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PREDICTION OF STUDENT LOAN DEFAULT RATES

ABWOGA SUSAN MANYA

Student Number: 071932

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**School of Finance and Applied Economics
Strathmore University
Nairobi, Kenya**

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SIGNED  DATE 23/11/2015

Abwoga Susan Manya,

071932.

This Research Project has been submitted for examination with my approval as the Supervisor.

SIGNED  DATE 23/11/2015

Dr Carolyn Njenga,

School of Finance and Applied Economics

Strathmore University

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ABSTRACT

This study focuses on the modeling of the probability of student loan default using certain characteristics of individual students. A generalized linear model is adopted in the rating of students' characteristics to parameterize their significance. Factors to be rated include the type of university attended, the type of loan taken by the student and the period of the course. By determining the probability of default, the amount of loss expected can be determined and thus create reserves to counter the adverse effects of high student loan defaults.

TABLE OF CONTENTS

DECLARATION.....	i
ACKNOWLEDGEMENT.....	ii
ABSTRACT.....	iii
LIST OF ABBREVIATIONS	iv
CHAPTER ONE	1
1. INTRODUCTION.....	1
1.1. Developments of HELB loan.....	1
1.2. Problem Statement.....	2
1.3. Research Objective	3
1.4. Research Questions.....	3
1.5. Rationale of the study	3
1.6. Chapter Summary	3
CHAPTER TWO	5
2. LITERATURE REVIEW.....	5
2.1. Background	5
2.2. Introduction.....	7
2.3. Type of Loan Programs.....	7
2.4. Importance of Human Capital.....	8
2.5. Ideal Student loan scheme.....	9
2.6. Student loans in developed countries	10
2.6.1. Reforms in United Kingdom	11
2.6.2. Reforms in United States of America	11
2.7. Student Loans in Africa.....	12
2.7.1. Higher Education Loan Board in Kenya	13
2.7.2. Strategic Plan 2009-2014	14
2.7.3. Challenges that HELB may face.....	15
CHAPTER THREE	16
3. METHODOLOGY	16
3.1. Introduction.....	16
3.2. Research Design	16
3.3. Population of the Study	16
3.4. Sample Size and Sampling Procedures	16
3.5. Data Collection.....	16

3.4. Research Procedure	17
3.5. Empirical Approach	17
3.5.1. Risk Prediction	17
3.5.2. Risk Segmentation.....	18
3.6. Measures used to quantify credit risk	19
3.6.1. Loan Loss Provisions	19
3.6.2. Actual Losses: Net Charge-offs.....	19
3.6.3. Loan Loss Provision Coverage Ratio	20
3.7. Chapter summary	20
CHAPTER FOUR.....	21
DATA PRESENTATION AND ANALYSIS OF FINDINGS	21
4.1. Introduction.....	21
4.2. Analysis of Data and Presentation of Findings	21
4.2.1. Box plot Analysis.....	21
4.2.2. Choosing exponential family	24
4.2.3. Coding and Statistical Tests	24
4.3. Summary and Interpretation of Findings.....	25
CHAPTER FIVE	27
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS.....	27
5.1. Summary.....	27
5.2. Conclusions.....	27
5.3. Recommendations	27
5.4. Limitations of the Study	28
5.5. Suggestions for Further Research	28
References	19

LIST OF ABBREVIATIONS

AIDS- Acquired Immunodeficiency Syndrome

FFEL- Federal Family Education Loan Program

HELB- Higher Education Loan Board

HELF-Higher Education Loan Fund

HIV-Human Immunodeficiency virus

JAB- Joint Admission Board

KRA-Kenya Revenue Authority

NHIF- National Health Insurance Fund

NSFAS- National Student Financial Aid Scheme

NSSF- National Social Security Fund

TSC- Teachers Service Commission

U.K – United Kingdom

U.S.A – United States of America

CHAPTER ONE

1. INTRODUCTION

In the presence of potential loss organizations can often reduce exposure to risk not just by the purchase of market insurance but also through expenditures on self-insurance that reduces the size of a loss or through expenditures on self-protection which is the reductions in the probability of a loss (Konrad, 2015). The government HELB¹ student loan in Kenya whose objective is to help needy students go through higher education has suffered default risks owing to different reasons. This requires that a significant margin be provided to cover the provision for such significant deviations from expected experience. Insurance companies may not be willing to take up this large risk as it may lead to huge losses considering the higher default rates in the recent past. Therefore this paper suggests a way of predicting student loan default that will help the HELB student loan to reduce the probability of loss due to default which is caused by underlying factors like death, disability and unemployment.

In the United States of America, the government has an ambitious agenda to make college education affordable and allow more Americans to earn degrees. They include increasing scholarships for students, adjusting the American Opportunity Tax Credit, increasing investments in community colleges and easing the student aid application process. However high default rates still continue to be a problem.

1.1. Developments of HELB loan

Since 1995, the government of Kenya has battled the notion that the HELB loan is a grant. Though an act of Parliament the Board was empowered to recover all outstanding loans given to the graduated student by the Government of Kenya since 1952 through HELF² (HELB, 2015). There has been a 30.4% increase in student loan recovery as recorded by HELB in 2012. This positive improvement has enabled them to disburse more funds to the growing student population (HELB, 2012). However the improvement does not make the scheme as sustainable as it is expected to be. The challenges owed to this shortcoming include, the increased need for financing due to double intakes by both public and private institutions of higher education. Furthermore, graduate

¹ Higher Education Loan Board

² Higher Education Loans Fund

unemployment which hampers efforts to make the HELB program sustainable and revolving fund. Students who migrate to other countries before repayment of the HELB loan also contribute to this deficiency in recovery. It is also important to note that the HIV/AIDS pandemic has played a role in the increased number of orphans in university.

In a bid to improve recovery of loans HELB has partnered with the Kenya Revenue Authority(KRA) through the KRA Act of 2005//2006 Finance Act which allows them to view data base information to trace university graduates not repaying loans. Most recently, HELB has information sharing agreements with the National Hospital Insurance Fund and the National Social Security Fund that it uses to track past loan beneficiaries. This measures are expected to be backed up by a law to be gazette by the education secretary in Chapter Six of the Kenyan Constitution to compel private sector employers to demand clearance certificates from HELB before hiring new employers. With all this measures put in place, the Government of Kenya plans to transform HELB into a bank and allow the agency to float bonds as part of its fundraising activities to meet rising demand of student loans (Herbling, 2015).

Herbling (2015) highlights how HELB signed banks to track borrowers with smart cards as means of ensuring their loans are recovered. This partnership with local banks involves developing the cards, which students will use to pay for tuition, upkeep, buy books and stationery. The HELB cards will have tap-and-go payment features based on the Near Field Communication technology. HELB has this year disbursed Sh5.4 billion in loans to about 125,000 students, whetting the appetite for Kenyan banks which stand to rake in millions in income from commission and fees charged on processing such payments.

