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Modelling delayed correlation between interest rates and equity market returns

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Dissertation submitted in partial fulfillment of the requirements for the Degree of Master of Science in Mathematical Finance at Strathmore University

Strathmore Institute of Mathematical Sciences
Strathmore University
Nairobi, Kenya

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Abstract

This study models the interaction between interest rates and equity markets using wavelet analysis. This approach facilitates assessment of the lead-lag relationships in an intuitive way considering variation across frequencies and over time. Analysis is done progressively on varying scales where the lower scales encompasses high frequency components of the data over shorter time scales whereas higher scales encompass low frequency components over longer time scales. The study uses daily data obtained from Kenya for the period October 2003 to October 2019. The Nairobi Securities Exchange 20 share index returns are used as a proxy for equity returns whereas the interbank rates represent interest rates. Three key findings emerge: (1) There is at least two-months delay in the correlation between interest rates and equity market returns, (2) The correlation is lower (correlation coefficients of 0.3 and below, including negative correlation coefficients) in the lower time scales of 4 to 8 days and higher (correlation coefficient of 0.3 and above) in higher time scales of 512 to 1,024 days, and (3) Equity returns lead interest rates in Kenya during the period of study. Unlike common practice of assuming joint stationarity of variables, the findings reiterate the need for modelling dynamic relationships considering delays, time variation and scaling over time horizons. Investors, portfolio managers and policy makers alike should therefore be cognisant of the dynamic nature of the relationship between interest rates and asset markets bearing in mind that changes in one variable may not have immediate impact on the other.

Keywords: *Wavelet Analysis, Interest rates, Equity returns, Delayed Correlation, Wavelet Coherence*

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Abbreviations

ARDL	Autoregressive Distributed Lag Model
BP	British Petroleum
CPI	Consumer Price Index
CWT	Continuous Wavelet Transform
DCCA	Detrended Cross-Correlation Analysis
DWT	Discrete Wavelet Transform
MODWT	Maximal Overlap Discrete Wavelet Transform
NASI	Nairobi Securities Exchange All Share Index
NSE	Nairobi Securities Exchange
NSE 20	Nairobi Securities Exchange 20 Share Index
S & P	Standard and Poor
US	United States of America
WT	Wavelet Transform

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Dedication

This thesis is dedicated to the Almighty God for the grace to successfully undertake the research and to my parents for their continued immense support and encouragement through out my academic journey.

Chapter 1

Introduction

1.1 Background to the study

Correlation is a well-established concept for quantifying the relationship between two variables. In it's traditional sense correlation represents a constant measure of the relationship or interconnection between two or more variables. Correlation plays a fundamental role in financial theory and practice, especially in portfolio selection and risk management. Portfolio managers seek to hold negatively correlated assets to minimize the risk of loss in the event of adverse movements in the market related to one asset class. Risk managers on the other hand are interested in understanding correlation as a basis for managing the risk of loss arising from a change in the historical relationships between risk factors; i.e correlation risk.

Traditional measures of correlation such as the pearson measure of correlation; rank correlation (Spearman's, Kendall Tau and Goodman and Kruskal's gamma); and the coefficients of tail dependence, have assumed joint stationarity of the correlated variables. Teng et al. (2016) in their study find that simply using a constant or deterministic correlation may lead to correlation risk, since market observations give evidence that correlation is hardly a deterministic quantity. They posit that a constant correlation which means that two variables are jointly stationary is generally not true in the real world. Furthermore, most financial time series have been observed to be stochastic with random variations Tsay (2002), Adhikari and Agrawal (2013), Mills and Markellos (2008), Taylor (2008). The implication of the random variations is that the correlation between the variables also do change over time. In some cases it has been noted that correlation between two stochastic processes is rarely instantaneous, often the impact of changes in an independent variable are reflected on the dependent variable after a certain lapse of time. (Loretan and English, 2000)

This lagged correlation between financial variables therefore presents a challenge for risk managers to adequately anticipate and manage the risk inherent in variations of correlation of the variables of interest. An understanding of the variations in correlation over specified time-scale therefore provides useful information for risk managers in developing their strategies against correlation risk.

Modelling of delayed correlation is an emerging area of study with relatively limited body of literature and the goal of this study is to provide an empirical perspective on the relationship between interest rates and equity market returns in the Kenyan context. Studies undertaken to model the interrelation between interest rates and stock markets have produced mixed results compounding the challenge even further for market practitioners, AL-Naif (2017), Alam and Uddin (2009), Aurangzeb (2012),Owolabi and Adegbite (2014), Elly and Oriwo (2013), Amata et al. (2016) and Chirchir (2014). In some cases no significant relationship has been established between interest rate and share prices but changes in interest rate have negative relationship with changes in share price whereas in other cases a positive bidirectional relationship between interest rate and stock prices has been recorded AL-Naif (2017). In most countries however, a significant negative relationship between either interest rates with share prices, changes in interest rate with changes in share prices or both has been reported Alam and Uddin (2009). Consistent results on negative correlation between interest rates and stock markets are obtained by AL-Naif (2017) in some Arabian countries.

Similarly, Aurangzeb (2012) finds that interest rates have significant and inverse relationship with the stock market. The study concludes that this is consistent with theoretical literature in finance which stipulates that an increase in the level of interest rates in an economy has a negative effect on the stock market since the higher interest rates create an opportunity for investors to shift from investments in stocks to fixed income instruments to maximize their returns. On the other hand when interest rates decline in an economy, fixed income instruments become less attractive to investor and instruments such as the stocks and real estate become more attractive leading to positive movement in stock markets.

A negative relationship between interest rate and capital market performance is also reported by Owolabi and Adegbite (2014) in Nigeria. The significance of interest rate's influence on capital market performance is reinforced with the assertion that interest rate alongside exchange rate together account for 92.6% of the variation in the market capitalization of the Nigerian capital market.

Literature on the relationship between interest rates and the stock market in Kenya point to the conclusion of a negative or inverse relationship between the two variables. Elly and Oriwo (2013) find that indeed a relationship exists between the two most prominent macroeconomic indicators (interest rate and inflation) and stock prices. They find that the three month (91 -day) treasury bill rate is negatively related with the Nairobi Securities Exchange All share Index (NASI) while inflation rate has weak positive relationship with the NASI. This relationship, they conclude, affects investment in securities markets more so for foreigners. Simillarly Amata et al. (2016) find negative relationship between interest rate and stock market volatility although it is established to be weakly significant.

The study by Chirchir (2014) also result in somewhat similar findings, specifically in terms of the direction of the relationship, he however points out that this relationship is not significant. Using the weighted average lending rate by commercial banks as a proxy for interest rates and stock prices (proxied by the NSE 20 share index) he finds that causal relationship between interest rate and share price is insignificant. As regards the sign of causality, negative causality exists in both directions.

From an empirical standpoint, it is evident that the correlation between variables is not instantaneous and may be time varying Loretan and English (2000). Despite the deep-rooted agreement of time-varying correlations between financial variables, decisions by portfolio managers and capital market practitioners are often based on traditional measures which assume joint stationarity of the variables. Researchers have employed various methodologies to model relationships between financial variables, in most cases, these studies focus on causality, existence and direction of relationship, Alam and Uddin (2009), Aurangzeb (2012), Owolabi and Adegbite (2014), Elly and Oriwo (2013), Amata et al. (2016), Chirchir (2014) and in some cases broad duality of the relationship AL-Naif (2017).

Delayed or lagged correlation presents an important concept for investors, risk managers, portfolio managers, policy makers and capital markets stakeholders in general. It refers to the assessment of the relationship between time series among profiles shifted in time. Essentially, it measures how variations in one variable at a given time may have an impact on another variable of interest at a given time in future. This study proposes the use of wavelet approach, where decomposition in both time and frequency domains of the time series is made possible facilitating an intuitive analysis of the lead -lag relationship between the variables. Wavelet analysis is a mathematical approach that enables splitting of a given signal (or series in this case) into several components each reflecting the evolution through time of the signal at a particular frequency and undertaking an inter and intra assessment of these components.

1.2 Problem Statement

Studies undertaken to assess the relationship between macroeconomic variables and more specifically interest rates and stock market performance often provide no elucidation on the behaviour of this relationship over different scales or the effect of delay on such relationships Alam and Uddin (2009), Aurangzeb (2012), Owolabi and Adegbite (2014), Elly and Oriwo (2013), Amata et al. (2016), AL-Naif (2017). Similar research on the relationship between interest rates and share prices in Kenya such as Chirchir (2014) have fallen victim of assuming stationarity. The research focuses almost entirely on causality and direction of the relationship. These approaches are therefore not adequate for risk and portfolio managers who would be keen to understand the dynamic relationship between financial variables to help them better manage correlation risk. Correlation risk, the variation in relationships between two variables, is a critical notion in risk management. Increases in correlation risk often result in substantial unexpected losses for many financial institutions some of which are systemically important as was evident during the global financial crisis between 2007 to 2009 (Meissner, 2013).

This research tackles the question of correlation risk by studying correlation between interest rates and equity market returns factoring in delay. By employing wavelet analysis the study analyses the lead-lag relationship between the variables as they vary across frequencies and evolve over time. For this study whereas delay refers to the period of time before the occurrence of an event following the occurrence of one event, lag measures the countable gap or interval between the occurrence of two events. In this analysis therefore time lag is used to measure delay and derive delayed correlation.

1.3 Research Objective

The objectives of this study are;

- *To test the relationship between interest rates and stock market returns over different time scales.*
- *To assess the effect of delay in the relationship between interest rates and equity market returns.*

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Chapter 2

Literature review

2.1 Modelling Delayed Relationships

Existing literature provide a starting point to the analysis of delayed correlation. Some of the models that have been employed to attempt to model the delayed correlation include, delayed correlation matrix approach, Lag Time Series Analysis and Auto - Regressive Distributed Lag Time Series Analysis, Cross Correlation, Deep- Learning, Spectral Analysis, Wavelet Analysis and Cointellation model. We assess these models under this literature review outlining the findings under the respective studies and outlining the shortfalls of the methodologies in the context of this study.

2.1.1 Delayed Correlation Matrix

Mayya and Amritkar (2006) explore the use of delayed correlation matrices to compute delayed correlation. Using daily prices of 406 S & P 500 companies for the period 1991-1996 they build a symmetrized delay correlation matrix (C^s) for daily price fluctuations given by the expression;

$$C^s_{ij} = \frac{1}{2T} \sum_{t=1}^{T} (\delta x_i(t)\delta x_j(t+\tau) + \delta x_i(t+\tau)\delta x_j(t)) \tag{2.1}$$

Where;

T is the length of the time series data

τ is the time delay

i and j are two time series

δx is the change in the value of the time series

The distribution of eigenvalues obtained from a diagonalized symmetrized correlation matrix is then calculated and the process is repeated for the delays. They find that the largest eigenvalue has a very low value for the matrices constructed for delay values greater than two days. The conclusion derived from this observation is that there is minimal or no correlation among different entities of the multivariate time series for the stock market data beyond two days.

