Modelling Depression Treatment and HIV Care Cascade Dynamics in Kenya

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Declaration and Approval

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Abstract

HIV/AIDS has become one of the major global health burdens and threat to public health. By the end of 2021, 38.4 million people globally were living with HIV and over 1.4 million people live with HIV in Kenya. The "HIV care cascade" serves as an individual-level tool for evaluating HIV care and treatment results and a population-level paradigm for estimating the percentage of HIV-positive individuals in a given region who are participating in each subsequent phase. Several factors have been highlighted to influence the HIV care cascade and among this is depression which influences the improvements in ART service provision; diagnosis of people living with HIV and AIDS (PLWHA), linkages to care, continued engagement in HIV care and retention in HIV care which are crucial in attaining the 95% on ART target in the sub-Saharan region. Thus, This study employed mathematical compartmental modeling to investigate the impact of depression treatment on the HIV care cascade dynamics in Kenya. A deterministic compartmental model of the depression and HIV care cascade was developed from a system of Ordinary Differential Equations (ODEs). The basic reproduction number was evaluated using the next generation matrix. The numerical results showed that improving depression treatment can positively influence the HIV care cascade, leading to improved outcomes, such as higher rates of testing, linkage, adherence, retention, and viral suppression. The study highlights the importance of integrating depression treatment into HIV care services and provides valuable insights for policymakers and healthcare providers on how to improve the HIV care cascade dynamics in Kenya.

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List of Abbreviations

AIDS	Acquired Immunodeficiency Syndrome
ART	Antiretroviral therapy
cART	Combination antiretroviral therapy
CDC	Centers for Disease Control and Prevention
CESD	Center for Epidemiologic Studies Depression Scale
CHMTs	County Health Management Teams
GHO	Global Health Observatory
GSP	Group support psychotherapy
HIV	Human Immunodeficiency Virus
KEMRI	Kenya Medical Research Institute
LTFU	lost to follow-up
MoH	Ministry of Health
NASCOP	National AIDS and STI Control Program
NSDCC	National Syendemic Diseases Control Council
PLWHA	People Living with HIV/AIDS
SSA	Sub-Saharan Africa
UNAIDS	Joint United Nations Programme on HIV and AIDS
USAID	United States Agency for International Development
WHO	World Health Organization (WHO)

Chapter 1: Introduction

1.1 Introduction

In this section of the research, the background of HIV/AIDS and depression is discussed to help in understanding what was studied and the reason for carrying out this research. It encompasses the statement of problem, objectives and justification. Additionally the scope of the study was also discussed.

1.2 Background Information

1.2.1 HIV and HIV Care Cascade

HIV (human immunodeficiency virus) is a virus that targets the immune system; if left untreated it can result to AIDS (acquired immunodeficiency syndrome). HIV/AIDS has become one of the major global health threat to the public health (Garcia and Guzman, 2021). By the end of 2021, 38.4 million people globally were living with HIV, 1.5 million people became newly infected with HIV and 650, 000 people died from AIDS-related illnesses (UNAIDS Fact sheet, 2022). Since the start of the epidemic, 84.2 million people have become infected with HIV. Among them, 39% were reported to have suffered from depression. Even though just 11% of the world's population lives in sub-Saharan Africa, the region is the epicenter of the HIV/AIDS epidemic. The prevalence of adult HIV is 0.7% globally but sub-Saharan Africa (SSA) contains two-thirds of all people living with HIV globally (USAID, 2001). The prevalence of HIV in sub-Saharan countries such as Kenya are among the highest in the world. The Kenyan government has adopted a "public health approach" to HIV scale-up in order to meet the UNAIDS 95-95-95 goals (diagnosing 95% of all people living with HIV, providing antiretroviral therapy [ART] to 95% of those diagnosed, and achieving viral suppression for 95% of those treated). Great strides have been made towards achieving these goals and engaging people living with HIV in care across the region. According to the 2020 Kenya HIV estimates, the diagnosis rate among adults aged 15-49 living with HIV is 89.1%, the ART treatment rate is 87.5%, and the viral load suppression rate is 83.2%.

The "HIV care cascade" also known as the HIV care continuum, serves as both an individual-level tool for evaluating care results and a population-level paradigm for estimating the percentage of HIV-positive individuals in a given region who are participating in each subsequent phase (Hogg, 2018). The cascade entails the stages of diagnosis of an HIV patients, linkage to care, retention in care and attaining viral load suppression. According to CDC data released in May (2021), the number of HIV-positive Americans aged 13 and above at the end of 2019 was estimated at 1.2 million. Of those 1.2 million persons: 87% were diagnosed, 50% were retained in care, 81% were linked to care and 57% had achieved viral suppression. To improve outcomes throughout the HIV treatment cascade, engagement in care is crucial (it is one of the care cascade's tools). The long-term objective of HIV treatment is viral load suppression, which is considered as a biological measure of ART adherence (Damulak et al., 2021). But regular attendance at HIV clinic appointments is also necessary for attaining prolonged viral suppression. Achieving viral load suppression is made possible by improvements in treatment effectiveness, a growth in population awareness of their HIV diagnoses, and utilization of medication. HIV appointment attendance is now more widely acknowledged as a crucial and distinctive sign of involvement in care along the HIV treatment continuum. Missed appointments at the HIV clinic have been linked to mortality, AIDS-defining diseases, failure to achieve viral suppression and retention in ART (Rodrigues et al., 2021). Therefore, it is crucial to comprehend how depressed symptoms impact HIV appointment adherence, viral suppression, and diagnosis over time in order to maximize outcomes throughout the HIV care cascade. Several factors have been highlighted to influence the HIV care cascade and among this is depression.