1.2. Problem Statement

Loans give students from poor families, who would otherwise be denied access to higher education on grounds of poverty, the chance to invest in their own future by providing them with financial aid when it is needed and allowing them to repay it when they can afford to do so. Increasing sustainability by ensuring that loan repayment from past cohorts of students help to finance financial support for the next generation. With the continued increase of students pursuing higher education, it is certain that student loans will be required to assist them. However some issues arise:

1. The HELB loan needs to secure and maintain adequate capitalization. Loan repayments from past students cannot totally eliminate the need for public funding, it only reduces.
2. High default rates attributed to low income, unemployment, illness or even death.

The above mentioned issues shed light on the crucial role the student HELB loan in Kenya plays and the importance of minimizing losses by designing the loan program to ensure maximum chances of recovery and ensuring that the collection process is as efficient as possible.

1.3. Research Objective

The purpose of this study was to determine the probability of student loan default by considering the student and loan characteristics and finally estimating the money to be set aside to meet future loss.

1.4. Research Questions

The study is guided by the following research questions:

1. Which rating factors are key in determining the probability of student loan default?
2. How can reserves be calculated using the probability of student loan default?

1.5. Rationale of the study

Here in Kenya the Higher Education Loan Board recently intended to establish a scheme as part of its risk management strategy that contributes to the sustainability of the fund. This however is a new product and historical data on management and performance is quite limited. Research on the area is therefore necessary to establish sound actuarial advice to set out the financial mechanisms as well management of the fund. The success of the proposed self-insurance together with current policies to recover outstanding loans will put HELB in a better position to issue out more loans to people and reduce pressure on National Government.

1.6. Chapter Summary

This chapter has presented the background of the study including the current situation in Kenya on student loans and challenges faced. This background informs the problem of the statement, research questions, and scope of the study and the importance of the study. Chapter two reviews the existing literature on student loan issues and reforms from books, journals and dissertations. Chapter three presents the research methodology and design to be used in the research. Chapter

four of the study presents the findings and interpretations from data collected and analyzed. Chapter five presents the discussion of findings, conclusions and recommendations of the study.

CHAPTER TWO

2. LITERATURE REVIEW

2.1. Background

The idea of student loan programmes dates from the 1940s and 1950s where the idea of students borrow from government funds to finance higher education (Woodhall, 2007). Students' loan programmes now exist in many developed and developing countries. Examples of student loan programmes which are financed from public funds or backed by government guarantees, were found in Japan, Scandinavia and the U.S.A. Other developed countries set up loan programmes in the 1960s, including Canada and several European countries. Student loans have recently been proposed in several other countries, including the U.K., and New Zealand among developed countries, and Tanzania, South Africa, Kenya, Uganda among the developing countries (Nyahende, 2013).

Friedman (1995) suggests that developing countries governments have resorted to provide various forms of interception and other possible subsidies to help improve access to education and ensure that there are sufficient graduates to push their economies forward. Technological advancements have further driven the need for skills, placing human capital as one of the most important determinants of economic development (Barr, 2010). Therefore in order to compete internationally, countries need mass high quality systems of tertiary education³. The demand for student loans in higher education is growing as existing means of sponsorship is pushed to its limit by a growing population and desire to pursue learning. However the Great Recession that ended in 2009 saw people with student loans having difficulty with repayment (Lorin, 2015). Debates have arisen to discuss possible ways of cost-sharing and equity in higher education for recovering a portion of the costs either of student living or tuition fees or both (Johnstone, 2004). Previous literature mainly originates from the developed countries such as the U.K, Scandinavian countries, Australia and the United States who have tried to expansively discuss the issue. Researchers⁴ have

³ In South Korea the participation rate in tertiary education in 2003 was 82 per cent, and total spending on tertiary education 2.6 per cent of GDP, double the average for the EU19 of 1.3 per cent; *private* spending on tertiary education in South Korea is significantly higher than *total* (public plus private) spending in any OECD country except the USA and Canada (OECD 2006, Table B2.1b, all figures for 2003).

⁴ Previous studies have investigated student loan schemes in many countries, for example, Australia (Chapman, 2006; Chapman & Ryan, 2002), Europe and the US (Johnstone, 1986), Africa (Johnstone, 2004; Johnstone & Aemero, 2001), South East Asia (Ziderman, 2004) and developing countries more generally (Woodhall, 1987).

considered the implications of cost sharing⁵ to several different notions of higher education equity. Although there is wide literature that covers the topics of financing student loan systems and repayment structures, this review will focus on challenges facing student loans in developed countries, student loan systems in developing countries and the possible effects of improving student loans in developing countries.

⁵ The shift of higher educational costs from exclusive or near exclusive financial reliance on government, or the taxpayer, to being with parents/or students.

2.2. Introduction

Chapman (2006) describes two main types of financial assistance, namely **means-tested grants** and **student loans**. Student loans are mainly supported by the government and take up a mortgage type form and income contingent form. A conventional, or “mortgage-type,” loan carries three contractual elements: (1) a *rate of interest* expressed as an annual percentage of the amount borrowed or still to be repaid (which may be fixed or may vary according to some index such as the government’s borrowing rate or the calculated annual rate of inflation); (2) a *repayment period*, or the amount of time the borrower has to repay the loan; and (3) a *repayment mode*, such as whether the payments are to be in equal monthly installments, or installments that begin small and increase over time, or some other arrangement that yields a stream of payments sufficient to amortize the loan at the contractual rate of interest. (Johnstone D. , 2003). An income contingent (or “contingent repayment”) loan carries a contractual obligation to repay some percentage of future earnings (sometimes per \$1000 borrowed) generally until the loan is repaid at the contractual rate of interest (whether subsidized, unsubsidized, or *premium*--that is, designed to generate a surplus) or until the borrower has repaid for a maximum number of years. The borrower who has repaid the maximum number of years without paying off his or her loan at the contractual rate of interest is released from further obligations and thus granted a subsidy, or an *effective grant*. This subsidy is given not on the basis of the current low income of the borrower’s family at the time of the original loan, but on the basis of the borrower’s own low income over an effective earning lifetime--that is, on the basis of his or her higher education never really “paying off” monetarily. (Johnstone D. , 2003). Most countries have been using the mortgage type form but debates of the cost borne by the government and taxpayers have led to talks towards income contingent loan systems.