Subsequently Mayya and Santhanam (2007) in their study on Correlations, Delays and Financial Time Series, using the delay correlation matrix prove the concept of delayed correlation. In the context of data from the Bombay Stock exchange they find that there are groups of stocks that remain fairly correlated for up to 3 days.

The delayed correlation matrix approach, like a majority of the models assessed in subsequent literature under this research lacks the ability to compartmentalize data to assess inter-temporal correlation. From the two studies the model is only effective in analysis of short-term data.

2.1.2 Lag Time Series Analysis and Auto -Regressive Distributed Lag Time Series Analysis

Hansen (2017) employs a distributed lag model to model the relationship between the U.S. retail gasoline prices and the Brendt European spot price for crude oil using data over the period 1991-2016.

Assuming the two variables are strictly exogenous, the coefficients of the distributed model are the effects of the independent variable (crude oil prices) on the regrassand/response variable (gasoline prices). From his analysis he is able to determine that a unit percentage change in crude oil prices leads to a contemporoeous change in retail gasoline prices of 0.24 percent, further increases are recorded over the following six weeks.

Further he estimates the cumulative multipliers and it is found that a unit percentage change in crude oil prices leads to a 0.56 percent cumulative change in retail prices of gasoline over a six - week period with a large part of this change occurring within the first four weeks. The analysis shows that the impact of changes in crude oil prices is not instantaneous but rather is incorporated into the retail gasoline prices over time.

Hansen (2017) Further combines the autoregressive and distributed lag models to obtain the dynamically sound model that is the Autoregressive Distributed Lag Model (ARDL). The study models the Phillips curve; a plot of the change in CPI inflation as a function of unemployment rate in the US context.

The model is estimated without the contemporaneous unemployment rate to enable its use in forecasting. The study notes that the coefficients on lagged changes in inflation are large and negative. This is an indication that three - month changes in inflation tend to be followed by a reversal over the subsequent six - month period. Unemployment rate has a negative effect on inflation, implying that when unemployment rate is high inflation rate will decline and when unemployment rates fall the inflation rates rise. These findings validate the quintessential Phillips curve relationship.

From the model, all four coefficients add to zero, this is an indication that the rate of unemployment has no impact on inflation rate changes in the long-run. The inference is that when the rate of unemployment is higher than the recorded long-run average and constant for several periods (or lower than the recorded long-run average and constant for several periods) then the net effect on inflation is zero.

The statistical significance of the overall effect is assessed through a joint statistical test on all four lags of the unemployment rate and the F statistic with a p-value of 0.03 confirms significance of the effect. This test, known as a granger causality test, provides evidence to the effect that the unemployment rate in the US "granger causes" inflation rates. This implies that the unemployment rate is helpful in forecasting inflation but not that it has a causal relationship with inflation in a strict sense.

The key advantage of lagged- time series analysis and the Auto -Regressive Distributed Lag time series analysis is their ability to establish the temporal ordering of variables as a way of delineating which of variables may be the likely cause of the other. Their limitations however lie in their discrete nature and their inability to model relationships in a dynamic context. Their applicability in this study is therefore limited.

2.1.3 Cross Correlation

Cross Correlation has been employed in various contexts to model delayed correlation and time delays. Lin et al. (2012) conduct a study on cross-correlations of stock markets based on Detrended Cross-Correlation Analysis (DCCA) and time-delay DCCA.

They find that the cross-correlation exponents and crossovers exhibit periodical uncertainty changes with a time delay. They also observe that the scaling behaviors in the three regions in the study coincide for a time-delay greater than (1 year).

The two key findings of their study is the manifestation of an absence of cross-correlation, where considered data tend to behave as two uncorrelated series with a cross-correlation exponent around 0.5 in one case. In the other case, as similar patterns in stock markets repeat year by year, crossovers still exist and cross-correlation exponents are slightly larger than 0.5 indicating the existence of a weakly persistent scaling behavior. Essentially, the cross-correlation exponents and crossovers have periodical uncertainty changes with time delays.

The linearity of the cross- correlation approach in addition to being a discrete model encompass the shortcomings associated with the model. From empirical studies, interest rates and capital markets performance metrics have been known to follow stochastic process Tsay (2002), Adhikari and Agrawal (2013), Mills and Markellos (2008), Taylor (2008).

2.1.4 Deep- Learning

Moews et al. (2019) explore the presence and the use of lagged correlations using a combination of deep feed-forward neural networks and exponential smoothing. The Lagged correlation is then deployed to predict trend change in stock market data. Their model uses gradients from a linear regression as input-features for feed-forward neural networks. Their findings show how deep learning approaches can be used in time series analysis as well as how complex interdependencies can be extract from useful features provided by linear regression derivatives.

A shortcoming of the model that they identify in their study is that the model is not effective in combating noise since there is no transfiguration in time and frequency to facilitate computation of local variations representation on different time scales. Essentially in the context of this study where the objectives relate to analysis of the relationship between two variables in both time and frequency domains the model falls short. As part of their recommendations Moews et al. (2019) posit that their model could be enhanced by use of wavelet analysis which would enable more elaborate extraction of relevant information from intervals in time series.

2.1.5 Spectral Analysis

Iacobucci (2005) shows that there is a Phillips relationship between inflation and unemployment, over the archetypal business cycle components (fourteen and six year), even if this does not show in the raw data. Moreover, through the use of phase spectrum analysis, it emerges in her research that unemployment is the leading variable while inflation is the lagging variable, with inflation being delayed for about one year.

A major limitation of the approach is noted as the inability to assess frequency component of the time series and its evolution over time given that the spectrum only depends on the frequency but does not depend on time. In essence, inspection of the spectrum would not be sufficient to detect a subsample interval if the data set has a specific frequency component that remains "activated" or dominant in the subsample. The study also recommends, among other potential methodologies the use of wavelet analysis to keep the information about time-localization.

2.1.6 Wavelet Analysis

Kim and In (2005) in their study on the relationship between stock returns and inflation conduct a regression analysis in the wavelet domain and establish varying correlations over different time periods. Their study finds that the wavelet correlation in the short term is positive (over a 1-month period) the rest of the wavelet scale they observe a negative relationship.

Ramsey and Lampart (1998) who are credited with introducing the concept of wavelet analysis, note that the wavelet approach facilitates the separation of scales of variation into a sequence of scales that can be decomposed orthogonally. They note varying correlation between variables of study subject to difference in time and frequency.

According Skoura (2019) the intuition behind the wavelet approach enables the assessment of lead - lag relationships in a way that determines the variation of such relationships across frequencies as well as evolution over time. Using wavelet analysis it is found that in the short and medium term the lead - lag relationship between the variables changes depending on the time period. Fundamentally, the study finds evidence that the lead - lag relationship between variables over time is dynamic in nature.

Wavelet Analysis is a relatively new tool in finance and economics and takes its roots from filtering methods and Fourier analysis, but overcome most of the limitations of these two methods. Its principal advantages derive from, first combined information from both time and frequency domain and second from its flexibility as it does not make strong assumptions concerning the data-generating process for series being considered Masset (2015). Another strength of Wavelet analysis has been noted as the ability to assess simultaneously the significance of the covariation at different frequencies and evolution of this significance over time.

2.1.7 Cointelation Model

Damghani (2013) introduced the cointelation model which is an amalgam of the correlation and co-integration models.

He first defines the stochastic differential equation of a leading stochastic process as;

$$\frac{dS_t}{S_t} = rdt + \sigma dW_t \tag{2.2}$$

where; $S_t(t \geq 0)$ is a stochastic process, $dW_t \sim \mathbb{N}(0, dt)$, dt is the change in time and r is the interest rate;

The correlation model is then defined by ;

$$\begin{aligned} \frac{dS_t}{S_t} &= rdt + \sigma dW_t \\ \frac{dS_{r,t}}{S_{r,t}} &= \sigma \left(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp} \right) \end{aligned} \tag{2.3}$$

$S_{r,t}(t \geq 0)$ is a stochastic process, σ is the volatility of the values of the stochastic process, $dW_t^{\perp} \sim \mathbb{N}(0, dt)$ is an independent Brownian motion, and ρ represents the correlation coefficient between the two stochastic processes with the correlation measure being the Pearson's correlation coefficient (defined as below for any two variables x and y);

$$\rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y}$$

whereas the cointegration model is defined by;

$$\begin{aligned} \frac{dS_t}{S_t} &= rdt + \sigma dW_t \\ dS_{g,t} &= \theta \left(S_t - S_{g,t} \right) dt + \sigma S_{g,t} dW_t^{\perp} \end{aligned} \tag{2.4}$$

S_{gt} is introduced in the equation to prevent the stochastic process from ever going negative.

$S_{r,t}(t \geq 0)$ is another stochastic process, $dW_t^{\perp} \sim \mathbb{N}(0, dt)$ and θ represents the mean reversion speed of the "lagging" stochastic process towards the leading stochastic process ($\theta \in [0, 1]$)

The cointelation model is derived from equation 2.3 and 2.4 and takes the form Damghani (2013);

$$\begin{aligned} \tfrac{dS_t}{S_t} &= rdt + \sigma dW_t \\ dS_{l,t} &= \theta \left(S_t - S_{l,t} \right) dt + \sigma S_{l,t} \left(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp} \right) \end{aligned} \tag{2.5}$$

ρ is defined as the correlation of the cointelation, θ, σ, dW_t and $dW_t^{\perp}$ are as defined above.

In the cointelation model two stochastic processes have the same long-term drift (similar to the cointegration model) but also have a model that has characteristics of the correlation model most of the time or at least in the short term.

This study by Damghani (2013) employs this model to assesses the correlation between British Petroleum's (BP) performance and performance of the oil market. The intuition is that since BP is an oil production company, its performance in the long- term would be linked strongly to the prices of oil. The analysis finds a discrepancy between short and long term correlation/interdependence of BP returns and oil prices. According to the paper the difference is attributed to the fact that oil prices and BP have the same drift in the long run but due to numerous noises that can only impact one of these assets and not the other, there is a weaker perceived relationship in the short run. The study concludes that the cointelation model is the best model in this scenario given the duality of the short-term and long-term relationship presented in the results and with limited literature in financial mathematics on measuring time-varying correlation.

The cointelation model does present a promising model for lagged/delayed correlation, its imperfection lies in its dependence on a mean reverting model. For a mean-reverting process in instances where the driving dynamic moves rapidly on average, the induced measured correlation for the lagging process returns increases drastically. This is because, in a mean-reverting process, the diffusion process always counteracts when the drift process significantly goes up (or down, as opposed to hesitatingly switching direction). In these situations the model returns a high measured correlation for returns.

