Strathmore UNIVERSITY

School of Computing and Engineering Sciences
Bachelor of Science in Electrical and Electronics Engineering
End of Semester Examinations
MAT 1101- Mathematics I

Date: 28^{th} *October*, 2021 Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

a) Show that $\sqrt{3}$ is indeed an irrational number. [3 Marks]

b) Given that $f(x) = (1-x)^{-1}$ and $g(x) = (5-x)^{0.5}$, find the domain and range of $g \circ f$. [3 Marks]

c) An arithmetic sequence has first term a and common difference d. It is given that the sum of the first four terms is more than the sum of the next four terms by 8. Also, the first term, third and sixth term of the sequence are three consecutive terms of a geometric progression. Find the exact values of a and d. [5 Marks]

d) Evaluate by partial fraction decomposition the integral [4 Marks]

$$\int_0^1 \frac{3x+5}{-3+2x^2-5x}\,dx$$

e) Express in $\sum$ notation hence evaluate the value of:
$21 + 24 + 27 + 30 + \cdots + 9000$. [2 Marks]

f) Find y' given that $x^y = y^x$. [4 Marks]

g) Find y' from first principles if $y = kx^2$ with k a constant. [3 Marks]

h) Find the area of the triangle with vertices $P(0,1,4)$, $Q(-5,9,2)$ and $R(7,2,8)$. [3 Marks]

i) Find y' given that $y = \frac{tan3x}{Cotx+e^{2x}}$ [3 Marks]

QUESTION TWO [15 Marks]

a) Show that $\int Cos^n x dx = \frac{1}{n} Cos^{n-1} x Sinx + \frac{n-1}{n} \int Cos^{n-2} x dx$ [5 Marks]

b) Evaluate $\int \frac{4x}{2x^2+1} dx$. [3 Marks]

c) By the method of partial fractions evaluate $\int \frac{4x^2+10x+4}{x(2x+1)} dx$ [5 Marks]

d) Evaluate $\int tan^2(x) dx$ [2 Marks]

QUESTION THREE [15 Marks]

a) Express $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$ in sigma (Σ) notation. [3 Marks]

b) The sum of the first 10 terms of an AP is 30 and the sum of the first 40 terms is 90. Find the first term of the AP and the least value of r such that the r^{th} term is negative. [3 Marks]

c) A geometric progression has positive terms. The sum of the first six terms is nine times the sum of the first three terms. The seventh term is 320. Find the common ratio and the first term. Find the smallest value of n such that the sum to first n terms of the progression exceeds 10^6 [5 Marks]

d) Give the general solution of a recurrence relation whose roots are $r_1 = r_2 = r_3 = \theta$ and $r_4 = \pi$. Evaluate this root at $\theta = 0, \pi = 1$ and $\theta = 1, \pi = 0$. [4 Marks]

QUESTION FOUR [15 Marks]

a) Evaluate $Lim_{x \to 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$ [5 Marks]

b) Find a vector $\mathbf{c}$ that is perpendicular to both $\mathbf{a} = \theta\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ (θ is a real number) [4 Marks]

c) Evaluate $\int xSin(3x) dx$. (*Use integration by parts*) [4 Marks]

d) Differentiate from first principles $y = \frac{2}{-x}$ [2 Marks]

QUESTION FIVE [15 Marks]

a) Given that $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, show that $\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$ [4 Marks]

b) Find the directional cosines of the line joining the points $A(-4, 1, 7)$ and $D(2, -3, 2)$ [3 Marks]

c) Show that $9^n + 3$ is divisible by 4 for $n \geq 1$ [4 Marks]

d) Find $y^{'}$ if $y = Cot^4(3x^2 + 2x + 1)$ [4 Marks]