2.3. Type of Loan Programs

Student loan programmes in developing countries are mostly used to assist students in living expenses and this accounts for a large percentage of the total student population. As previously mentioned most loans are the mortgage type loan as compared to the income contingent loan which happens to be more favorable to graduates with lower incomes (Albrecht & Ziderman, 1995). However we have the *graduated* nominal payments where borrowers pay equal rather than nominal amounts, thus ensuring that the first payments are not necessarily large in real terms in relation to others. In Kenya, the loan scheme has been of mortgage type. Similarly, in Malawi,

Tanzania, Nigeria and South Africa. Ghana however has an income contingent scheme. Internationally similar programmes are run in Australia (income contingent), Sweden (income contingent), Morocco (mortgage), Sri Lanka (mortgage), Philippines (mortgage) and the UK (mortgage).

The South African government established National Student Financial Aid Scheme (NSFAS) in 1996 to support higher education through borrowing. The Soviet Union initially provided free higher education but since its split private institutions have opened and state institutions have been allowed to charge fees to students who fall below acceptable scores on entrance examinations but are willing to pay to study (Owino, 2003). Bain (1997) mentions that about 10 percent of the full time students at Moscow State University pay from \$1500 to \$ 4000 in tuition per year. In Kenya these loans are for students joining public or private universities within the East African Community directly from high school either through the Joint Admissions Board (JAB) or as self-sponsored. Amounts awarded range between 35,000 minimum and 60,000 maximum based on the level of need. The loan attracts interest of 4% per annum and the students are required to repay their loans on completion of their studies. Undergraduate loan repayment starts within one year of completion of studies or within such a period as the Board decides to recall the loan whichever is earlier. However, you can make voluntary payments before or after you complete your studies to reduce your loan balance. (HELB, 2015).

2.4. Importance of Human Capital

Thurow (1996) argues that human capital as a resource to production is key today than in the past. The conventional production function states:

$$Q = f(K, L, M)$$

Equation 1: Production function

where output, Q , is related to inputs of capital, K , labour, L , and raw materials, M , via the production function f . Historically, the 19th century considered raw materials to be critical. Almost all the largest US firms were involved with raw materials in one way or another. Today, value cost; the steel used in a modern car costs less than the electronics. Initially countries with a larger capital stock were richer and so, through higher savings, could invest more than poorer

countries, thus further increasing their capital stock. With today's worldwide capital markets, domestic investment is less constrained by domestic savings: investment by an entrepreneur in Thailand is not constrained by Thai domestic savings, since he can borrow elsewhere.

Technology remains a critical determinant of relative economic performance but with the free flow of information, technology is no longer dominant in particular countries. Johnstone (2003) therefore argues that the remaining variable, L , thus assumes increasing importance. Furthermore the combination of technological advance and international competitive pressures makes education a more important source of economic performance than ever.

The coalition government saw the importance of education in Kenya and went ahead to make primary education free. Later on secondary school tuition fee was subsidized to allow students from needy backgrounds to access education. According to Kenya National Bureau of Statistics (2013) the number of students enrolling into institutions of higher education continues to increase from 21,132 in 2008 to 37,672 in 2013 in private universities and 97,107 in 2008 to 201,690 in 2013 as recorded in public universities. This increase can be attributed to the elevation of 15 colleges to full universities by the coalition government in 2013, bringing to a total of 22 public universities. The challenge however has been to offer courses that will equip graduates with relevant skills and knowledge to participate in nation building towards achieving Vision 2030. In view of the Kenya Vision 2030 the Government intends to put in place measures that will raise the national GDP growth rate from the current 5.1% to over 10% by 2017 by creating more employment opportunities and bringing more equitable development in all regions of the country. HELB can support this by financing quality education and training. With quality education comes active participation in economic activities. However as much as many are enrolling into these institutions the question of how many are HELB sponsored is key. It is therefore important for HELB to ensure they recover their previous loans and sustain their scheme by reducing the probability of loss occurring.

2.5. Ideal Student loan scheme

For HELB to come up with an efficient self-protection and self-insurance scheme, an in-house audit must be done. Ideally the scheme should mirror the following characteristics as suggested by Woodhall (2004).

1. There should be an efficient institutional management, including adequate systems for the selection of borrowers, the disbursement of loans, record-keeping, data storage, and data processing. The challenge HELB is currently facing is keeping track of its clients after they leave school due to many reasons. There are many measure being put in place like (1) information sharing agreements the National Hospital Insurance Fund and the National Social Security Fund, (2) signing with banks to track borrowers with smart cards as means of ensuring their loans are recovered. Sound financial management, including setting appropriate interest rates to cover inflation, thus maintaining the capital value of the loan fund and covering administrative costs.
3. Effective criteria and mechanisms for determining eligibility for loans, for targeting subsidies, and for deferring or forgiving loan repayments.
4. Adequate legal frameworks to ensure that loan recovery is legally enforceable (e.g. law to be gazette by the Kenyan government in Chapter Six of the Constitution to compel private sector employers to demand clearance certificates form HELB before hiring new employers.
5. Effective loan collection machinery, using either commercial banks, the income tax system (as in Australia, the U.K., and several other developed countries), national insurance mechanisms (as in Ghana), or employers (as in Kenya and South Africa) to ensure high rates of repayment and to minimize default.
6. Information and publicity to ensure that recipients understand and accept the underlying principles and consequent obligations for the borrowing and repayment of loans. HELB has gone out of its way to explain the difference between a loan, scholarship and bursary in their website and main offices.

2.6. Student loans in developed countries

Johnstone (1986) suggests that internationally the United Kingdom, Germany, France, Sweden and the United States have adequate information on higher education costs and student support to facilitate a comparison analysis on their successes and weaknesses. The International Comparative Higher Education Finance and Accessibility Project at the State University in New York at Buffalo supports Johnstone's work on the basis of cost-sharing between students, parents, taxpayers and institutions(donors or philanthropists).

2.6.1. Reforms in United Kingdom

Barr (2011) cites how initially there were no tuition fees for students in the United Kingdom plus their living expenses were covered by tax-funded grant, mortgage type repayments and parental contribution. Furthermore he notes from 1998 things changed and students now had to pay £1,125 between 2003 and 2004 and the burden of living expenses transformed to parent contribution and an income contingent loan. The income contingent approach protect access because the loan has built-in insurance against inability to repay. Later on a White Paper government proposal was published January 2003 proposed that:

- From 2006 universities will be free to set fees between 0-£3,000;
- The system of income-contingent loans will continue in its current form to cover living costs but will be extended to cover all fees, i.e. a system of deferred charges;
- Grants for poor students will be restored; and,
- Student numbers will rise, increasing participation from 43 per cent to 50 per cent;

The White Paper also provides an additional £194 million to promote access.

More recently the Toty proposal seeks to abolish tuition charges altogether, keep participation at 43% and save the £194 million spending from the White Paper proposal.