1.2.2 Depression on PLWHA

In SSA, where access to mental health care is frequently restricted, depression is a major cause of sickness and disability and is particularly common among individuals with HIV in Kenya and other SSA countries (Stockton et al., 2021). Depression affects (18-30)% of patients receiving HIV care in Africa, and estimates from Kenya range from (1-19)%. It is believed that the high prevalence of depression among individuals living with HIV is brought on by dealing with the HIV diagnosis, illness symptoms, grief, difficulties in relationships, stigma and discrimination, concurrent impoverishment, ART adverse reactions, COVID-19 lockdown, fear of death, and infection-related inflammatory reactions. In individuals living with HIV, depression may worsen existing disease states and lead to poorer health outcomes (Stockton et al., 2021). Along with lower quality of life, reduced economic output, social isolation, and cognitive impairment, depression can also have these effects. Depression in HIV-positive individuals has the potential to aggravate HIV-related morbidity and death, particularly in settings with limited resources.

Depression is a prevalent mental condition and is estimated to the primary cause of global disease burden by 2030. People living with HIV/AIDS (PLWHA) are at least two folds likely to experience this neuro-psychiatric disorder compared to the general population (Lopez and Mathers, 2006). In low resourced settings of SSA, where HIV/AIDS burden is enormous, prevalence of depressive symptoms is high among PLWHA on combination antiretroviral therapy (cART) and in mixed/untreated groups. In Kenya, studies conducted among PLWHA have reported prevalence of major depressive disorder (depression and anxiety) as 32% and other depressive disorders as 15% (Nyongesa et al., 2019). Despite recent improvements in ART service provision; diagnosis of PLWHA, linkages to care, and continued engagement in HIV care in sub-Saharan Africa (SSA) remain challenging; improving retention in HIV care will be crucial to attaining the 95% on ART target in the sub-Saharan region. Human resource and organizational obstacles, travel distance, lack of support, discrimination, worry about HIV status disclosure, and mental diseases including depression are some of the factors that contribute to disengagement in care among PLWHA (Harries et al., 2010).

Disparities in depression prevalence, treatment, and remission caused by mental comorbidities and substance use in HIV-positive people are poorly understood. Low rates to no rates of depression treatment have been reported in SSA (Kulisewa et al., 2019). Though a single-centre randomised controlled trial done in Uganda showed that Group support psychotherapy (GSP) is a culturally sensitive strategy that seeks to treat depression by fostering social support, imparting coping mechanisms, and fostering the development of income-generating abilities (Nakimuli-Mpungu et al., 2020). Resolving the mental health needs of individuals living with HIV ought to be a crucial part of the successful implementation, scale-up, and accomplishment of ”treat all” objectives in SSA and beyond. This is because mental health disorders are prevalent among PLWHA, frequently under-diagnosed and under-treated in low-resource environments, and associated with sub-optimal HIV therapeutic efficacy.

1.3 Motivation for Research

In Kenya and other sub-Saharan African (SSA) nations, where access to mental health care is frequently restricted, depression is particularly widespread among PLHIV and is a substantial contributor to disease and disability, according to (Harries et al., 2010). A growing corpus of research in SSA is starting to show that people who are depressed are less likely to start or be connected to ART care. Furthermore, in SSA, low ART adherence is also linked to depression. On the relationship between depression medication and regular diagnosis, linkage to care, retention in HIV care, and viral suppression, however, little study has been done in Kenya. As a result, more research is required to define the relationship between depression, participation in HIV care, and viral suppression in the SSA region. However, no studies have evaluated the relative importance of subsets of depressive symptoms in relation to HIV care cascade. Thus, the primary objective of this study was to model depression treatment and HIV care cascade dynamics in Kenya. Identifying the most relevant depressive symptoms related to missed HIV care visits can inform tailored treatments that not only relieve depression but also improve retention in HIV care.

1.4 Problem Statement

HIV continues to be a significant global public health challenge, with over 38.4 million people living with the disease. By the end of 2021, over 1.4 million people were living with HIV/AIDS in Kenya. HIV/AIDS has also affected the global human development of African countries through its devastating impact on health and demographic indicators such as life expectancy at birth, healthcare assistance, age and sex distribution, and economic indicators like income, workforce, economic growth, education, and knowledge. Although understanding the HIV care cascade is vital in evaluating care results among the percentage of HIV-positive individuals, it also helps in achieving the 95:95:95 UNAIDS target

However, today Kenya is faced with mental health issues such as depression, which is a common mental disorder. Globally, it is estimated that 5% of adults suffer from depression. Depression and anxiety disorders are the leading mental illnesses diagnosed in Kenya. The rates of depression among PLWHA are three times greater than those found in the general population. In this study, a model for depression and HIV care cascade dynamics was developed and analysed to understand how depression affects the achievement of the 95:95:95 target.