2.2 Summary of Literature Review

The objective of the study is to model delayed correlation, the variables under study are interest rates and equity market returns. Models of delayed correlation can be dichotomized into traditional models and emerging models. In our study, traditional models include delayed correlation matrices, Lag time series analysis including Auto -Regressive Distributed Lag (ARDL) models and Cross Correlation analysis. Emerging models include Spectral Analysis, Deep- Learning, Wavelet Analysis and the Cointellation model. Overall, the traditional models have deep rooted limitations which emerging models are increasingly seeking to address.

The delayed correlation matrix approach, for instance, lacks the ability to compartmentalize data to assess inter-temporal correlation Mayya and Amritkar (2006). Though well suited for establishing the temporal ordering of variables as and studying causality, lag-time series analysis and ARDL models perform dismally in studying dynamic relationships Hansen (2017). Cross Correlation models - both Detrended Cross-Correlation Analysis (DCCA) and time-delay DCCA assist in revealing time-variation but are only found to be capable of studying linear dynamics Lin et al. (2012).

Innovative approaches to study delayed correlation have also registered various shortfalls. For instance, spectral analysis Iacobucci (2005) which does facilitate the characterization of the frequency of a time series, ignores the time component making it difficult to apply for investment and policy choices. Deep feed-forward neural networks and exponential smoothing Moews et al. (2019) fail to effectively combat noise in the time series since there is no transfiguration in time and frequency to facilitate localization to different time scales. The cointelation model an amalgam of the correlation and co-integration model introduced by Damghani (2013) presents a promising model for lagged/delayed correlation but the reliance of the model on a mean reverting process renders it inaccurate in modelling correlation over time. For this study the wavelet approach is adjudged as the most appropriate methodology given its suitability for analysis of dynamic time-series and the ability to facilitate a decomposition in both time and frequency domain in doing so.

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Chapter 3

Methodology

Based on the review of literature, this study employs the wavelet approach to evaluate correlation in a time - varying context. The methodology overcomes most of the limitations of both the traditional and emerging models through it's ability to facilitate decomposition in both time and frequency domain. The model does not make strong assumptions concerning the data-generating process for series being considered Masset (2015) and has the ability to assess simultaneously the significance of the covariation at different frequencies and evolution of this significance over time.

3.1 Data Description

This research uses the interbank lending rate as a proxy for interest rates. Equity returns are measured using the change in Nairobi Securities Exchange 20 share index (NSE 20). The interbank lending rate is the overnight lending rate between banks and it defines the short end of the yield curve. It serves as a good proxy and benchmark for interest rates given that it tends to be driven by monetary policy and expectations on monetary policy. The NSE 20 share index on the other hand is the benchmark index for the Kenyan equity market and is obtained by computing the geometric mean of the prices of the NSE's top 20 stocks. The top 20 stocks account for over 80 % of the NSE's market capitalization and the index gives a good barometer of stock market activity in Kenya.

The sample for both variables comprises of 16 year daily data (from October 2003 to October 2019) totalling to 3316 observations per variable. The choice of the duration of study is based on availability of data and is also informed by the need for sufficiently long-term, high frequency data needed to provide meaningful statistical and empirical insights.

The interbank lending rate data was obtained from the Central Bank of Kenya whereas data on the NSE 20 share index was sourced from the Nairobi Securities Exchange being the reliable primary sources of these datasets. The quality of the data was then enhanced through cleaning, interpolation and sorting using Microsoft Excel before analysis was done using R software, version 3.5.0. The tests in the analysis were conducted multiple times and consistently yielded the same results. The report was documented using Latex version 5.0.3.

3.2 Wavelet Approach

The study uses the Daubechies wavelet, Daubechies (1992) and deploys the maximal overlap discrete wavelet transform to the bivariate time-series allowing for the decomposition of the time series into scales with each scale representing an investment horizon Marco et al. (2014). We work with a least asymmetric wavelet filter that has eight coefficients with non-zero values i.e the length of the filter is equal to eight, the boundary conditions are also reflected. We then apply the maximal overlap discrete wavelet transform up to a level $J \leq 8$ this produces a single vector of smooth coefficients S_8, that represents the underlying smooth behaviour of the data at the the lowest frequency scale, and eight vectors of details coefficients, d_8, d_7, d_6, d_5, d_4, d_3, d_2, d_1. The eight vectors represent progressively finer scale deviations from the smooth behaviour.

With $J \leq 8$ we reconstruct eight wavelet details vectors D_8, D_7, D_6 D_5, D_4, D_3, D_2, D_1 and one wavelet smooth vector, S_8, each associated with a particular scale 2^{j-1}. We then calculate wavelet variances for the both time series which is employed in calculation of scale correlation analysis. This allows the restriction of the variation in the variables under study in an indicated specific scale j and assessment of the interaction between the variables at each of the scales. The study also introduces a time lag between the variables and computes the cross correlation between the variables with the delay introduced by the lag. This study uses the Bayesian information criterion to determine the appropriate lag for the analysis. This is achieved using the R software and the process involves first estimation of a vector autoregressive model that fits the data and then identifying the appropriate lag by deploying the Bayesian information criterion.

From the analysis we are able to assess the dependence between the two time series over different time and frequency i.e to determine variation in correlation over different time periods. The analysis also enables assessment of whether the correlation is stronger over short time periods or over long time periods. In addition, with the introduction of the lag we determine the delay in correlation (or lack thereof) between the two time series. The wavelet approach incorporates tapering in order to avoid spurious correlations at large lags.

Graphical representations of the outputs are also presented in line with principles of Ramsey (2002) who argues that visual inspection of two variables is a good exploratory tool for assessing delay over time or phase variations between variables.

3.2.1 Wavelet

A wavelet is a mathematical function which enables splitting of a given signal (or series in this case) into several components each reflecting the evolution through time of the signal at a particular frequency Masset (2015). It is predicated on a function called *the mother wavelet* denoted as $\psi(t)$. This function is defined on the real axis and must satisfy two conditions:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \tag{3.1}$$

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1 \tag{3.2}$$

Jointly, these conditions imply that at least some coefficients of the wavelet function must be different from zero and that these departures from zero must cancel out.

Additional conditions imposed in order to conduct wavelet analysis relate to admissibility, where a wavelet function is admissible if its Fourier transform;

$$\Psi(f) = \int_{-\infty}^{\infty} \psi(t)e^{-i2\pi ft}dt, \tag{3.3}$$

is such that;

$$C_\Psi = \int_0^\infty \frac{|\Psi(t)|^2}{f} df \quad \text{satisfies} \quad 0 \leq C_\Psi \leq \infty \tag{3.4}$$

3.2.2 Wavelet Transform

A wavelet transform (WT) is the decomposition of a series into a set of basis functions consisting of contractions, expansions, and translations of a mother function $\psi(t)$, called the wavelet, Daubechies (1990). It is is a mapping from $L^2(R) \to L^2(R^2)$, but one with superior time-frequency localization as compared with the short time Fourier transform. It inherently offers a generalization of the short time Fourier transform.

The continuous wavelet transform (CWT) is a non-numerical approach that provides an over-complete representation of a series by letting the translation and scale parameter of the wavelets vary continuously (Akansu and Haddad, 2001).

It is defined as;

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \leftrightarrow \psi_{ab}(\Omega) = \sqrt{a}\psi(a\Omega)e^{-jb\Omega} \tag{3.5}$$

at a scale (a>0) $a \in \mathbb{R}^{+*}$ and translational value $b \in \mathbb{R}$ where $t = time$, $\Omega = frequency$ and j is the number of transform levels, $j = 0, 1, 2, \ldots, J-1$

The continuous wavelet transform maps a function $f(t)$ onto time-scale space by;

$$W_f(a,b) = \int_{-\infty}^{\infty} \psi_{ab}(t) f(t) dt = \{< \psi_{ab}(t), f(t) >\} \tag{3.6}$$

Where;

$$f(t) = \underbrace{\frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_0^\infty \frac{dadb}{a^2}}_{\text{summation}}$$

$$\underbrace{W_f(a,b)}_{\text{coefficients}}$$

$$\underbrace{\psi_{ab}(t)}_{\text{Wavelet}}$$

In Discrete Wavelet Transform wavelets are discretely sampled. Its key advantage is that unlike Fourier transforms it has temporal resolution where it captures both frequency and location information (location in time).

The mother wavelet $\psi(t)$ is shifted and scaled by powers of two;

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{t - b2^j}{2^j}\right) \tag{3.7}$$

where j is the scale parameter and is an integer.

The analysis in this paper employs a modified version of the discrete wavelet transform, the Maximal Overlap Discrete Wavelet Transform (MODWT). The advantanges of MODWT include; the fact that down-sampling is not used in this approach, resulting in vectors of equal length for all the wavelet coefficients at all scales, which corresponds to the length of transformed time series. The approach therefore is not restricted to sample sizes that are powers of two Akansu and Haddad (2001). This makes the MODWT a translation-invariant type of transform and it is not sensitive to the choice of the starting point of the examined process and which according to Brassarote et al. (2018) makes it appropriate for identifying stationary and non-stationary behaviors in signals. This is an important feature in the context of this analysis that aims to assess the relationship in a time varying context.

The Maximal Overlap Discrete Wavelet Transform (MODWT) differs from the conventional discrete wavelet transform (DWT) in the sense that it is not orthogonal and it is eminently redundant in the sense that a level J_0 transform consists of $(J_0 + 1)\,N$ values rather than just N. The fact that MOWDT is redundant enables alignment of the coefficients from the decomposed wavelet and scaling at each scale with the initial data set. This facilitates a juxtaposition of the time series and its decomposition to facilitate comparison. By being redundant the effectiveness of the degrees of freedom of the wavelet coefficients at each scale is increased resulting in a reduction of the variance of the computational estimates. MODWT aligns wavelet coefficients at each time point with the original data index enabling simultaneous analysis of localized signal variation with respect to scale and time, and the temporal relation to events (Ozaydin and Alak, 2018).

MODWT is defined naturally for all sample sizes, N, Percival and Walden (2000). Given an integer J such that $2^J < N$ where N is the number of data points, the original time series represented by the vector $X(n)$ where $n = 1, 2, \cdots, N$, can be decomposed on a hierarchy of scales by details, $D_j(n)$, and a smooth part, $S_J(n)$, that shift along with X:

$$X(n) = S_J(n) + \sum_{j=1}^{J} D_j(n) \tag{3.8}$$

with $S_j(n)$ generated by the recursive relationship

$$S_{j-1}(n) = S_J(n) + D_J(n) \tag{3.9}$$

The MODWT details $D_j(n)$ represent changes on a scale of $\tau = 2^{J-1}$, while the $S_j(n)$ represents the smooth or approximation wavelet averages on a scale of $\tau_J = 2^{J-1}$.

MODWT enables us to optimize both time and frequency resolution by choosing the best window width for a particular frequency band (scale) Kestin et al. (1998). The benefit derived from this is the ability to localize the two time series into low frequencies over long time periods and high frequencies over short time periods.