2.6.2. Reforms in United States of America

The government of the United States of America set a national goal to have their country have the highest number of graduates from collage by 2020. Statistically, the high costs of joining collage has forced many students to take up expensive loans that prove to be overwhelming for them. The high default rates are then attributed to serious illness, unemployment or general drop out from the course.

The Middle Class Task Force chaired by the Vice President came up with a way student borrowers can repay their loans more comfortably. Under the new law, students who enrol from 2014 and onwards can choose to:

- **Limit Payments to 10 Percent of Income:** Borrowers choosing the income-based repayment plan will pay no more than 10 percent of their income above a basic living

allowance from the initial 15 percent under current law. This reform helps more than 1 million borrowers to be eligible to reduce their monthly payments.

- **Forgive Any Remaining Debt after 20 Years, or after 10 Years for Those in Public Service:** Borrowers who take responsibility for their loans and make their monthly payments will see their remaining balance forgiven after 20 years of payments, reduced from 25 years in current law. Teachers, nurses and those in the military will benefit most through forgiveness of debt after 10 years.
- **Fully Funded by Student Loan Reforms:** These new initiatives are funded by ending the current subsidies given to financial institutions that make guaranteed federal student loans. Starting July 1, all new loans will be direct loans delivered and collected by private companies under performance-based contracts with the Department of Education. According to the non-partisan Congressional Budget Office, ending these wasteful subsidies will free up nearly \$68 billion for college affordability and deficit reduction over the next 11 years(The White House, 2014).

2.7. Student Loans in Africa

Albrecht (1992) notes that the guaranteeing of student loans by the government in many countries reduces the risk to the lender and lowers the interest rate charged for borrowing funds to attend college. Johnstone et al. (1998) however points out imminent shortcomings like absence of credit-worthiness and collateral in other cases from needy students in poor countries. Developing countries like Kenya experience such incidences and in the event of default find it difficult to recover such loans and end up writing them off. This then puts a strain on the fund as it slowly ceases being a revolving fund.

According to the World Bank (2010) most countries budgetary practices remain largely traditional. This means university operating budgets use the previous year(s) as a baseline and make incremental changes based on general considerations such as the country's economic performance, government revenues, inflation rates, or institutional growth. Thus, in spite of the magnitude of financial needs confronting the institutions, their leeway, when considering a significant adjustment to their allocated amount, is minimal. Consequently, budget discussions are limited to fine-tuning the internal distribution of these fixed allocations among staff salaries, student services, staff development, and operational expenses. There are, in addition, other problems in budget

management, such as the lack of transparency in decision-making, fragmentation in budget responsibilities and the absence of measures for curbing out-of-control budgets in higher education. In Kenya and Rwanda, cost is per student while governments in Ethiopia, Ghana, Mozambique, and South Africa, supplement the core budgets of universities with competitive funds to stimulate qualitative improvements, research, and partnerships.

The World Bank research notes different forms of cost sharing are being implemented in most African countries. The report records as of 2009, at least 26 countries in Africa charge either tuition fees or other types of fees such as examination fees, registration fees, identity card fees, library fees, and management information system fees. The report states that countries, like Uganda, are implementing dual-track tuition policies whereby a certain number of free (or very low-cost) university places are awarded based on criteria such as academic excellence, income level, or positive discrimination, while other places are available on a tuition fee-paying basis or deferred-tuition policy. Even in some Francophone countries, such as Benin, where free higher education had long been considered a right, some public universities have chosen to charge fees for professional programs or programs of excellence.

The common problem however is cost recovery for student loans due to interest rates that are set far too low, grace periods and repayment periods that are unnecessarily long and exacerbate the losses, and loans that are implemented in such a way that students are frequently unaware that they are incurring a real repayment obligation. In addition, legal systems often make debt collection expensive, and record keeping cannot adequately keep track of students or graduates. Finally, insufficient numbers of jobs in African economies challenge the ability of university graduates to repay their loans (World Bank, 2010).

2.7.1. Higher Education Loan Board in Kenya

According to the 2012 annual report by HELB, The Board disbursed Kshs. 4,570,235,000 during the year compared to Kshs. 3,632,992,945 in 2010/2011. It is a notable increase but the number of students seeking financial support from HELB continues to rise as a result of the rapidly growing number of Kenyans seeking university education. This is because HELB caters for both public and private universities in the country. The challenge of recovering loans however still remains to be a challenge as it hinders more funds from being disbursed. The board has therefore developed

strategies to strengthen the recovery functions. The main one being enhanced collaboration with the Kenya Revenue Authority (KRA) as provided by the amendments to the KRA Act made by the 2005/2006 Finance Act which gave HELB the mandate to liaise with KRA with the view to accessing its data base to trace university graduates not repaying their loans. We also look to strengthening liaison with other institutions such as the like NHIF, TSC, NSSF and Government ministries to help in loan recovery the loaned. Other strategies include intensified loan recovery campaigns through both the print and the electronic media, initiating partnership with credit reference bureaus with the view to enhancing loan recovery, strengthening awareness among employers regarding their obligations on loan recovery as spelt out in the HELB ACT. Loan repayment methods have increased to incorporate the changes in technology. Apart from the commonly used cheque and cash deposits, the loaned can now pay via PesaPoint and Equity, ATMs, credit and debit cards and M-payment services.

2.7.2. Strategic Plan 2009-2014

The HELB strategic plan from 2009 to 2014 had the following objectives:

1. To finance all qualifying students.
2. To improve financial base by mobilizing funding from external stakeholders to bridge the financing deficit between loaning levels and available resources.
3. To improve productivity of the human capital by anchoring functional work-plans on an effective Performance Management System.
4. To formulate and implement communication strategy.
5. Carry out continual improvement of infrastructure.

According to the HELB (2013) an improvement was noted through networking with partners that included KPLC, NSSF, Nairobi Water, NHIF, Alumni and taking other measures which made the recovery to realize more than 10 % annual growth. Unfortunately most of the plans they had were not achieved. HELB only managed to finance 39% of the enrolled students to the universities as opposed to the intended 100%. Alternative loans were availed at 12% for the postgraduate students. More effort was made to Partner with banks like Kenya Commercial Bank and Commercial Bank of Africa to avail loans to students and their parents who could not qualify for

the HELB loans. The plan was not vigorously pursued. Few donors were identified among them, Visa Oshwal, Ford Foundation, Funzo Kenya, the strategy was not vigorously pursued due to lack of internal required capacity. The plan was to pursue government to allocate more funds to finance higher education, the contribution from the Government totaled Kshs.2.365 billion in 2013/14 against the requested amount of Kshs 14.5 billion which is equivalent to 15% of the expected amount.