1.5 Objectives

1.5.1 General Objective

The main aim of this study was to assess the impact of depression treatment on HIV care Cascade Dynamics in Kenya using a deterministic compartmental model.

1.5.2 Specific Objectives

The specific objectives for this study were:

1. To develop HIV care Cascade compartmental model in presence of depression treatment.
2. To perform mathematical analysis to assess the epidemiological suitability of the parameters.
3. To analyze the compartmental Model to assess how depression treatment impact HIV care Cascade Dynamics in Kenya.

1.6 Scope of the Study

Whilst the immediate negative health problems related to the HIV/AIDS has been well documented, the impact of the virus on the mental health of the population is poorly understood. The aim of this study was to report on HIV care cascade dynamics in presence of depression treatment.

The study focuses on interplay between HIV and depression state in Kenya. As such it makes use of the rich HIV data and depression treatment statistics and literature in Kenya to quantify the two diseases. This is a comprehensive study of the entire country so as to give context to HIV treatment cascade and depression treatment in Kenya. The the next generation matrix method was used to assess depression treatment in HIV care cascade.

1.7 Significance of the Study

It is evident in every HIV comprehensive treatment center that treatment failure and poor adherence to treatment among PLWHA is not only increasing but also poses a major threat to the milestones already made towards global success in HIV treatment (Gupta and Neogi, 2020). People today not only battle with HIV and its effects but also immensely battle with mental illness like depression. This co-morbidity threatens the achievement of 95:95:95 target; Kenya's HIV vision by 2025 in order to end AIDS by 2030. This compliance can be realized when mental health of PLWHA forms part of care and this requires baseline effects of Mental health on HIV retention in care which the study sought to determine.

This study provides information on the patterns of HIV/AIDS poor retention in care and impact of depression in Kenya, to support regional HIV prevention and achieving the care cascade (95-95-95 target). The findings would be also of major importance in highlighting the specific interventions such as Antidepressant medications and lifestyle changes (regular exercise and exposure to sunlight) to reduce probability of disengagement with care. These results aids the respective County Health Management Teams (CHMTs) in setting up structural interventions for depression disorder to improve retention in care among PLWHA in Kenya. It can also be used by Disease modelling and research institutions such as Kenya Medical Research Institute (KEMRI), National Syndemic Disease Control Council (NSDCC), National AIDS and STI Control Programme (NASCOP), and Centers for Disease Control and Prevention (CDC) for implementation purposes so as to benefit the general society.

Chapter 2: Literature Review

2.1 Introduction

In this chapter a review of the relevant literature related to the present study are presented. The chapter is divided into sections according to the variables of the study. The first section is an introduction to the chapter which is followed by the discussion of HIV/AIDS and depression in the second section. The third section looks at depression and HIV care cascade which is followed by mental health treatment in Kenya. Finally, the fifth section presents literature review on mathematical modelling of HIV/AIDS.

2.2 HIV AIDS and Depression

Kenya is one of the countries in SSA with a high prevalence of HIV/AIDS. The virus has affected the socio-economic and health status of the country, leading to a rise in depression among individuals infected with HIV/AIDS. This section of the literature review aims to examine the existing literature on HIV/AIDS and depression, including prevalence rates.

According to estimates, depression affects 15–22% of the world's population on a lifelong basis (Kulisewa et al., 2019). Depression is thought to be two to three times more prevalent in those with chronic illnesses like HIV. The most frequent psychiatric side effect of HIV illness is mood problems, especially depression. Although some research indicate that depression is no more prevalent in HIV+ individuals than in persons who are at risk for HIV infection, a significant meta-analysis of ten research papers found that the risk of depression was twice as high in HIV+ people as it was in those who were at risk for HIV but were not actually infected (Ciesla and Roberts, 2001). One meta study estimated a high lifetime prevalence of depressive disorders in HIV+ individuals (Ciesla and Roberts, 2001). A study by Nall et al. (2019) on factors affecting HIV testing among youth in Kenya showed that depression and social support served as barriers to HIV testing. Similarly, a cross-sectional study conducted among 595 HIV-positive patients in Uganda found that 44% of the participants had symptoms of depression (Kinyanda et al., 2013). Another study conducted in the western part of Kenya reported a prevalence rate of 30.9% (Singla et al., 2017).