3.2.3 Wavelet Analysis of Correlation and Cross Correlation

MOWDT ensures the wavelet coefficients vectors are of equal length. Therefore, for a series x_t , $t = 1, 2 \ldots, N$, $j = 1, 2 \ldots, j^m$, coefficients vectors of the wavelet and one vector of scaling coefficients of length N, the wavelet correlation $\rho_{xy}(j)$ between time series x_t and y_t at scale j is defined as;

$$\rho_{xy}(j) = \frac{\text{cov}\,(W_x(j,t)W_y(j,t))}{[\text{Var}\,(W_x(j,t))\;\text{var}\,(W_y(j,t))]^{\frac{1}{2}}} \equiv \frac{\gamma_{xy}(j)}{v_x(j)v_y(j)} \tag{3.10}$$

where $v_x^2(j)$ and $\gamma_{xy}(j)$ denote wavelet variance and covariance, respectively. The wavelet correlation estimator is thus written as:

$$\hat{\rho}_{xy}(j) \equiv \frac{\hat{\gamma}_{xy}(j)}{\hat{v}_x(j)\hat{v}_y(j)} \tag{3.11}$$

where $\hat{\gamma}_{xy}(j)$ is the wavelet covariance estimator and $\hat{v}_x(j)^2$ and $\hat{v}_y(j)^2$ are estimators of wavelet variance at scale j for time series x_t and y_t

The wavelet cross-correlation is computed in exactly the same way but introduces defined time- lag between the two variables under study. The appropriate lag in this study is determined using the Bayesian information criterion.

$$\rho_{xy}(j) = \frac{\text{cov}\,(W_x(j,t)W_y(j,t+\tau))}{[\text{Var}\,(W_x(j,t))\,\text{var}\,(W_y(j,t+\tau))]^{\frac{1}{2}}} \tag{3.12}$$

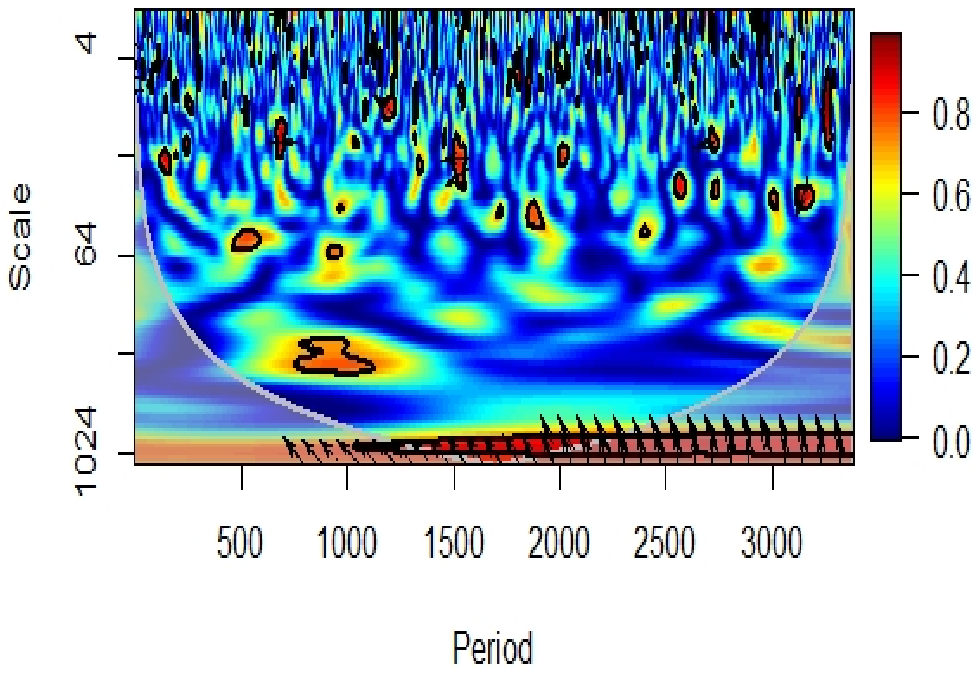
Where τ is the time lag.

3.2.4 Wavelet Coherence

To further assess the variation in correlation the study assess the coherence of the resulting wavelets from the study. From this assessment we determine when the variables are moving in the same direction, when they move in the opposite direction or when a particular variable is leading or lagging.

To assess this we use a wavelet coherence map, a sample of which is displayed figure 3.1, which locates periods in time-frequency spectrum where there is co-movement between two time series.

Figure 3.1: Sample Wavelet Coherence Map



Shades of red in the map represent regions with high interrelation, while the blue coloration signify regions of lower interrelation between the series. Right pointing arrows indicate that the variables are in phase and left pointing arrows are an indication of an anti - phase relationship between the variables. Right-down or left-up pointing arrows indicate that the first variable is leading, while right-up or left-down pointing arrows show that the second variable is leading. Time (number of days) is displayed on the horizontal axis, while the vertical axis shows the frequency (the lower the frequency, the higher the scale)

Shades of red in the map represent regions with high interrelation, while the blue coloration signify regions of lower interrelation between the series. The blue area beyond the areas of significant interrelation (shown by the concave decreasing lines along the left and right vertical axes) represent time and frequencies with no dependence or zero correlation between the two time series.

Arrows in the wavelet coherence map represents the lead/lag relationship between the series under study. A zero phase difference means that the two time series move together on a particular scale. Right pointing arrows indicate that the variables are in phase and left pointing arrows are an indication of an anti - phase relationship between the variables.

Two series are in phase if they move in the same direction and are anti-phase if they move in the opposite direction. Right-down or left-up pointing arrows indicate that the first variable is leading, while right-up or left-down pointing arrows show that the second variable is leading.

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Chapter 4

Data Analysis and Findings

4.1 Descriptive Analysis

4.1.1 Central Tendency, Dispersion, Covariance and Correlation

The analysis commences from the first principles, where we compute the mean, variance, covariance and correlation of the two variables. We observe a generally low variance of the equity returns, implicitly, a low standard deviation, indicating that the stock market in Kenya has been relatively stable over the period of study. The average daily equity return over the period of the study is relatively low standing at 0.0178 %, the average interbank lending rate over the period is 6.57 %

The cursory covariance and correlation of the two variables during the period of study are found to be positive. This is contrary to financial theory where interest rates have been deduced to have a negative relationship with stock markets performance. A negative covariance and correlation is however noted between the interest rates and the price level of the equity market as measured by the value of the NSE 20 share index.

Table 4.1: Data Descriptions

Variable	Value
Interbank Lending rate mean	6.5705
Equity Returns mean	0.0002
Interbank Lending rate variance	24.3224
Equity Returns variance	0.0001
Covariance between Interbank rate and Equity Returns	0.0011
Correlation between Interbank rate and Equity Returns	0.0254
Covariance between Interbank rate and NSE 20 share index	-155.2088
Correlation between Interbank rate and NSE 20 share index	-0.0374

4.1.2 Trend and Distribution

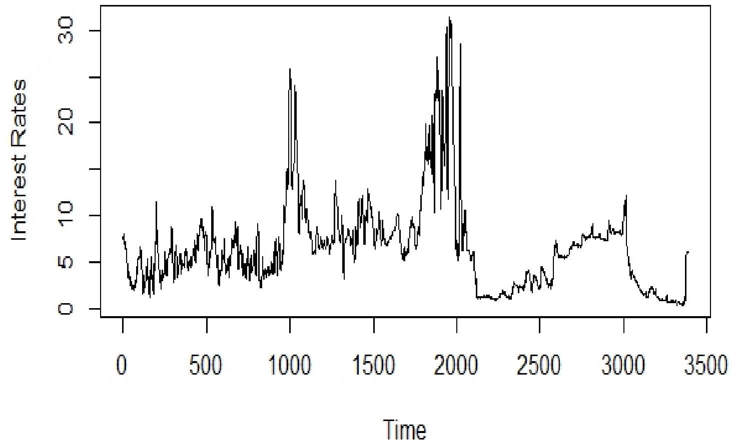


Figure 4.1: Interest Rates between October 2003 to October 2019

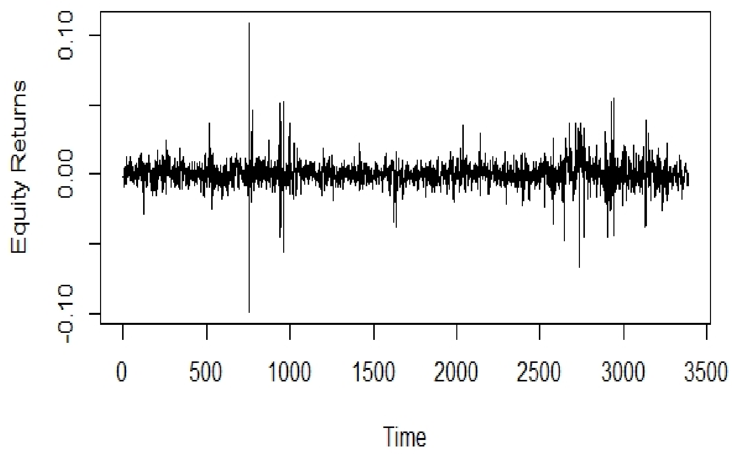


Figure 4.2: Equity Returns between October 2003 to October 2019

An assessment of the trend characteristics of the two time series reveals some common stylized facts about financial time series. The level of interest rates and the equity returns over the period of study display a mean reverting tendency, with the values of the variables oscillating around a specific long term average. In addition, the plot of the time series record non - constant variability from the long term average over the period of study.

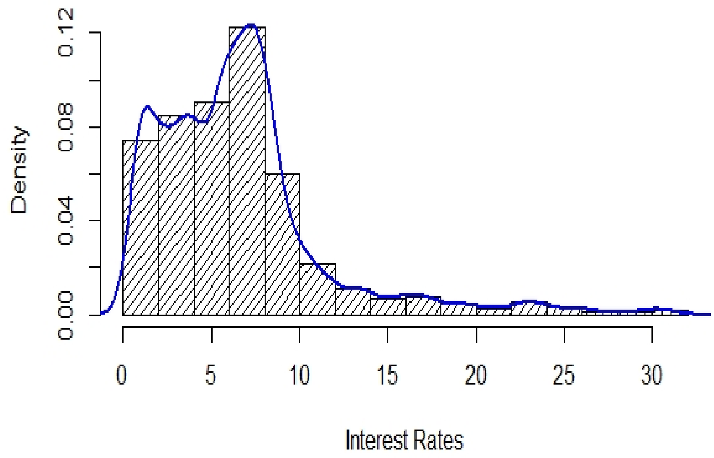


Figure 4.3: Histogram of Interest Rates between October 2003 to October 2019

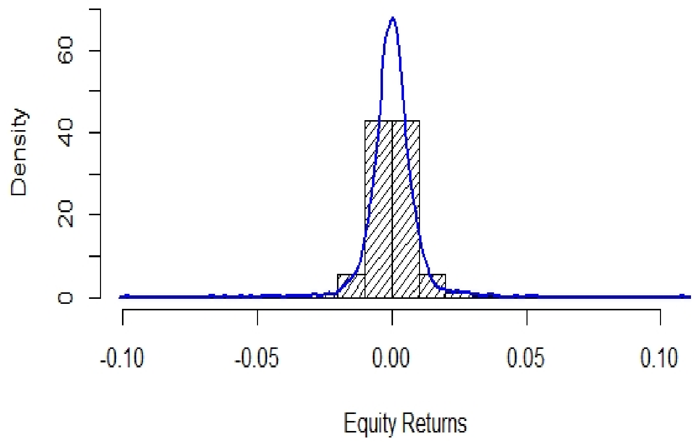


Figure 4.4: Histogram of Equity Returns between October 2003 to October 2019

The level of interest rates is found to have a non - normal distribution and positive skewness, implying that there have been a few outliers in the values of the interest rates over the period of study. The value of the interest rates also point to a bimodal situation with the presence of two visible peaks, this could be evidence of regime change during the period of study. It was also determined that Central Bank of Kenya rates influenced the interbank lending rates, distinctively high levels of interbank rates were recorded in November 2011 and September 2015 which coincided with the highest central bank rates during the respectively years.