2.7.3. Challenges that HELB may face

In their 2013 to 2018 strategic plan HELB mentions with rising domestic unemployment, Kenyans are seeking alternative employment opportunities outside the country which has affected the effectiveness of loan recovery from these employees. They continue to note the rising inflation rates due to increase in fuel and food prices has led to Kenyans placing first priority to the purchase of basic needs, leading to low repayment of HELB loans and at the same time increasing demand for HELB loans due to insufficient/inadequate family income. Inflation increases the cost of living. Currently inflation is at 7.76% and the macroeconomic policies are geared toward maintaining it below double digit. The Kenyan economy has been growing at a very slow pace of 2.8% in 2009 to 5.1% in 2012 against a target of 6.2%. The forecast is that it will grow at more than 10% by 2017.

The high population growth rate together with increased rural urban migration in search for employment which is unmatched with an equal fast creation of economic opportunities. This has led to a high unemployment rate amongst the youth and widespread poverty. This in turn increases the demand side for higher education financing both in number of applicants and the amount allocated to applicants. HELB however is not able to fully provide its loans because of overreliance on Government capitation for funding and inadequate funding due to limited student funding budgetary allocation. The government concentrates more on provision of funding to basic education up to secondary school level. This leaves the post-secondary education with inadequate funding. This is due to the belief that provision of basic education is more beneficial to the society while for higher education is more beneficial to the individual. There are also other high priority service provision areas such as security and infrastructure.

CHAPTER THREE

3. METHODOLOGY

3.1. Introduction

This chapter documents the systematic steps to be taken to solve the research problem discussed earlier (Kothari 2004). The methodology the research design, target population, sampling design, sample size, data collection and analysis. The study is designed to test individual student characteristics against loan repayment and default behaviour. This would provide a reserve using actuarial techniques to cover the student fund against future loss.

3.2. Research Design

Kothari (2004) defines a research design as the arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure. The study has a descriptive design as it seeks to describe the characteristics of a particular individual in relation to default or of a group how much should be reserved to cover possible loss due to default of the loan. It also has an exploratory design since the future probability of student loan that is not known is to be determined.

3.3. Population of the Study

The population of interest constituted of student who took up Federal student loans in 2008. The period is significant considering the financial crisis that led to many student loan defaults between 2008 and 2011. The data was available from The National Student Loan Data System (NSLDS) and Integrated Postsecondary Education Data System (IPEDS) in the United States of America.

3.4. Sample Size and Sampling Procedures

The study focused on undergraduate Federal student loan borrowers since it is the main provider in the USA and it has audited annual cohort default rate reports that are readily available.

3.5. Data Collection

The study used secondary data. The secondary data was quantitative in nature and collected from the National Student Loan Data System (NSLDS) and Integrated Postsecondary Education Data System (IPEDS) in the United States of America. All data was sourced from the internet.

3.4. Research Procedure

This research will aim to yield probability of default given factors like total amount of loan given, the age of student, period of study, gender and the institution of study among others. Finally determine the amount of future loss expected on defaulted student loans. This is to develop a default management program by preventing occurrence of default long before the loan beneficiary graduates.

It will be done in the following way:

Step 1: Identify experienced default which involves collecting secondary data from The National Student Loan Data System (NSLDS) and Integrated Postsecondary Education Data System (IPEDS) database of student loan beneficiaries whose loans were in default.

Step 2: Use a generalised linear model to determine probability distribution function of past default rates and identify which key factors affecting student loan default.

Step 3: Quantify the amount of expected loss by using measures like loan loss provisions, net charge offs and loan loss provision coverage ratios to determine reserve amounts on defaulted loans.

3.5. Empirical Approach

The loss event in the study is default of loan repayment. The prediction of risk of loss is key to assessing the probability of a loss event in a fixed period of time. Let Y be our **risk variable**, and the probability that Y takes a given value 0 or 1 will be the **risk predictor**. Consider individual risk and introduce risk variable Y_i , where subscript i indicates the individual. Therefore Y_i is equal to 1 if the individual i defaults in their loan repayment and 0 if they repay their loan. Thus, the occurrence of default can be predicted by finding the probability of $Y_i = 1$, or by finding the expected value of Y_i .

3.5.1. Risk Prediction

Risk prediction is based on individual characteristics. These can include personal demographic of individual i , such as age, gender, occupation, attitude as well as the amount of loan taken and the university attended. Conditional prediction of risk is done by taking into account individual characteristics.

Since Y takes only two values, 0 and 1, the conditional and marginal probabilities given X= x is denoted by:

$$P[Y = 1 | X = x] = p_1(x), \quad P[Y = 0 | X = x] = p_0(x)$$

Equation 2: Conditional and marginal probabilities

and the marginal distribution function of risk variable Y is denoted by :

$$P[Y = 1] = p_1, \quad P[Y = 0] = p_0$$

Equation 3: Marginal distribution function

We shall know whether the loss event has occurred or not at the end of the observational period, updating information on individual characteristics. Then the final updating yields:

$$P[Y = 1 | Y, X] = Y$$

Equation 4: Probability of loss

The expected value of Y can be used as a risk predictor. If the covariate values are taken into consideration we obtain the expected value of Y conditional on the covariate values of a given individual. For Y, the conditional expectation of Y is equal to the conditional probability that the probability that the individual will be a 'good' risk:

$$E(Y) = P[Y = 1] = p_1, \quad E(Y|X = x) = P[Y = 1|X = x] = p_1(x)$$

The property of iterated expectations relates marginal probability and marginal expectation and conditional probability and conditional expectation. The theorem tells us that for any random variable, the marginal expectation is equal to the expectation of the conditional expectation.

$$p_1 = \int p_1(x)f(x)dx \Leftrightarrow EY = E[E(Y|X)]$$

3.5.2. Risk Segmentation

The outcome of the risk prediction will be a partition of the population of HELB loan takers into two categories of 'good' and 'bad' expected risks, depending on whether or not the predicted probability of outcome 1 or 0 is sufficiently high or low respectively. The information available to HELB includes the amount of loan advanced, age of student, whether self-sponsored or under Joint Admission Board, the institution the student attends and the study period. If the HELB student loan providers are to decide whether to accept or reject an application for a student loan, given the category of loan requested and the available set of covariates representing the person's personal characteristics, the decision is a dichotomous variable. A denotes a subset of the set χ of all possible

covariate values. A contains information on amount of loan advanced, age of student, the institution the student attends and the study period.

The decision- making process is accept application if

$$X \in A$$

reject application if

$$X \notin A$$

Therefore what is important in the study is to determine the partition in order to minimize expected loss or maximize the expected gain. However the selection of applicants can lead to two types of errors, accepting application of a 'bad' risk individual or reject the application of a 'good' risk individual. Ultimately this means the loan will either never be repaid, or a trustworthy customer will be denied.

