STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BSC ACTUARIAL SCIENCE

END OF SEMESTER EXAMS

BSA 3110 - Actuarial modeling II

DATE: 23rd July 2019 **TIME: 2 HOURS**

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION 1 (30 MARKS)

a. Briefly define the following terms and symbols
   i. μ_{40} (2 marks)
   ii. Censoring (2 marks)
   iii. Graduation (2 marks)

b. A mortality table, which obeys Gompertz' Law for older ages, has:

$$\mu_{70} = 0.025330 \ \ and \ \ \mu_{90} = 0.126255$$

Find the probability that a life aged 60 will survive for 20 years. (4 marks)

c. In a certain population, the force of mortality is given by:

	μ_x
$60 < x \leq 70$	0.01
$70 < x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. (3 marks)

d. Show that the Kaplan-Meier and Nelson-Aalen estimates are the same (5 marks)

e. The covariates for the *i* th observed life are (56, 183, 40) representing (age last birthday at the start of the study, height in *cm*, daily dose of drug A in *mg*). Using the regression parameters β = (0.0172, 0.0028, –0.0306), calculate $\lambda(t; z_i)$ in terms of $\lambda_0(t)$. (6 marks)

f. Give an example of a situation in which the hazard function may be expected to follow each of the following distributions:
   i. Exponential (1 mark)
   ii. decreasing Weibull (1 mark)
   iii. Gompertz-Makeham (1 mark)
   iv. log-logistic. (1 mark)

g. In a mortality investigation covering a 5-year period, where the force of mortality can be assumed to be constant, there were 46 deaths and the population remained approximately constant at 7,500. Estimate the force of mortality. (2 marks)

QUESTION 2

a. A mortality investigation covers the period 1 January 2001 to 31 December 2003. In this investigation, the age label used is "age next birthday". Give the range of dates for which table above contribute to E_{34}^{c} at each age where they make a contribution. Assume that the day of entry counts in the exposed to risk but the day of exit does not. Clearly show your calculations. (8 marks)

	Date of Birth	Date of Joining	Date of Exit	Reason for Exit
A	25.04.69	07.08.99	30.10.02	Death
B	01.07.69	12.09.02	-	-
C	04.09.68	22.07.03	4.12.03	Withdrawal

b. Discuss in detail any three methods of graduation (12 marks)

QUESTION 3

20 heart transplant patients are observed until they die from surgery related complications. The data in the table overleaf is collected.

a. Define $n, m, t_j, d_j, c_j, and\, n_j$ for these data, assuming that censoring occurs just after the failures were observed (5 marks)
b. Calculate the Nelson Aalen estimate of $F(t)$ (6 marks)
c. Using Greenwood's formula, estimate $\text{var}\left(\tilde{F}(16)\right)$ (4 marks)

Day	**Event**
4	Patient 2 dies from surgery related complications
5	Patient 7 dies from surgery related complications
6	Patient 1 is discharged by relatives for treatment abroad
7	Patient’s 3 oxygen mask malfunctions leading to her death
8	Patient 4 dies from pneumonia
10	Patient 13 and 17 die from surgery related complications
11	Patient 3 dies from surgery related complications
13	Patient 5 dies from surgery related complications
14	Patient 6 dies from malaria
15	Patient 8 dies from surgery related complications and patient 9 escapes due to amounting hospital bill
17	Patient 10 and 11 die from surgery related complications
18	Patient 12 and 15 from surgery related complications while patient 16 is transferred to a private hospital
19	Patient 14 is taken by family to recuperate at home
21	Patient 18 dies from surgery related complications
22	Patient 19 dies from surgery related complications

c. A large computer company always maintains a workforce of exactly 5,000 young workers, immediately replacing any worker who leaves. Use the Poisson model to calculate the probability that there will be fewer than 3 deaths during any 6 month period, assuming that all workers experience a constant force of mortality of 0.0008 per annum. (5 marks)

Question 4

a. Discuss the following types of censoring mechanisms (4 marks)
   1. Type II Censoring
   2. Interval Censoring

b. The following data was obtained from a mortality investigation.

x	E_x	d_x	$\hat{q}_x$
30	70,000	39	0.000557
31	66,672	43	0.000645
32	68,375	34	0.000497
33	65,420	31	0.000474
34	61,779	23	0.000372
0.000757	66,091	50	0.000757

Assuming that $\log_e\left(\frac{q_x}{p_x}\right) = \hat{\beta}_0 + \hat{\beta}_1 x$ obtain the graduated estimates $\overset{0}{q}_x$.

Hint: $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$

$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$ (12 marks)

c. Determine whether the above graduated rates are smooth (4 marks)

QUESTION 5

a. Show that the density function of the future life time random variable T_x is given by: (10 marks)

$$f_x(t) = {}_t p_x \mu_{x+t}, \quad for\ 0 \le t < \omega - x$$

b. Prove that, under Gompertz's Law, the probability of survival from age x to $x+t$, ${}_t p_x$ is given by: (6 marks)

$$_t p_x = \left[\exp\left(\frac{-B}{\ln c}\right)\right]^{c^x(c^t-1)}$$

c. For a certain population, estimates of survival are available as follows:

$$_1 p_{50} = 0.995$$

$$_2 p_{50} = 0.989$$

Calculate B and c consistent with these observations. (4 marks)