Many medical practitioners also view that depression will inevitably follow an HIV+ diagnosis. Similarly, various research shows that receiving the diagnosis will undoubtedly cause anxiety and distress, sometimes to a point where it interferes with functioning and may even result in suicidal ideation, this type of situation-specific emotional reaction is comparable to depression Bylsma et al., 2008. A person who is distressed after receiving an HIV diagnosis may need therapy, most likely for an adaptation response, but supportive and other forms of counseling work better to manage the discomfort than drugs cause (Harrington et al., 2021). According to Nanni et al. (2015), depression may exacerbate pre-existing illness states and decrease health outcomes in those living with HIV. Previous studies have shown that depression increases the risk of mortality and is linked to greater HIV viral loads and lower CD4 cell counts(Olatunji et al., 2006; Harrington et al., 2021). It also hastens the development of AIDS. Although, HIV can cause brain damage to sub-cortical regions, leading in HIV dementia, which can cause symptoms that are misinterpreted for clinical depression.

A study in Kilifi, Kenya, shows that the prevalence of depressive symptoms among adults living with HIV on the Kenyan coast is high (Nyongesa et al., 2019). Similarly, Mochache (2016) showed that there is a high prevalence of depression among PLWHA; but the patients were more likely to be depressed if they were food insecure, had low income, adopted maladaptive coping styles, experienced a stressful event, intimate partner violence, had opportunistic infections and experienced side effects of antiretroviral.

Meffert et al. (2019) showed that PLWHA have high prevalence of depression on the Center for Epidemiologic Studies Depression Scale (CESD). Similarly, depression symptoms were independently associated with increases in viral load which highlights the need for comprehensive treatment of depression. According to a South African study, those with severe depression were more likely than people without depression to undergo late testing (Rane et al., 2018). A study done outside of SSA in Nepal discovered a link between depression among female sex workers and not using HIV testing and counseling (Shrestha et al., 2017). In a study published in 2019 by Tran et al. (2019) they used bibliometric and scientometric analysis to present worldwide research trends and interests, identify research gaps in the literature, and propose many implications for depression in HIV-positive people. Even though this topic has received a lot of attention and has been examined in depth, additional efforts should be made to overcome the gaps in empirical research in developing nations and biological analysis of the relationship between HIV and depression.

2.3 Depression and HIV care Cascade

Mental health issues have the potential to have an influence on all aspects of HIV care, from HIV prevention methods to diagnosis and retention in ART programs, if they are not recognized and treated. The World Health Organization (WHO) highlights the significance of neuro-cognitive disorders, substance use disorders, and depression notwithstanding the possibility that any mental health condition could have a negligible connection with HIV. Though, literature shows that depression has the biggest disease burden worldwide and in SSA.

Every step of the HIV care cascade lowers outcomes for people with HIV who are depressed. People who have both HIV and depression are less likely to seek care, stay in care, and even survive longer. Despite the fact that the prevalence of HIV is highest in Africa, there is a global correlation between depression and even worse HIV outcomes. Depression is associated with delayed HIV diagnosis, lack of HIV care after diagnosis, delayed initiation of HIV treatment, and non-adherence to treatment (Goin et al., 2020). By preventing access to the enormous health benefits that combination antiretroviral therapy (cART) delivers, a lack of an HIV diagnosis endangers the wellbeing of PLWHA. Because a significant part of new HIV infections are caused by people who are unaware of their HIV status, the lack of HIV diagnosis poses an additional public health risk. Depression-related mental health impairment can prevent people from getting regular HIV tests, knowing their HIV status, connecting to care for HIV, continuing in care, starting cART, and adhering to cART to attain HIV viral suppression.

Although depressive symptoms have frequently been associated with poor drug adherence among HIV/AIDS patients, their relationship to care retention is less well understood. (Zuniga et al., 2016). Depression may provide a significant obstacle to effective retention in HIV primary care. The occurrence of depression is associated with low rates of HIV care linkage and retention, according to research. In one Malawian study, depression in HIV-positive individuals has the potential to aggravate HIV-related morbidity and death, particularly in settings with limited resources. Worldwide, it has been demonstrated that depression is a significant obstacle to linkage to care, retention, ART adherence, and eventually long-term viral suppression. A growing corpus of research in SSA is starting to show that people who are depressed are less likely to start or be connected to ART care. Furthermore, in SSA, low ART adherence is also linked to depression. There has, however, been little study done in SSA on the relationship between depression and regular retention in HIV care and viral suppression. Therefore, more research is required to define the relationship between depression, participation in HIV care, and viral suppression in the sub-Saharan Stockton et al. (2021).

Depression is one of the most powerful indicators of poor cART drug adherence, according to research. According to a large meta-analysis on relationship between depressive disorders and cART nonadherence , the likelihood of achieving good cART adherence was lower among those with depressive symptoms than among those without depressive symptoms (Uthman et al., 2014). This solid result held true in low-, middle-, and high-income nations. Positive HIV health outcomes (such as an increasing viral load, lower CD4+ levels, and an increase in opportunistic diseases) are definitely caused by depression impairment, which contributes to poorer healthcare behaviors across the HIV care continuum. However, there is also research that points to a direct biological link between mental health issues and worse HIV health outcomes, particularly in the case of depression. Therefore, the overwhelming body of data suggests that depression disorders frequently obstruct prompt HIV care linkage as well as long-term treatment retention.