3.6. Measures used to quantify credit risk

Measures used to quantify credit risk include loan loss provisions, net charge offs and loan loss provision coverage ratios.

3.6.1. Loan Loss Provisions

The loan loss provision or reserve would represent HELB's best estimate of future loan losses. Suppose that HELB extends Ksh.50, 000, four-year loan to a male student in university X. If one year later after graduation the borrower runs into financial problems, using the characteristics of the student, HELB will create a loan loss provision. If they believe the borrower will only repay 60 percent of the borrowed amount, then HELB will record a loan loss provision of

$$Ksh. 50,000 * (100\% - 60\%) = Ksh. 20,000$$

3.6.2. Actual Losses: Net Charge-offs

It is possible that after creating a loan loss provision for a worrisome loan, HELB will discover how much the borrower is actually able to repay. At that moment they will record a net charge-off -- the amount of the loan that will never be repaid. In the earlier example, suppose that HELB is

only able to collect Ksh.10, 000 from the borrower. In this situation the net charge-off would equal Kshs.40, 000 -- an amount even greater than the original loan loss provision.

3.6.3. Loan Loss Provision Coverage Ratio

The loan loss provision coverage ratio is an indicator of how protected the loan provider is against future losses. A higher ratio means they can withstand future losses better, including unexpected losses beyond the loan loss provision. The ratio is calculated as follows:

$$\text{Coverage Ratio} = \frac{\text{Pretax income} + \text{Loan Loss Provision}}{\text{Net Charge-offs}}$$

Equation 5: Coverage Ratio

In the earlier example suppose that HELB reported pre-tax income of Ksh.2, 500,000 along with a loan loss provision of Ksh.800, 000 and net charge-offs of Kshs.500, 000. Its loan loss provision coverage ratio would equal 6.6.

$$\frac{\text{Ksh. 2,500,000} + \text{Ksh. 800,000}}{\text{Ksh. 500,000}} = 6.6$$

3.7. Chapter summary

This chapter presents the research methodology of the study. It shows the research design, data collection, population and sampling techniques and data analysis processes adopted. Chapter four presents the findings and results of data collected and analysed.

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS OF FINDINGS

4.1. Introduction

In this chapter, the focus is on the presentation of data and interpretation of the findings. It presents the generalized linear model analysis results.

4.2. Analysis of Data and Presentation of Findings

Data of number of government student loans and number of defaults in various Universities in the United States tabulated by university attended, type of loan taken & number of students' entering repayment was given. The data points totalled to 4055.

Default rate was obtained by dividing number of defaults by number of students entered into repayment with the objective of relating default rate to the risk factors given, in order to determine the best model to use to establish which factors affect default rate.

4.2.1. Box plot Analysis

To relate default rates to the risk factors, boxplots were used.

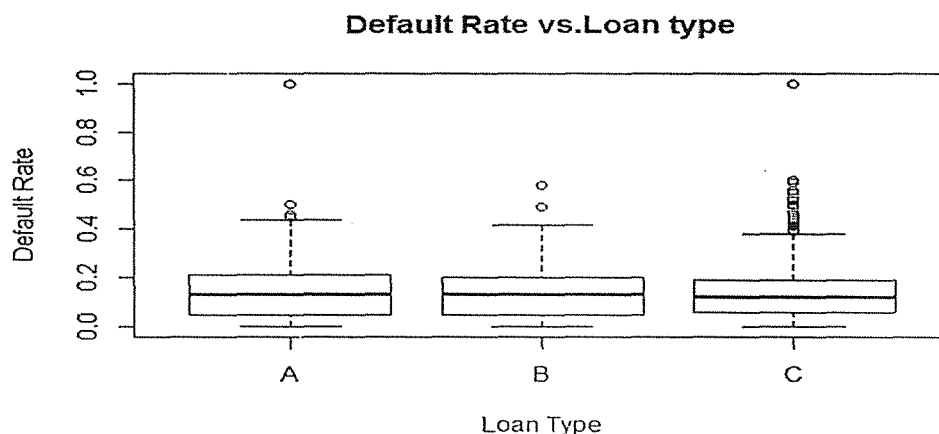


Figure 1: Default rate vs Loan Type Taken

Loan Type category were three and included Federal Family Education Loan(C), Direct only loan (A) and one that constitutes both(B). From the analysis the type of loan taken does not affect default rates as observed in figure 1.

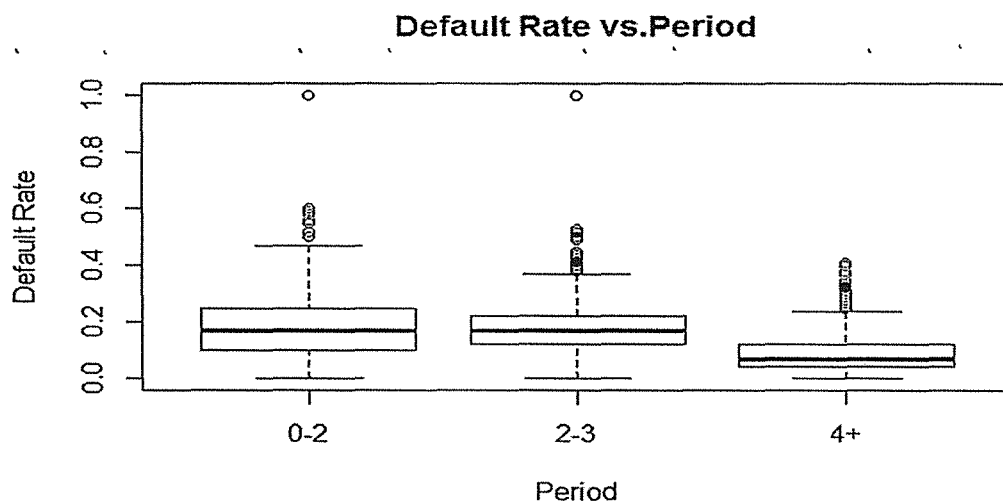


Figure 2: Default rate vs. Period of education

From the above plot it was seen that default rates vary with the period taken to complete a selected course. Students who take a short time in college have a higher probability of default.

