data Analysis

Here, we determine the optimal duration (term) for an investable bond for issuing mortgage backed securities, using the dynamic programming approach discussed above.

Our model's key output was the time to expiration of the bonds, that minimizes cost to the issuer.

In addition to the assumptions listed in chapter 3 above, we further assume that the size of each bond issue is the same, for all credit classes. We also assume that the bonds are issued as one class, of equal term.

Adopting the PSA concept of increasing prepayments for 8.33% of the term of issue of the class, we carried out the tests below, using the 100% PSA rate of 6% and did a sensitivity test with the 50% PSA rate, 3%.

4.1. Description of analysis.

We created all cash flows associated with the issue of the class of securities. We then appropriated the cash flows arising to each class of securities in a sequential manner and calculated the optimal duration (time to expiration) as described in chapter 3 above

4.2. Results:

4.2.1. Scenario 1: 100% PSA

ASSUMPTIONS	
Parameter	Value
No of Tranches	4
Pool Size	203,300,000,000
Unit bond Value (KSH)	50,000
PSA month of constance	5.50
PSA Rate	6%
Risk Free Rate	7%

Table 4.1. Source: Author (2017)

FURTHER ASSUMPTIONS AND OPTIMISATION						
Credit Rating	Loss multiple	Interest Spread	Interest Rate	No of Units	Term	
AAA	6	1.00%	8.00%	225,888.8889	5.5	
AA	5	2.00%	9.00%	225,888.8889	5.5	
A	4	3.50%	10.50%	225,888.8889	5.5	
BBB	3	6.00%	13.00%	225,888.8889	5.5	
BB	2	7.00%	14.00%	-		
B	1.5	7.00%	14.00%	-		
C	0	7.00%	14.00%	-		

Table 4.2. Source: Author (2017)

CHARACTERISTICS OF THE BONDS THAT ARE ISSUED							
Tranche	No of units	Bond value	Discount Rate	Coupon rate	Term of bond	Terminal Bond Value	
AAA	225,889	11,294,450,000	0.08	0.08	5.50	17,246,292,293.5	
AA	225,889	11,294,450,000	0.09	0.09	5.50	18,143,072,074.2	
A	225,889	11,294,450,000	0.11	0.11	5.50	19,559,494,035.7	
BBB	225,889	11,294,450,000	0.13	0.13	5.50	22,120,580,833.1	
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-

Table 4.3. Source: Author (2017)

	Time to Expiration						
	AAA	AA	A	BBB	BB	B	C
MONTHS		9	20	53	66	0	0
YEARS		0.75	1.666666667	4.416666667	5.5	0	0

Table 4.4. Source: Author (2017)

Weighted Average Life	19.05596602
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Table 4.5. Source: Author (2017)

4.2.2. Scenario 2: 50% PSA

ASSUMPTIONS	
Parameter	Value
No of Tranches	4
Pool Size	203,300,000,000
Unit bond Value (KSH)	50,000
PSA month of constance	8.50
PSA Rate	3%
Risk Free Rate	7%

Table 4.6. Source: Author (2017)

FURTHER ASSUMPTIONS AND OPTIMISATION					
Credit Rating	Loss multiple	Interest Spread	Interest Rate	No of Units	Term
AAA	6	1.00%	8.00%	225,888.8889	8
AA	5	2.00%	9.00%	225,888.8889	8
A	4	3.50%	10.50%	225,888.8889	8
BBB	3	6.00%	13.00%	225,888.8889	8
BB	2	7.00%	14.00%	-	-
B	1.5	7.00%	14.00%	-	-
C	0	7.00%	14.00%	-	-

Table 4.7. Source: Author (2017)

CHARACTERISTICS OF THE BONDS THAT ARE ISSUED							
Tranche	No of units	Bond value	Discount Rate	Coupon rate	Term of bond	Terminal Bond Valu	
AAA	225,889	11,294,450,000	0.08	0.08	8.50	21,725,361,35	
AA	225,889	11,294,450,000	0.09	0.09	8.50	23,495,804,48	
A	225,889	11,294,450,000	0.11	0.11	8.50	26,390,307,48	
BBB	225,889	11,294,450,000	0.13	0.13	8.50	31,917,719,72	
	-	-	-	-	-	-	
	-	-	-	-	-	-	
	-	-	-	-	-	-	

Table 4.8. Source: Author (2017)

	Time to Expiration						
	AAA	AA	A	BBB	BB	B	C
MONTHS		18	41	90	102	0	0
YEARS		1.5	3.416666667	7.5	8.5	0	0

Table 4.9. Source: Author (2017)

Weighted Average Life
39.29222385

Table 4.10 Source: Author (2017)

4.3. Summary of Results

4.3.1. Scenario 1: 100% PSA

The cost of issue is 31,899,186,887.83 (31 billion)

The time to expiration of the first bond is 0.75 years, while that of the entire issue is 5.5 years.

Here we have a weighted average life of 19.05 months (3.27 years).

4.3.2. Scenario 2; 50% PSA

The cost of issue is 58,351,393,046.82 (58 billion)

The time to expiration of the first bond is 1.5 years, while that of the entire issue is 8.5 years.

Here we have a weighted average life of 39.29 months (3.27 years).

5. Discussions and Conclusions

5.1. Summary

We can see that prepayment has a very big effect on the cost of issuing the bonds. A change of 3% in the rate of issuance caused an 82% increase in the cost of the class of securities.

A faster prepayment of mortgages by the mortgagees, would lead to a minimization of the cost of borrowing.

Also, we notice that the weighted average life is lower for the prepayment rate of 6%. This means that a faster prepayment makes the issuance a more secure one, compared to the one with a higher weighted average life, making it less secure for investors. This is also evident in the time to expiration of the first bond, as well as the term of the entire issue.

If the government were to guarantee these securities, we would recommend an expansionary economic environment after the issue, to encourage increased prepayment to minimize cost to the government as well as minimize the risk it faces of having to repay investors.

5.2. Conclusion:

This research has found that should one class of mortgage backed securities be issued, the optimal duration for the issues, assuming a 100% PSA prepayment standard, would be 0.75 years for the first tranche, 1.67 for the second tranche, 4.42 years for the third and 5.5 years for the fourth tranche.

We recommend further research into the recommended time for scheduling an issuance, as our sensitivity analysis showed that an expansionary economy is more favorable than a contracting economy.

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