Patient retention in care is fundamental to meeting the UNAIDS 95-95-95 targets by 2025: to have 95% of people living with HIV know their status; 95% of those who know their status on antiretroviral therapy (ART); and 95% of those on ART having undetectable viral loads. Studies have observed a substantial drop-out of people living with HIV across HIV care cascade (Patsis et al., 2020; Kiplagat et al., 2018). The long-term success of ART regimens is threatened by the gaps in the HIV treatment cascade Fox and Rosen (2015). According to a pooled analysis of 37 studies conducted in sub-Saharan Africa, only 57% of individuals who knew their status completed an ART eligibility screening, 66% of those who were eligible started treatment, and 65% of those starting treatment were kept on ART (Kranzer et al., 2012). Similarly, from a study by Patsis et al. (2020), alcohol consumption was associated with significant delays in ART initiation and reduced retention in care for patients enrolling in HIV care and treatment programs in East Africa.

According to CDC (2019), the overall health of HIV-infected people and for the prevention of HIV transmission, regular HIV care is essential. This includes starting and maintaining antiretroviral therapy (ART). Recent studies have demonstrated that over a 2-year period, HIV-infected individuals who got continued, regularly scheduled care had considerably lower viral loads, greater CD4 cell counts, and lower morbidity and mortality than those who missed even one medical appointment (Genberg et al., 2018; Sabin et al., 2020). Regular, continuous HIV care has other advantages, such as promoting safer sexual practices, reducing the risk of developing AIDS, reducing hospitalization rates, and enhancing general health. Patients entering care from different HIV-testing programs demonstrates differences in retention in HIV care over time beyond disease severity.

Various studies have done current estimates of retention among HIV-infected patients on antiretroviral therapy (ART) in Africa and consider patients who are lost to follow-up (LTFU) as well as those who die shortly after their last clinic visit to be no longer in care and to represent limitations in access to care. Whereas, a study by Geng et al. (2010) showed that patient retention for the clinic population assuming lost patients were not in care was 82.3%, 68.9%, and 60.1% at 1, 2 and 3 years. Similarly, van der Kop et al. (2018) showed that 609 of the 775 HIV patients (78.6%, 95% CI: 75.7% to 81.5%) were retained in care at 12 months. Retention in care was 16% greater than retention in clinic. Having a higher baseline CD4 count and participating in the trial versus the cohort study only were associated with a reduced risk of attrition. Other factors that influence retention include depression and point of diagnosis (Genberg et al., 2018).

Treatment failure is most frequently attributed to non-adherence to ART. As a result, keeping patients on ART is crucial for public health since it is correlated with medication access and adherence (Mukumbang et al., 2017). Major obstacles stand in the way of lowering new HIV infections, addressing health inequities, and enhancing health outcomes: non-adherence to ART. Thus, a key component of ART programs is retention in treatment, or the ability of PLHIV to adhere to crucial parts of care, such as attending regular follow-up appointments, scheduled laboratory testing, and other monitoring activities as prescribed by the healthcare practitioner. Several factors influence retention in care among persons in HIV ART. The individual level barriers to retention in care included side effects, gaining weight, belief in faith healing, and use of herbal remedies and alcohol (Jopling et al., 2020). Other barriers are interpersonal barriers such as stigma and nondisclosure of HIV status were reported. Factors affecting retention in care at the institutional level are inadequate space in the clinic, long waiting times, long travel distances, and shortage of third-line drugs. Food shortages and patient mobility were reported as community barriers to retention in care.

2.4 Mental Health Treatment in Kenya

Our state of mind includes all aspects of our physical, psychological, emotional, and social wellbeing, according to the CDC (2021). It affects the way we feel, think, and act. It also affects how we react under duress, communicate with others, and make choices. Important components of general health are both physical and emotional well-being. For example, depression increases the risk of many various physical health conditions, particularly chronic illnesses like diabetes, heart disease, and stroke. Similar to chronic physical conditions, having mental illness increases the risk of getting it. A startling 1 in 4 people in Kenya who seek medical attention have a mental health disorder, according to the World Health Organization (2021). In addition to rising incidence of drug and alcohol use disorders, depression is a widespread condition.

A World Health Organization report ranked Kenya the fifth among African countries with the highest number of depression cases. The majority of people in Africa who suffer from significant mental illnesses cannot get the necessary care. As a result of poor health, psychological disability, and early mortality, there is a significant burden of mental illness and significant access gaps to healthcare. COVID-19, which has led to an increase in mental health-related problems, has not helped the situation either (Muller et al., 2020). The lack of understanding of mental health diseases and, specifically, the symptoms connected with each, is a significant problem Kenya encounter. The WHO research also reveals that Kenya was one of the few countries without a dedicated budget for mental health, with a government expenditure amounting to just 0.01% of the overall budget. According to the Auditor General's 2017 audit on mental health, this equates to a lack of suitable facilities. An astounding 22 out of Kenya's 47 counties lack mental facilities.