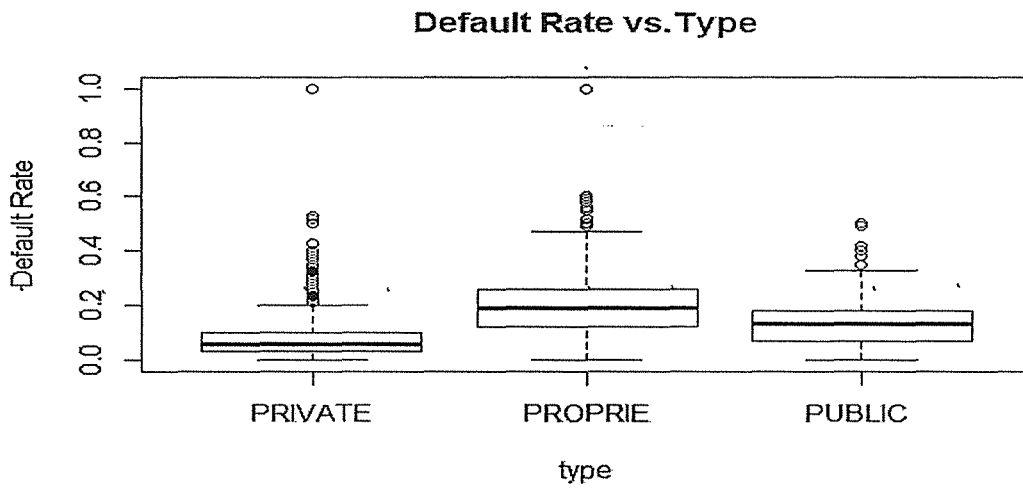


Figure 3: Default rate vs. Type of university

The type of universities identified from data collection included private universities, proprietary universities which are profitmaking and public universities. From the box plot, students from proprietary universities/ colleges have the highest probability of loan default.

From the boxplot the key risk factors to consider in the model are the type of university& period of course taken. The loan type taken could be ignored.

4.2.2. Choosing exponential family

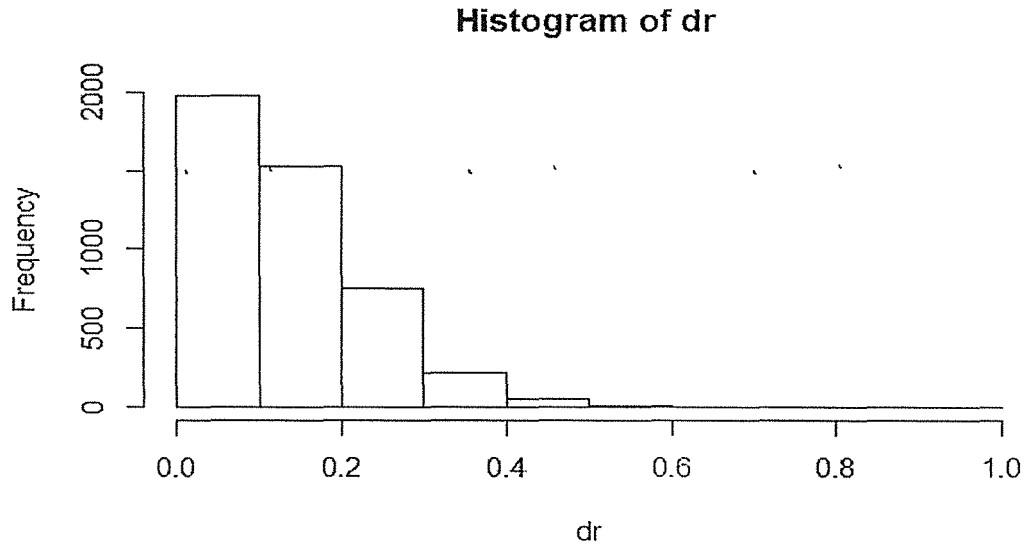


Figure 4: Histogram of past default rates

Having assumed that the default rates take only positive real values, I used a normal distribution of the default rates and also considered the fact that the student loan default rates are random, independent, skewed and the mean is approximately normally distributed.

4.2.3. Coding and Statistical Tests

RStudio was used to generate the generalized linear model and obtained results as shown in the table below.

Model	Parameters	Deviance	Degrees of Freedom	deviance	Degrees of Freedom	2(p-q)
modelS		3975.8	4324	1516.5	2	4
model1	Type	2459.3	4322	-822.9	0	0
model2	Period	3282.2	4322	1269.8	4	8
model3	Type+Period	2012.4	4318	18.7	4	8
model4	Type+Period+Type*Period	1993.7	4314	1993.7	4314	8628

Source: R-Script & Excel File

From the table, deviance generally reduces as parameters are added to the model. (The smaller the deviance, the better the model fits the data)

Model S is the saturated model, model with as many parameters as observations, with null deviance of 3975.8 on 4324 degrees of freedom.

Model 1, which has type of university as a parameter has a residual deviance of 2459.3 on 4322 degrees of freedom whereas Model 3, with Period of degree as parameter, has a residual deviance of 3282.2 on 4322 degrees of freedom. Model 1 in this case best fits the data compared to Model 2.

Model 3 with Type of University and Period of degree as parameter has a lower residual deviance, of 2012.4 and 4318 degrees of freedom, compared to the other two models. Model 3 is therefore an improvement of both Model 1 and Model 2.

Model 4 with Type of University, Period of degree and interaction of Type and Period has a lower residual deviance compared to Model 3. The result can be confirmed by carrying out a Chi-square test between Model 3 and Model 4 and you obtain 0.00000000000161 as the p-value which is less than the critical value of 0.05, therefore null hypothesis that the two models are significantly similar is rejected.

Adding the parameter involving interaction between Type of University and Period adds a significant value to the model. Therefore model 4 is an over-parameterized model and is the best model.

Also, conducting a chi-square test between Model 1 and Model 3 (*with a null hypothesis stating that the models are significantly similar*) you obtain a p-value less than 0.05. Therefore reject the null hypothesis, and this implies that Model 3 is significantly different from Model 1. Using Q-Q Normal plot, you find that Model 3 fits the data better than Model 1.

4.3. Summary and Interpretation of Findings

In order to calculate the probability of default, the following equation can be used:

$$P(\text{loss}) = \text{Intercept} + \text{Type of university} + \text{Period of education} + \text{Loan type}$$

The intercept generally represents the probability of loan default for a student who goes to any of the institutions and privately pays for their education. The table below is a summary of the different combination of student characteristics and their possible default rates:

Student	Type of University	Loan Type	Period of Course	Probability of default
Student A	Proprietary	FFEL	0-2	0.300099
Student B	Proprietary	Direct	2-3	0.187686
Student C	Proprietary	Both	4+	0.220283
Student D	Private	FFEL	0-2	0.196163
Student E	Private	Direct	2-3	0.152226
Student F	Private	Both	4+	0.116347
Student G	Public	FFEL	0-2	0.184118
Student H	Public	Direct	2-3	0.140181
Student I	Public	Both	4+	0.104302

From the above analysis, it can be concluded that the students with the highest probability of loan default are those who go to proprietary colleges, take up FFEL loans and are in school for at most 2 years. This may be explained by the fact that these colleges provide short courses and mainly related to business. The students with the lowest probability of loan default on the other hand go to public colleges, have both loans and are in school for more than four years. These students mostly undertake science courses which require longer periods and are mainly expensive and provided by public colleges.