Both the level of interest rates and the equity returns are found to be leptokurtic implying that the data sets for the two variables contain more extreme values that would be found in normally distributed data.

4.1.3 Causality

The p-value on Granger causality test on whether the level of interest rates Granger causes equity returns is 0.6274 which is greater than α of 0.05 we therefore fail to reject the null hypothesis and conclude that the level of interest rates does not Granger cause Equity returns.

Table 4.2: Granger Causality Test (Level of Interest Rates and Equity Returns)

Granger causality H_0: Level of interest rates does not granger-cause Equity Returns
F-Test = 0.78814, df1 = 9, df2 = 6716, p-value = 0.6274

Similarly, we conclude that Equity returns do not Granger cause the level of interest rates from the p-value of 0.09368 which is greater than 0.05.

Table 4.3: Granger Causality Test (Equity Returns and Level of Interest Rates)

Granger causality H_0: Equity Returns do not granger-cause Level of Interest Rates
F-Test = 1.6567, df1 = 9, df2 = 6716, p-value = 0.09368

Similar to existing literature an initial analysis of the descriptive statistics provide mixed or inconclusive results. We find a negative correlation between the level of interest rate and stock prices as proxied by the level of the NSE 20 share index, there is however a positive relationship between interest rates levels and the equity returns as measured by the changes in the 20 share index. Trend and distribution analysis however confirms some of the stylized facts about financial time series data in both cases.

4.2 Wavelet Analysis

We begin with the specification of the daubechies least asymmetric (LA) wavelet based on eight non-zero coefficients with a filter of $L = 8$. We then define the number of scales $J \leq 8$ in which case we reconstruct eight wavelet details vectors D_8, D_7, D_6 D_5, D_4, D_3, D_2, D_1 and one wavelet smooth vector, S_8, each associated with a particular scale 2^{j-1}. The vector S_8, represents the intrinsic smooth behaviour of the time series at higher scales, and the eight vector d_8, d_7, d_6, d_5, d_4, d_3, d_2, d_1, represent progressively finer scale deviations from the smooth behaviour.

A scale is a frequency band, lower scales in the analysis refer to a compressed wavelet, that trace abrupt changes or high-frequency components of the data. The higher scales on the other hand are composed of the stretched version of a wavelet and the corresponding coefficients represent low-frequency components of the data (Partal, 2012).

4.2.1 Wavelet Variance

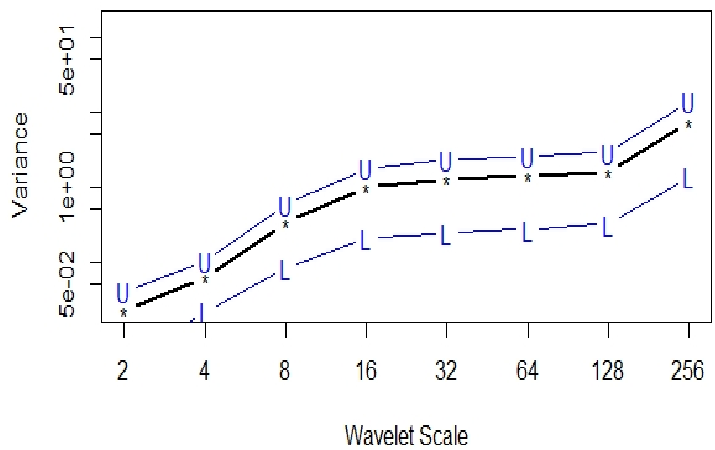
Wavelet variance decomposes the variance of the time series into components associated with different scales. The decomposition enables identification of periods of high and low variability. To compute the variance we deploy the MODWT using the specified variable and the defined wavelet filter and number of scales. Further using the MODWT and the wavelet filter we find the brick wall of the variable and then compute Wavelet Gaussian Variance and Wavelet Non- Gaussian Variance. The outcomes of the analysis is highlighted in table 4.4.

Table 4.4: Wavelet Variance - Interest Rates

Scale	Wavelet Variance	Lower	Upper
d1	0.0225	0.0054	0.0397
d2	0.0610	0.0212	0.1007
d3	0.3313	0.0809	0.5816
d4	0.9833	0.2106	1.7559
d5	1.2425	0.2390	2.2461
d6	1.4010	0.2703	2.5317
d7	1.5813	0.3166	2.8459
d8	7.3504	1.3410	13.3598
s8	84.7430	15.2786	154.2074

Column 1 shows the variance multiple of the time series at a given scale, column 2 and 3 show lower and upper limits of the 95% confidence interval respectively.

Figure 4.5: Wavelet Variance - Interest Rates



Plot showing the variance multiple of the time series at a given scale and the upper and lower limits of the 95% confidence interval.

The blue lines show the upper and lower limits of the 95% confidence interval. The black line connects the value of the correlation multiple between the given time series at a certain scale.

The variability of the interbank lending rate rises steadily over the period of study taking higher and higher values over time. In essence the interbank lending rate displays the highest variability over a longer time period compared to the medium and short terms.

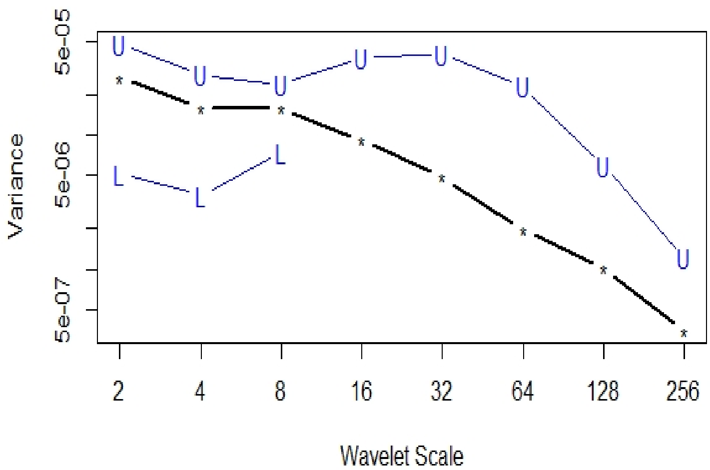
The variability is highest in 1.4 to 2 year scale (scale 8) indicating that the interest rates are more volatile in the long term compared to the short term. The variance of the lending rate is less than 0.05 in the intra week scale (scale 1).

Table 4.5: Wavelet Variance - Equity Returns

Scale	Wavelet Variance	Lower	Upper
d1	0.00003	0.00001	0.00005
d2	0.00002	0.00000	0.00003
d3	0.00002	0.00001	0.00002
d4	0.00001	-0.00002	0.00004
d5	0.00000	-0.00003	0.00004
d6	0.00000	-0.00002	0.00002
d7	0.00000	0.00000	0.00001
d8	0.00000	0.00000	0.00000
s8	0.00000	-0.00001	0.00001

Column 1 shows the variance multiple of the time series at a given scale, column 2 and 3 show lower and upper limits of the 95% confidence interval respectively.

Figure 4.6: Wavelet Variance - Equity Returns



Plot showing the variance multiple of the time series at a given scale and the upper and lower limits of the 95% confidence interval.

The variance of equity returns is generally low and is lowest in the 1.4 to 2 year scale. This is an indication that the average equity returns were generally stable over the period of study and more so over a the longer time period.

4.2.2 Wavelet Covariance and Correlation

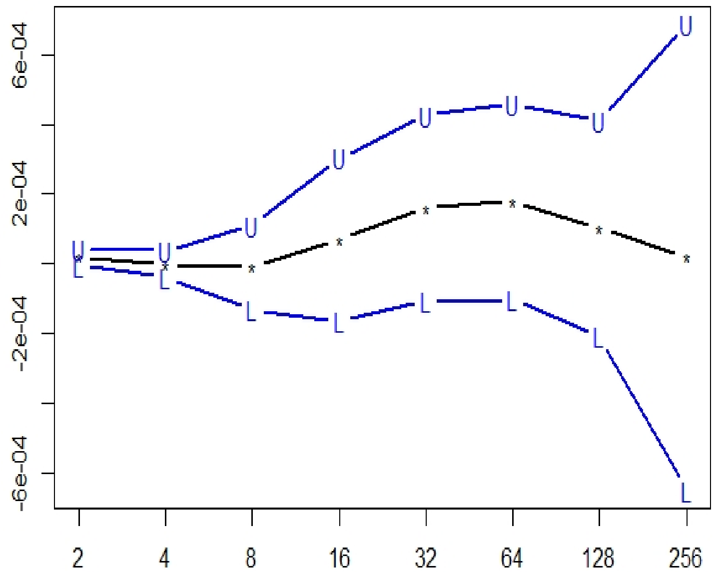
Similar to wavelet variance, wavelet covariance offers a scale by scale decomposition of the usual covariance between time series, in this case the interbank lending rate and equity returns.

Table 4.6: Wavelet Covariance, Interbank Lending Rate vs Equity Returns

Scale	Wavelet Covariance	Lower	Upper
d1	0.0000	0.0000	0.0000
d2	0.0000	0.0000	0.0000
d3	0.0000	-0.0001	0.0001
d4	0.0001	-0.0002	0.0003
d5	0.0002	-0.0001	0.0004
d6	0.0002	-0.0001	0.0005
d7	0.0001	-0.0002	0.0004
d8	0.0000	-0.0006	0.0007
s8	-0.0010	-0.0044	0.0024

Column 1 shows the covariance multiple between the time series at a given scale, column 2 and 3 show lower and upper limits of the 95% confidence interval respectively.