The lack of a defined mental health response strategy is largely due to the misconception that other diseases pose the greatest threats to the nation, which is more and more disproved by data on the causes of death in the country (Walker and Vearey, 2022). The stigma surrounding mental health prevents patients from receiving the compassion and understanding that have long been extended to the sick in African society. Thus, the goal of this series is to promote policy reform and increase awareness of mental health. Given the context, it is obvious that, in order to close the gap, especially in a resource-constrained economy, mental health should be prioritized in public health (Katsonga-Phiri et al., 2021).

2.5 Use of ODEs in Modelling Depression and HIV

Differential equation (DE) models can be used to explain a variety of biological systems and processes. In study of infectious diseases like HIV dynamics and cellular kinetics, differentiable equation modeling is a crucial tool. Miao et al. (2009) demonstrated the use of ODEs in modeling infectious disease by using the HIV viral fitness experiment. Various ODE models have been developed to understand the dynamics of depression and its treatment in Kenya. For example, a model was developed to simulate the impact of cognitive-behavioral therapy (CBT) on depression in adolescents in Kenya (Gichuru et al., 2017). The model used a system of nonlinear ODEs to capture the changes in depression scores over time. The results showed that CBT was effective in reducing depression scores, and the model provided insights into the optimal timing and duration of CBT. Similarly, Rao and Luo (2021) developed a compartmental model with susceptible population, HIV infected persons who are not diagnosed, susceptible population ,HIV infected individuals who are diagnosed and those who proceed to AIDS compartment. They employed the ODEs and Stochastic Differential equations to model HIV/AIDS infection in the presence of the diagnosed and un-diagnosed infected populations. The research showed that the two models were closely associated in modelling disease dynamics.

Bhunu et al. (2011) analyzed the effects of a rise in the proportion of HIV-positive people who are no longer engaged in sexual activity on the AIDS epidemic in sub-Saharan Africa. They created a model of HIV/AIDS that includes several categories of sexual activity levels and known HIV statuses, which are influenced by HIV/AIDS awareness campaigns. The entire population is classified into the following compartments: Susceptibles (S); HIV positive individuals who are unaware of their status; HIV positive individuals who are aware of their status but have increased their risky sexual behavior as a result of knowing their status; and HIV positive individuals who are sexually inactive and AIDS persons.

Additionally, Waema and Olowofeso (2015) studied an age-structured stochastic method to model HIV transmission diseases. This approach used models based on mother-to-child transmission (MTCT), heterosexual transmission, and combined case (integrating all groups and the two modes of transmission). The growing variance and constant expectation were the results of the S(t) sensitive model. It was established that heterosexual models and mother-to-child transmission are particular instances of the mixed model. Similar stochastic approaches have been used by Tengaa et al. (2020); Bershteyn et al. (2018); Mugo et al. (2022)

Simangunsong and Mungkasi (2021) also developed an HIV compartmental model with the Susceptible, Infected, and AIDS cases (SIA). They employed the fourth order Runge-Kutta technique, one of the very accurate numerical approaches that may be used to handle initial value issues of non-linear ordinary differential equations, to solve the SIA model. They concluded that the Runge-Kutta technique effectively solves the HIV-AIDS transmission framework, and based on the numerical solutions, the susceptible sub-population, the infected sub-population, the AIDS cases sub-population, and the total population will never go extinct and approach constants, which means that they each reach their equilibrium points in a longer time (approaches to infinity).

Chapter 3: Research Methodology

3.1 Introduction

In this chapter, the methods that was used to assess the research questions, sources of data, method to be used, parameterization and analysis are presented. The mathematical model for depression treatment and the HIV care cascade was developed based on a system of ordinary differential equations (ODEs) that captures the dynamics of the disease treatment, and outcomes. The model was constructed based on existing epidemiological data and clinical evidence for the diseases in Kenya.

3.2 Source of Data

In this research data to be used was obtained from literature sources. The study was relying on a variety of data sources, including epidemiological data from national health surveys, clinical data from hospitals and health clinics, and published literature on depression and HIV. Parameter estimations were collected from previous studies which was used for simulation of estimates and modelling.

3.3 Compartmental Model

The deterministic compartmental model of the depression and HIV epidemic was developed as illustrated in Figure 3.1. In this proposed study, before constructing the depression and HIV compartmental model, the total human population N(t) was separated into nine mutually exclusive classes such that:

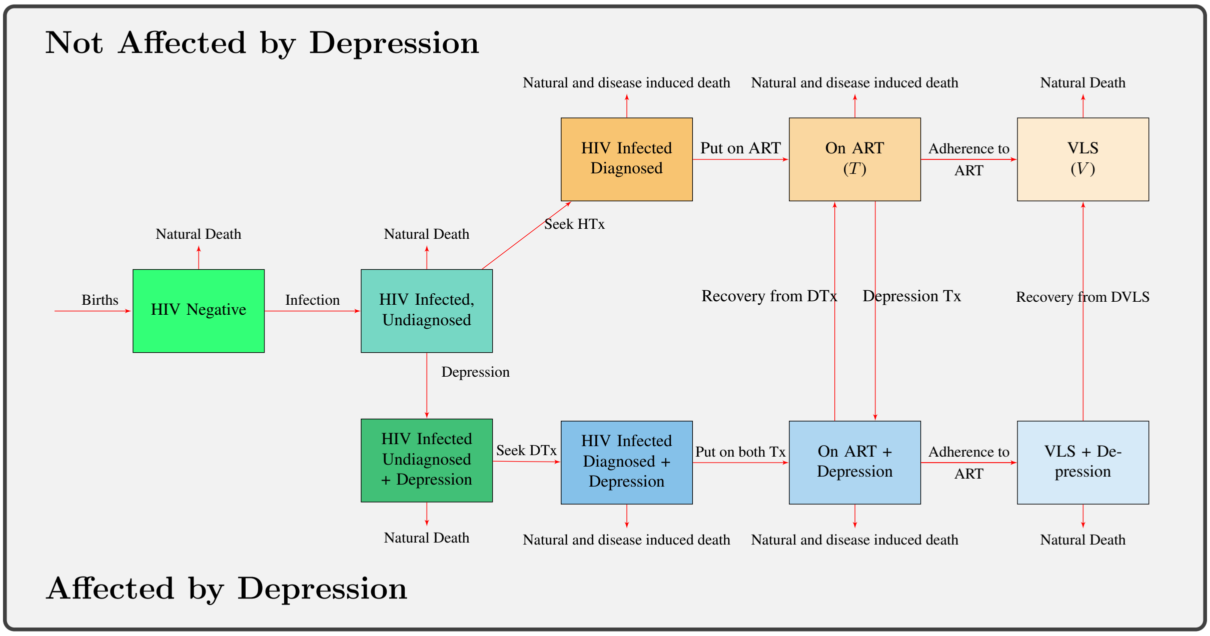

Figure 3.1: Model diagram for HIV and Depression Compartmental Model

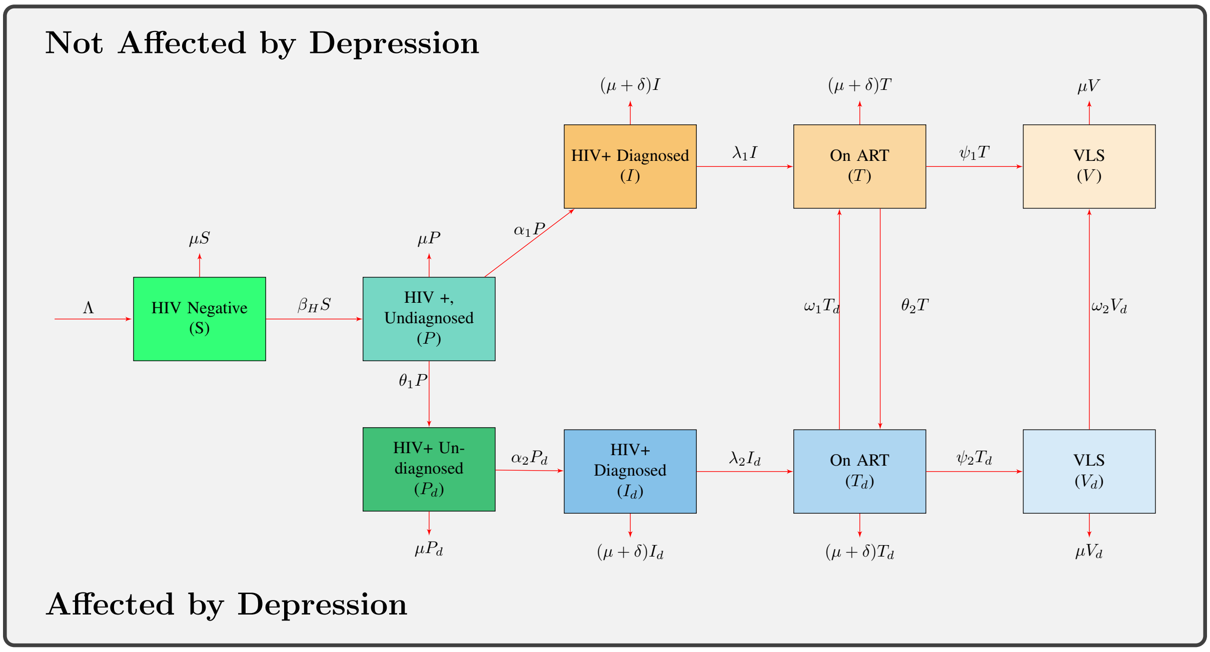

Figure 3.2: Parameterized Model diagram for HIV and Depression Compartmental Model

Table 3.1: Table of the Model States

List of States	
State	Descriptions
S	HIV Negative susceptible class representing a group of susceptible individuals free of both HIV and depression.
P	HIV Positive class representing HIV positive individuals that are not aware of their HIV statuses (They have not been tested).
P_d	HIV Depressed Positive class which gives a group of individuals who are HIV positive, affected by depression and that are not aware of their HIV statuses (They have not been tested).
I	HIV Diagnosed class depicting a group of individuals who are diagnosed as HIV positive and are not affected by depression.
I_d	HIV Diagnosed Depressed class depicting a group of individuals who are HIV positive and diagnosed, who are affected by depression.
T	HIV On-ART class representing a group of individuals who are HIV positive and they are taking ART treatment who are not affected by depression.
T_d	HIV Depressed On-ART class representing a group of individuals who are HIV positive on ART but are affected by depression.
V	HIV + VLS compartment depicting a group of individuals who are HIV positive on ART and achieved Viral Load Suppression (VLS) but they are not affected by depression.
V_d	HIV + Depressed VLS compartment depicting a group of individuals who are HIV positive on ART and achieved Viral Load Suppression (VLS) but are affected by depression.