Using the above probabilities, the amount of anticipated loss can be estimated by multiply the probability of default and the amount of loan taken.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1. Summary

Dealing with student loan default has become an important issue worldwide. Consequently this has spurred interest in education policy reforms in USA, U.K, Australia and other parts of the world. Besides the policy reforms, the study proposes an analytical way of dealing with future default. The study examined the key rating factors that help determine default behaviour among undergraduate students.

Data was collected for 4 years beginning 2008 and ending 2011 which is the financial crisis period. At this point in time student loan default rate was rampant in most parts of the world. Box plots were constructed to give a pictorial view of the possible factors to use in determine student loan default rate. Using the identified rating factors, generalized linear model was then run to determine the probability of loan default and statistics tests using p-value was carried out to differentiate the models used. The key rating factors identified were type of university and period of education.

5.2. Conclusions

From the findings of this research, the following conclusion are made. Generally students who stay in school for a short period of time seem to have the highest probability of default regardless of the university they attend. I think this is attributed to the lack of commitment and seriousness of the course undertaken. This is the total opposite for students who take more than four years because of the seriousness required from the course undertaken and probably the anticipated future pay is higher for this category of students.

The type of loan taken may not be a significant rating factor because different students have different needs however affects the total amount of debt the student has. This is important to the government to determine the amount of loss in case default is to occur.

5.3. Recommendations

Based on the findings of this study, the following recommendations arise. First, institutions that provide student loans need to come up with other rating factors to determine eligibility of a loan

and amount of loan to be granted. Qualitative factors need to be taken into consideration, for example the background of the individual, the need of the student.

One of the reasons for student loan default is unemployment. Unless governments provide more jobs for their growing workforce, the problem of default will still be recurrent. For example teach more entrepreneurial skills in the universities to avoid dependence on white collar jobs.

Also provide suitable payment options for the students to relieve them of financial strain at an early age. Providing more scholarships and philanthropists contribution to education can reduce the number of students taking up student loans.

5.4. Limitations of the Study

This study has various limitations. First, the study covered a few years, precisely only four years. The findings may not be elaborate enough and applicable in the future. The results given by this study are therefore limited to the three years of the data collected.

Secondly, the study had few rating factors due to nature of data recording by the data source. Finally, the study does not taking into account the possible effects of qualitative data. Age, gender, occupation data was not available and these factors have proved to affect loan default behaviour.

5.5. Suggestions for Further Research

The findings of this study can be improved if the study is expanded to cover a longer period of time. Further research can be carried out on the same topic but using data across a longer period of time, with the assumption that more data will provide better results.

The study can also be expanded to cover more countries with the aim of determining whether the same characteristics are the same worldwide.

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APPENDICES

R-Script Codes

```
lc=sus["Loan.Code"]
dr=sus["Dr"]
nd=sus["Df.no"]
nr=sus["No.rp"]
pd=sus["Period"]
tp=sus["Type"]
boxplot(dr~lc, weights = nd, main = "Default Rate vs.Loan type", xlab="Loan Type", ylab = "Default Rate")
boxplot(dr~pd, weights = nd, main = "Default Rate vs.Period", xlab="Period", ylab = "Default Rate")
boxplot(dr~tp, weights = nd, main = "Default Rate vs.Type", xlab="type", ylab = "Default Rate")
library(MASS)
fitdistr(dr, 'normal')$loglik
fitdistr(dr, 'Poisson')$loglik
fitdistr(dr, 'gamma')$loglik
fitdistr(dr, 'log-normal')$loglik
fitdistr(dr, 'geometric')$loglik
fitdistr(dr, 'weibull')$loglik
fitdistr(dr, 'exponential')$loglik
fitdistr(dr, 'cauchy')$loglik
fitdistr(dr, 'beta')$loglik
fitdistr(dr, 't')$loglik
fitdistr(dr, 'binomial')$loglik
hist(dr)
###saturated model
b <- factor(1:length(dr))
modelS=glm(dr~b, weights=nd, family =gaussian(), subset = nd>0)
summary(modelS)
```

```

####Model with Type
model1=glm(dr~tp, weights=nd, family =gaussian(), subset = nd>0)
summary(model1)

####Model with Period
model2=glm(dr~pd, weights=nd, family =gaussian(), subset = nd>0)
summary(model2)

####Model with Type+Period
model3=glm(dr~tp+pd, weights=nd, family =gaussian(), subset = nd>0)
summary(model3)

####Model with Tpye+Period+Type*Period
model4=glm(dr~tp+pd+lc, weights=nd, family =gaussian(), subset = nd>0)
summary(model4)

####Model with Tpye+Period^(2)
model5=glm(dr~tp+pd^2, weights=nd, family =gaussian(), subset = nd>0)
summary(model5)

plot(residuals(model5, type = "deviance"))
plot(residuals(model5, type = "pearson"))

plot(residuals(model1, type = "deviance"))
plot(residuals(model1, type = "pearson"))

plot(residuals(model2, type = "deviance"))

```

```
plot(residuals(model2, type = "pearson"))
```

```
plot(residuals(model3, type = "deviance"))
```

```
plot(residuals(model3, type = "pearson"))
```

```
plot(residuals(model4, type = "deviance"))
```

```
plot(residuals(model4, type = "pearson"))
```

```
### if plot of pearson residuals is identical to plot of deviance residuals
```

```
###then data is normally distributed
```

```
anova(model1,model2, test="Chisq")
```

```
### Null Hypothesis: Model1 & Model3 are equal
```

```
### If P-value is less than 0.05, reject Null Hypothesis
```

```
anova(model2,model3,test="Chisq")
```

```
### Null Hypothesis: Model2 & Model3 are equal
```

```
### If P-value is less than 0.05, reject Null Hypothesis
```

```
anova(model3,model4,test="Chisq")
```

```
### Null Hypothesis: Model3 & Model4 are equal
```

```
### If P-value is less than 0.05, reject Null Hypothesis
```

```
anova(model1,model3,test="Chisq")
```

```
### Null Hypothesis: Model3 & Model4 are equal
```

```
### If P-value is less than 0.05, reject Null Hypothesis
```

```
qqplot(dr,residuals(model1, type = "pearson"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model1, type = "deviance"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model2, type = "pearson"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model2, type = "deviance"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model3, type = "pearson"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model3, type = "deviance"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model4, type = "pearson"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
qqplot(dr,residuals(model4, type = "deviance"))
```

```
qqnorm(dr); qqline(dr, col = 2)
```

```
summary(model4)
```

```
anova(model4)
```