Figure 4.7: Wavelet Covariance, Interbank Lending Rates vs Equity Returns

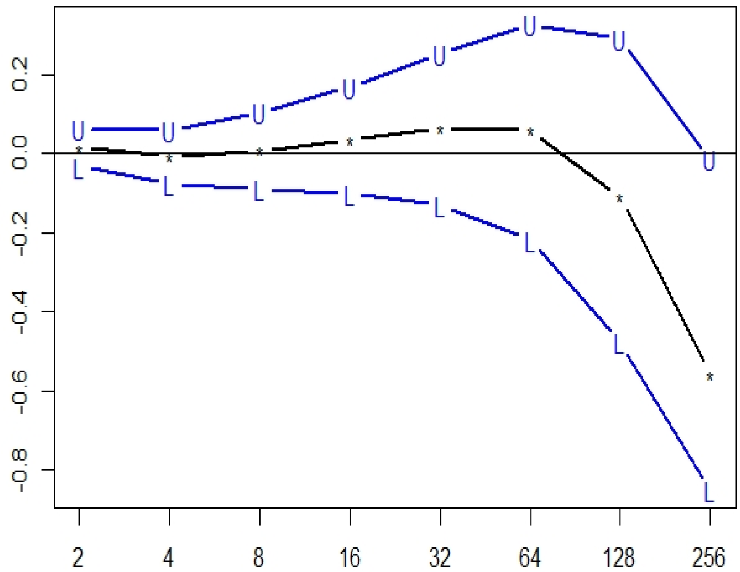


Plot showing the covariance multiple between the time series at a given scale and the upper and lower limits of the 95% confidence interval

The covariance between the two variables takes on values close to or not significantly higher than zero over the study period both in the short and long term. A similar outcome is obtained from the analysis of correlation between the two variables, (*see Appendix A.1*). Intrinsically, the wavelet correlation is the wavelet covariance standardized at each scale, the shape of a plot of the wavelet correlation is similar to that of the wavelet covariance although the magnitudes may differ.

An assessment of the wavelet correlation between the level of interest rates and the price levels measured by the NSE 20 share index however returns significant negative values in the longer term with correlation values as low as -0.6

Figure 4.8: Wavelet Correlation, Interbank Lending Rates vs NSE 20 Share Index

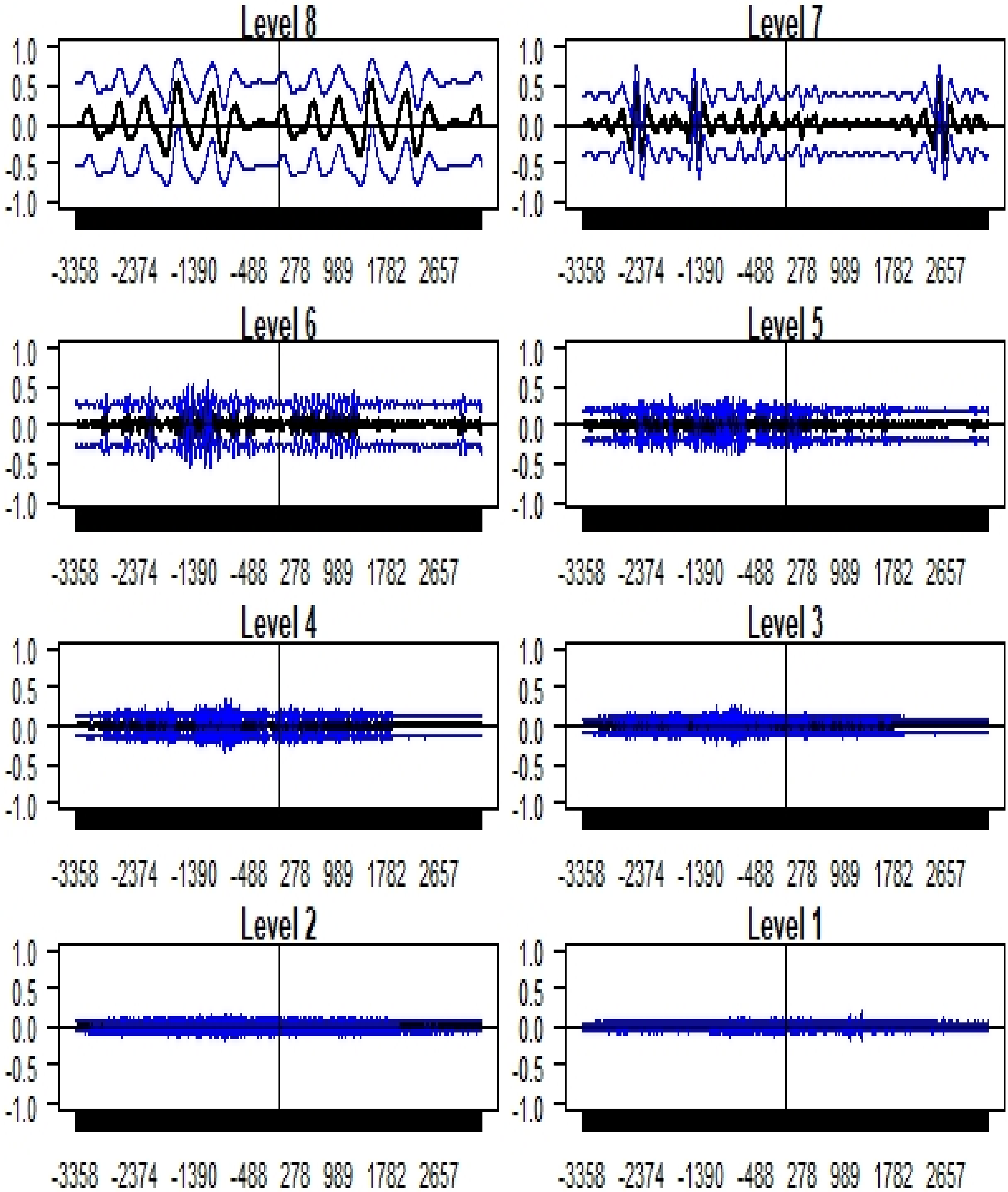


Plot showing the correlation multiple between the time series at a given scale and the upper and lower limits of the 95% confidence interval

4.2.3 Wavelet Cross Correlation

Cross-correlation is a measure of similarity of two series as a function of the lag of one relative to the other. The following graphs present cross-correlations between interest rates and stock market index at different time scales.

Figure 4.9: Wavelet Cross Correlation, Interbank Lending Rates vs Equity Returns



Level 1 shows cross correlation of data scaled into the high frequency over a short period of time whereas Level 8 represents the highest scale with the low frequency and longer time frame. The horizontal axis shows lags in terms of number of days.

Figure 4.9 shows the lagged relationship between the two variables across the various time scales, the lagged variable is the equity returns. The horizontal axis in the plots shows the range of the lags being assessed while the eight levels represent the eight scales. It is evident that the relationship between interest rates and equity returns begins to become significant from the fifth level/scale which corresponds to the period between 64-128 days at all lags. The interrelation becomes even more significant in subsequent scales and is highest at the highest scale in the analysis being the 1.4 -2 year scale.

4.2.4 Wavelet Coherence

Coherence is a measure of the degree of relationship, as a function of frequency, between two time series. Through the use of the wavelet coherence map, we assess the directional co movement of the two variables, we also infer the lead - lag relationship between the variables over the various time scales.

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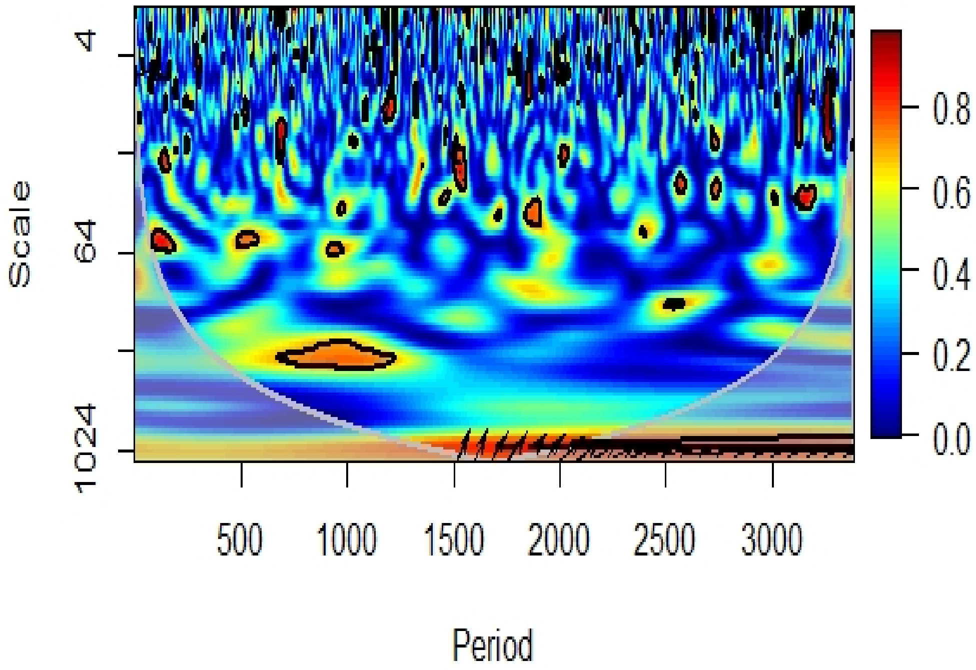
Figure 4.10: Wavelet Coherence - Interbank Lending Rate vs Equity Returns



The predominantly blue colour shows low correlation between the two variables. There are however yellow and orange patches in the map signifying positive but non-strong correlation between the variables. The arrows in the figure are right and upward pointing indicating that the two variables are in phase but equity returns is the leading variable in the model. The horizontal axis shows the frequency with regards to number of days

Figure 4.10 is the wavelet coherence map for interest rates and equity returns, time is displayed on the horizontal axis, while the vertical axis shows the frequency (the lower the frequency, the higher the scale).

It is evident from the plot that the interrelation between interest rates and equity returns is relatively low. There is no significant positive interrelation between the variables as evidence by lack of a completely red patches. There are however orange patches in the map signifying positive but non-strong correlation between the variables. The arrows in the map are right pointing indicating that the two variables are in phase, essentially interest rates and equity returns move in the same direction, that is during the period of assessment the equity returns were higher when interest rates were high. The arrows are also upward pointing an indication that interest rates proxied by the interbank lending rate is the leading variable in the model.

A similar situation where there is no significant interrelation is recorded in the assessment of the wavelet coherence between interest rates and the price level in the market measured by the value of the NSE 20 share index. The two variables were however found to be anti-phase with the arrows in the map (*see Appendix A.2*) being left pointing, interest rates were still noted to be the leading variable in the bivariate assessment.

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Chapter 5

Conclusion and Recommendations

5.1 Discussions of results

From a time decomposed correlation perspective, there is evidence of delayed correlation between interest rates levels and equity market performance in Kenya. Both the correlation and cross correlation assessment find that correlation increases with increase in the scales. There is lower correlation (correlation coefficients of 0.3 and below, including negative correlation coefficients) in the lower scale, i.e the intra week scale covering 4-8 days but higher correlation (correlation coefficients of 0.3 and above) in higher scales between 512 to 1024 days.

In the analysis the interrelation between both the interest rate and equity returns and interest rates and level of prices in the market measured by the level of the NSE 20 share index become significant at the fifth scale corresponding to periods between 64-128 days. In the lower scales the relationship is not significantly different from zero. Results form wavelet coherence assessment points to evidence that the equity market returns precedes changes in interest rates over the entire study period. The relationship between the interest rates and the price levels however shows that interest rates lead the changes in the price levels measured by the level of the NSE 20 share index.

Regarding the direction of the correlation, the study finds that on average, correlation between interest rates and and the share index, especially over a long time horizon (coarse scales), is negative. This is in line with the findings of Aurangzeb (2012), Owolabi and Adegbite (2014), Elly and Oriwo (2013), Amata et al. (2016), Chirchir (2014) and Toda and Yamamoto (1995). The finding is also in line with financial theory in general which asserts that the negative relationship is based on the fact that a rise in interest rate will lead to higher borrowing costs, lower future profits, increase in discount rate for equity investors; and then stock prices decrease AL-Naif (2017). On the other hand the correlation between interest rates and equity returns were found to be not significantly different from zero over the various time scales with small but positive correlation figures recorded in the medium term, on average however the wavelet correlation is still negative.

5.2 Conclusion

The results from this research present conclusive evidence of delay in interrelation between interest rates and equity markets (both equity returns and the price level measured by the level of the share index) with correlation being lowest in the intra week scale and highest in the 1.4-2 year scale. The wavelet methodology further facilitates an analysis that narrows down to the time period when the interrelation between these variables starts becoming significant, where we note that the correlation between the variables becomes significant after 64-128 days. The lead-lag relationship and the phase differences of the two variation finds that the two variables are in phase but equity returns is the leading variable meaning that variations in the market returns can be used by observers and market actors to infer changes in interest rates.

Short falls of existing approaches used in modelling delayed correlation outlined in the review of literature under this study have been addressed through the wavelet approach which facilitates a comprehensive assessment of the inter temporal relationship between the time series in a dynamic way.

5.3 Recommendations

The findings in this research are important to policy makers in assessing the effect that the changes in interest rates may have on stock markets and markets in general. In the context of this study, policy makers should be cognisant of the fact that changes in interest rates at a given point in time will not have immediate impact on markets (lower correlation in the lower scales), these impacts may only manifest after a given period of time (higher correlation in the higher time scales). The results and the wavelet approach in general are also relevant to investors and portfolio managers in enabling them to design appropriate strategies to mitigate correlation risk factoring in delays that may be inherent in relationships not only for variables covered under this study but also other variables that may be relevant to their respective investment strategies.

The study provides a good starting point for policy makers, portfolio managers and other market participants to probe further the dynamics of this relationship in a broader context. This study recommends the deployment of wavelet analysis in further research that would incorporate additional variables such as inflation and exchange rates that have a bearing on stock market performance. This will facilitate a closer recreation of macroeconomic context in the model to improve its robustness.

The study also recommends the deployment of wavelet analysis to assess the reaction of banking sector stocks to changes in interest rates and other fundamentals in the banking sector. Empirical evidence points to evident and almost immediate reaction of prices of banking sector stocks to reports on fundamentals for example through semi-annual or annual reports, a study that incorporates a decomposition in both time and frequency domains would provide more insights on the relationship between these variables.

It is further recommended that wavelet analysis be used in assessment of cross jurisdictional impact of various factors, where an assessment can be undertaken to determine how shocks in markets or macroeconomic performance in one country may affect performance of markets in another country.

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Appendix A

A.1 Wavelet Correlation-Interbank Lending Rate vs Equity Returns

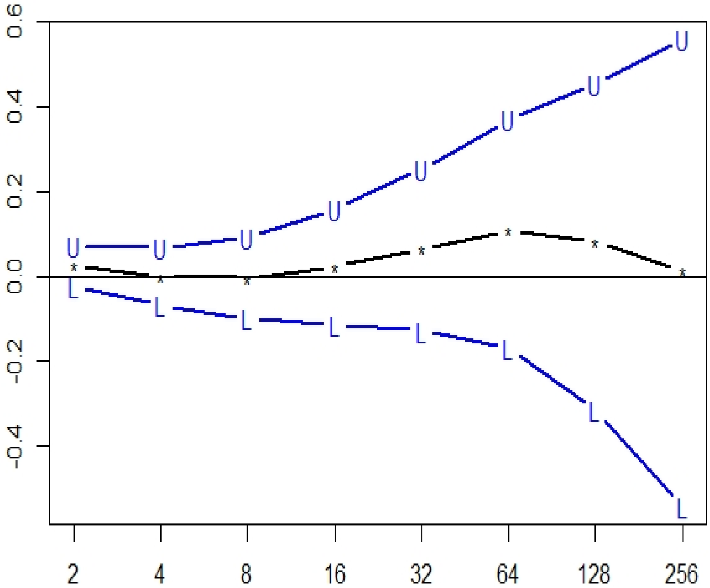
Through this assessment we visualize how correlation between interest rates and stock market performance changes with the change in time and frequency. We analyse the correlation between Interbank Lending Rate and NSE 20 share index on different scales.

Table A.1: Wavelet Correlation, Interbank Lending Rates vs Equity Returns

Scale	Wavelet Correlation	Lower	Upper
d1	0.0240	-0.0236	0.0716
d2	-0.0006	-0.0680	0.0668
d3	-0.0042	-0.0995	0.0912
d4	0.0230	-0.1124	0.1576
d5	0.0649	-0.1284	0.2534
d6	0.1076	-0.1703	0.3696
d7	0.0805	-0.3167	0.4537
d8	0.0119	-0.5426	0.5592
s8	-0.1071	-0.8452	0.7715

Column 1 shows the correlation multiple between the time series at a given scale, column 2 and 3 show lower and upper limits of the 95% confidence interval respectively.

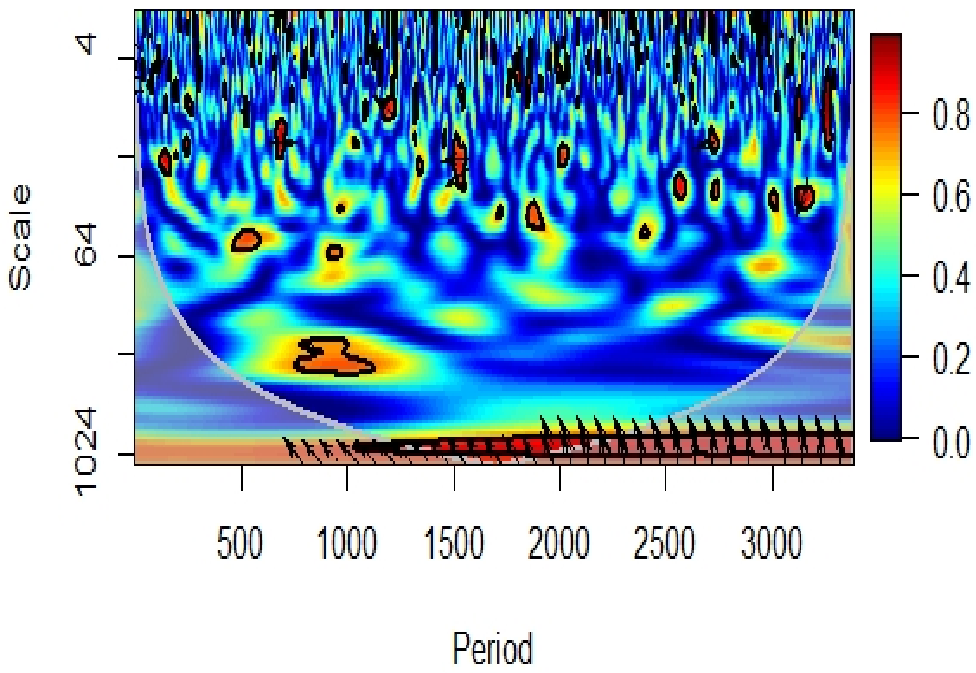
Figure A.1: Wavelet Correlation, Interbank Lending Rates vs Equity Returns



Plot showing the correlation multiple between the time series at a given scale and the upper and lower limits of the 95% confidence interval

A.2 Wavelet Coherence - Interest Rates and NSE 20 Share Index

Figure A.2: Wavelet Coherence - Interbank Lending Rate vs NSE 20 Share Index



The predominantly blue colour shows low correlation between the two variables. There are however yellow and orange patches in the map signifying positive but non-strong correlation between the variables. The arrows in the figure are left and upward pointing indicating that the two variables are in phase and interbank lending rate is the leading variable in the model. The horizontal axis shows the frequency with regards to number of days

Appendix B

B.1 R Codes

```
library(waveslim)
library(readxl)
Research_Data <- read_excel("C:/Users/brian/Desktop/MMF/Thesis␣-␣1/
Research␣DataR.xlsx")
attach(Research_Data)
A=Interbank
B=NSE20RET
C=NSE20
#Define the number of scales
J <- 8
#Choose the type of wavelet filter: la8 or d4
wf <- "la8"
## Descriptive Statistics
mean(A)
mean(B)
var(A)
var(B)
cov(A,B)
cor(A,B)

# Trend Analysis

plot.ts(A, xlab="Time", ylab="Interest␣Rates")

title('Interest␣Rates␣between␣October␣2003␣and␣October␣2019')

plot.ts(B, xlab="Time", ylab="Equity␣Returns")
```

```
title('Equity␣Returns␣between␣October␣2003␣and␣October␣2019')

#Distribution

hist(A, density=20, breaks=20, prob=TRUE,
     xlab="Interest␣Rates",
     main="Histogram␣of␣Interest␣Rates")
lines(density(A), col="blue", lwd=2)

hist(B, density=20, breaks=20, prob=TRUE,
     xlab="Equity␣Returns", ylim=c(0, 70),
     main="Histogram␣of␣Equity␣Returns")
lines(density(B), col="blue", lwd=2)

#Causality

library(zoo)
library(lmtest)
library(vars)

VAR = cbind (A,B)

VARX = VAR(VAR, type="const", lag.max =10, ic ="SC")
causality(VARX, cause = "A") $Granger

causality(VARX, cause = "B") $Granger

## WAVELET VARIANCE

## VARIABLE A

##Calculate MODWT for variable A
A.modwt <- modwt(A, wf, J)
# Find Brick Wall for variable A
```

```
A.modwt.bw <- brick.wall(A.modwt, wf)
# Calculate Variance on Non-Gaussian
WVARA1 <- wave.variance(A.modwt.bw, type = "nongaussian")
# Calculate Variance on Gaussian
WVARA2 <- wave.variance(A.modwt.bw, type = "gaussian")
# Show results for Wavelet Non-Gaussian Variance
WVARA1
# Saving results onto your computer in csv format
write.table(WVARA1, file = "WVARA1_NonGaus.csv", sep = ",")
# Show results for Wavelet Gaussian Variance
WVARA2
# Saving results onto your computer in csv format
write.table(WVARA2, file = "WVARA2_Gaus.csv", sep = ",")
# covariance for variable A.
## The next code is to create the graphs
par(mfrow=c(1,1), las=0, mar=c(5,4,4,2)+.1);

minWVARA<-min(WVARA2[,1][which(WVARA2[,1] > 0)])
maxWVARA<-max(WVARA1[,3])

matplot(2^(1:J), WVARA2[-(J+1),], type="b", log="xy",xaxt='n',
        ylim=c(minWVARA,maxWVARA), pch="*LU", lty=1, lwd=2,
        col=c(1), xlab="Wavelet␣Scale", ylab="Variance");
matlines(2^(1:J), as.matrix(WVARA1[-(J+1),2:3]),
         type="b", pch="LU", lty=1, col=4)
axis(side=1, at=2^(1:J))
#legend("topleft",c("Wavelet variance", "Gaussian CI", "Non-Gaussian CI")
, lty=1, col=c(1,8,8), bty="n")

title('Wavelet␣Variance␣-␣Interbank␣Lending␣Rate')

## VARIABLE B
# Calculate MODWT for variable B
B.modwt <- modwt(B, wf, J)
```

```
# Find Brick Wall for variable B
B.modwt.bw <- brick.wall(B.modwt, wf)
# Calculate Variance on Non-Gaussian
WVARB1<-wave.variance(B.modwt.bw, type="nongaussian")
# Calculate Variance on Gaussian
WVARB2<-wave.variance(B.modwt.bw, type="gaussian")
# Show results for Wavelet Non-Gaussian Variance
WVARB1
# Save results into csv file
write.table(WVARB1, file="WVARB1_NonGaus.csv",sep=",", digits=4)
# Show results for Wavelet Gaussian Variance
WVARB2
# Save results into csv file
write.table(WVARB2, file="WVARB2_NonGaus.csv",sep=",")

## The next code is to create the graphs
par(mfrow=c(1,1), las=0, mar=c(5,4,4,2)+.1);

minWVARB<-min(WVARB2[,1][which(WVARB2[,1] > 0)])
maxWVARB<-max(WVARB1[,3])

matplot(2^(1:J), WVARB2[-(J+1),], type="b", log="xy",
        xaxt='n', ylim=c(minWVARB, maxWVARB), pch="*LU",
        lty=1, lwd=2, col=c(1,4,4), xlab="Wavelet␣Scale", ylab="Variance")

matlines(2^(1:J), as.matrix(WVARB1[-(J+1),2:3]),
         type="b", pch="LU", lty=1, col=4)
axis(side=1, at=2^(1:J))

#legend("bottomleft",c("Wavelet variance", "Gaussian CI", "Non-Gaussian CI
), lty=1, col=c(1,8,8), bty="n")

title('Wavelet␣Variance␣-␣Equity␣Returns')
```

```
#VARIABLE C

# Calculate MODWT for variable B
C.modwt <- modwt(C, wf, J)
# Find Brick Wall for variable B
C.modwt.bw <- brick.wall(C.modwt, wf)
# Calculate Variance on Non-Gaussian
WVARC1<-wave.variance(C.modwt.bw, type="nongaussian")
# Calculate Variance on Gaussian
WVARC2<-wave.variance(C.modwt.bw, type="gaussian")
# Show results for Wavelet Non-Gaussian Variance
WVARC1
# Save results into csv file
write.table(WVARC1, file="WVARC1_NonGaus.csv",sep=",", digits=4)
# Show results for Wavelet Gaussian Variance
WVARC2
# Save results into csv file
write.table(WVARC2, file="WVARC2_NonGaus.csv",sep=",")

## The next code is to create the graphs
par(mfrow=c(1,1), las=0, mar=c(5,4,4,2)+.1);

minWVARC<-min(WVARC2[,1][which(WVARC2[,1] > 0)])
maxWVARC<-max(WVARC1[,3])

matplot(2^(1:J), WVARC2[-(J+1),], type="b", log="xy",
    xaxt='n', ylim=c(minWVARC, maxWVARC), pch="*LU",
    lty=1, lwd=2, col=c(1,4,4), xlab="Wavelet␣Scale", ylab="Variance");

matlines(2^(1:J), as.matrix(WVARC1[-(J+1),2:3]),
         type="b", pch="LU", lty=1, col=4)
axis(side=1, at=2^(1:J))

#legend("bottomleft",c("Wavelet variance", "Gaussian CI", "Non-Gaussian CI
```

```
, lty=1, col=c(1,8,8), bty="n")

title('Wavelet␣Variance␣-␣NSE␣20␣share␣index')

## WAVELET COVARIANCE

#Calculate Wavelet Covariance
WCOV <- wave.covariance(A.modwt.bw, B.modwt.bw)
# Show results for Wavelet Covariance
WCOV
# Saving results onto your computer in csv format
write.table(WCOV, file="WCOV.csv",sep=",", digits=4)

## The next code is to create the graphs

par(mfrow=c(1,1), las=0, pty="m", mar=c(2,3,1,0)+.1)

matplot(2^(1:J), WCOV[-(J+1),], type="b", log="x",
  pch="*LU", xaxt="n", lty=1, lwd=2, col=c(1,4,4),
  xlab="Wavelet␣Scale", ylab="Covariance",
  main="Wavelet␣Covariance␣-␣Interbank␣Rate␣and␣Equity␣Returns");

axis(side=1, at=2^(1:J));

## WAVELET CORRELATION (INTERBANK RATE AND EQUITY RETURNS)

#Calculate the total number of observations
N<- length(A)
#Calculate Wavelet Correlation
WCOR<-wave.correlation(A.modwt.bw, B.modwt.bw, N, 0.975)
#Show results for Wavelet Correlation
WCOR
#Saving results onto your computer in csv format
write.table(WCOR, file="WCOR.csv",sep=",")
```

```
## The next code is to generate graphs
matplot(2^(1:J), WCOR[-(J+1),], type="b", log="x",
  pch="*LU", xaxt="n", lty=1, lwd=2, col=c(1,4,4),
  xlab="Wavelet␣Scale", ylab="Correlation",
  main="Wavelet␣Correlation␣-␣Interbank␣Rate␣and␣Equity␣Returns");
abline(h=0)
axis(side=1, at=2^(1:J));

## WAVELET CORRELATION (INTERBANK RATE AND NSE 20 share Index)

#Calculate the total number of observations
N<- length(A)
#Calculate Wavelet Correlation
WCORC<-wave.correlation(A.modwt.bw, C.modwt.bw, N, 0.975)
#Show results for Wavelet Correlation
WCORC
#Saving results onto your computer in csv format
write.table(WCORC, file="WCORC.csv",sep=",")
## The next code is to generate graphs
matplot(2^(1:J), WCORC[-(J+1),], type="b", log="x",
 pch="*LU", xaxt="n", lty=1, lwd=2, col=c(1,4,4),
 xlab="Wavelet␣Scale", ylab="Correlation",
 main="Wavelet␣Correlation␣-␣Interbank␣Rate␣and␣NSE␣20␣share␣Index");
abline(h=0)
axis(side=1, at=2^(1:J));

## WAVELET CROSSCORRELATION


#Define number of lags (selected through BIC)
lmax<- 3358

returns.cross.cor<- NULL
for(i in 1:J) {
```

```
  spin<- spin.correlation(A.modwt.bw[[i]], B.modwt.bw[[i]], lmax)
  returns.cross.cor<- cbind(returns.cross.cor, spin)
}

returns.cross.cor<- -ts(as.matrix(returns.cross.cor),
start=-lmax, freq=1)

dimnames(returns.cross.cor)<- list(NULL, paste("Level", 1:J))

## The next code is to create the graphs

lags<- length(-lmax:lmax)

lower.ci<-tanh(atanh(returns.cross.cor) - qnorm(0.975)/
sqrt(matrix(trunc(N/2^(1:J)),
nrow=lags, ncol=J, byrow=TRUE) -3))

upper.ci<- tanh(atanh(returns.cross.cor) + qnorm(0.975) /
sqrt(matrix(trunc(N/2^(1:J)),
nrow=lags, ncol=J, byrow=TRUE) -3))

par(mfrow=c(4,2), las=1, pty="m", mar=c(2,3,1,0)+.1)

for(i in J:1) {
  plot(returns.cross.cor[,i], ylim=c(-1,1), xaxt="n", xlab="Lag", ylab="",
       main=dimnames(returns.cross.cor)[[2]][i], lwd=2)
  axis(side=1, at=seq(-lmax,lmax,by=1))
  lines(lower.ci[,i], lty=1,col=4)
  lines(upper.ci[,i], lty=1, col=4)
  abline(h=0, v=0)
}


## WAVELET COHERENCE (INTERBANK RATE AND EQUITY RETURNS)
```

```r
library(biwavelet)
# Define two sets of variables with time stamps
t1 = cbind(Date, Interbank)
t2 = cbind(Date, NSE20RET)
t1
t2

nrands = 500

wtc.AB = wtc( t1,  t2, nrands = 500)



par(mfrow=c(1,1))
# Plotting a graph
par(oma = c(0, 0, 0, 1), mar = c(5, 4, 5, 5) + 0.1)

plot(wtc.AB, plot.phase = TRUE,
lty.coi = 1, col.coi = "grey", lwd.coi = 2,
lwd.sig = 2,
arrow.lwd = 0.03, arrow.len = 0.12, ylab = "Scale", xlab = "Period",
plot.cb = TRUE,
main = "Wavelet Coherence: Interbank rate vs Equity Returns")

# Adding grid lines

## n = length(t1[, 1])
## abline(v = seq(200, n, 200), h = 1:16, col = "brown", lty = 1, lwd = 1)

# Defining x labels
axis(side = 3, at = c(seq(0, n, 200)), labels = c(seq(2003, 2019, 1)))

## WAVELET COHERENCE (INTERBANK RATE AND NSE 20 share Index)
```

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```r
library(biwavelet)
# Define two sets of variables with time stamps
t1 = cbind(Date, Interbank)
t3 = cbind(Date, NSE20)
t1
t3

nrands = 500

wtc.AC = wtc( t1,  t3, nrands = 500)


par(mfrow=c(1,1))
# Plotting a graph
par(oma = c(0, 0, 0, 1), mar = c(5, 4, 5, 5) + 0.1)

plot(wtc.AC, plot.phase = TRUE,
lty.coi = 1, col.coi = "grey", lwd.coi = 2,
lwd.sig = 2,
arrow.lwd = 0.03, arrow.len = 0.12, ylab = "Scale", xlab = "Period",
plot.cb = TRUE,
main = "Wavelet␣Coherence:␣Interbank␣rate␣vs␣NSE␣20␣Share␣Index")

# Adding grid lines

## n = length(t1[, 1])
## abline(v = seq(200, n, 200), h = 1:16, col = "brown", lty = 1, lwd = 1)

# Defining x labels
axis(side = 3, at = c(seq(0, n, 200)), labels = c(seq(2003, 2019, 1)))
```