Table 3.2: Table of Model Parameters

Parameter List			
Parameter	Descriptions	Values	Source
Λ	Recruitment rate.	0.028	World Bank Estimate 2021
β	HIV infection rate	0.037	NSDCC 2023 Estimates
γ_1	Relative HIV infectiousness of an HIV-infected undiagnosed individual	1.8	Assumed
γ_2	Relative HIV infectiousness of an HIV-infected undiagnosed co-infected individual	1.7	Assumed
γ_3	Relative HIV infectiousness of an HIV-infected diagnosed individual	1.0	Assumed
γ_4	Relative HIV infectiousness of an HIV-infected diagnosed co-infected individual	1.0	Assumed
θ_1	Rate at which HIV infected undiagnosed persons get depression	0.4725	Ng'ang'a (2011)
θ_2	Rate at which persons on treatment get depression	0.39	Tran et al. (2019)
α_1	HIV Undiagnosed individuals seek HIV testing.	0.056	Waruiru et al. (2014)
α_2	HIV Undiagnosed individuals seek depression treatment.	0.028^*	Assumed
λ_1	Diagnosed individuals put on HIV treatment (ART).	0.92	UNAIDS
λ_2	Diagnosed and depressed individuals put on ART and depression treatment	0.138	Nyongesa et al. (2019)
ψ_1	Adherence to ART.	0.906	KENPHIA
ψ_2	Adherence to ART for depressed individuals.	0.52548	Uthman et al. (2014)
ω_1	Recovery from Depression among OnART Persons.	Varies	Assumed
ω_2	Recovery from Depression among VLS Individuals.	Varies	Assumed
δ	Disease induced death.	0.015714286	NACC Estimates 2021
μ	Natural death.	0.007	World Bank Estimate 2021

3.4 Model Description

The compartmental model begins with a human population of susceptible who are free of HIV and depression, denoted by S. This population is replenished at constant rate Λ through sexual maturity or immigration. Upon effective contact with individuals infected with the HIV virus, the new infectives progress into the infected classes P (HIV positive undiagnosed) at rate β_H where β_H denotes the force of infection. Individuals in classes P and T get depressed at the rate θ_1 and θ_2 and move to class P_d and T_d, respectively. The class P_d was assumed to be as a result of depression caused by the high frequency associated with chronic illness. These individuals seek depression treatment at the rate α_2 and in the process they are diagnosed with HIV and move to class I_d (HIV positive diagnosed affected by depression). Individuals in class P seek HIV testing at the rate α_1 and move to class I (HIV positive diagnosed). HIV infection is known to promote or enhance the development of depression condition. It was assumed that individuals who are diagnosed with HIV (class I) start ART treatment at the rate λ_1 and move to class T. Similarly, individuals who are diagnosed with HIV and have depression, class I_d, are put on both ART and depression treatments at the rate λ_2 and move to class T_d. Also, all persons who are put on depression and ART treatments, adhere to depression treatment and they are cured of depression at the rate ω_1 and move to being on ART treatment, class T. The HIV-infected and un-depressed individuals undergoing ART treatment (T), adhere to ART treatmnet at the rate ψ_1 and they achieve viral load suppression. Also, HIV-infected and depressed individuals undergoing ART treatment (T_d) adhere to ART treatmnet at the rate ψ_2 and move to class V$_d$. Finally, all persons On ART affected by depression going for regular VLS check are screened and treated for depression and they adhere to depression treatment at the rate ω_2 and they achieve viral load suppression. Individuals in classes I, I_d, T and T_d die at a disease induced rate of δ. In the research, it is assumed that individuals in all classes are also subject to a natural mortality at the rate of μ.

The force of infection β_H depends on the probability of transmission per contact β, the proportion of infected individuals in each category P, P_d, I and I_d. Individuals in receipt of ART have reduced viral load and are therefore assumed to be less infectious relative to infectives not in receipt and initiating ART. Similarly, individuals who have achieved viral load suppression are assumed to be less infectious. Persons in classes P, P_d, I and I_d have weaker immune systems and are therefore likely to carry higher viral loads than their counterparts. This infectiousness is modeled by $\gamma_1 > 1$, $\gamma_2 > 1$ and $\gamma_3 > 1$ respectively. The total sexually active variable population at time t is given by $N(t) = S(t) + P(t) + P_d(t) + I(t) + I_d(t) + T(t) + T_d(t) + V(t) + V_d(t)$. Assuming homogeneous mixing, the time dependent force of infection for HIV is given by

$$\beta_H = \beta \left[\frac{\gamma_1 P + \gamma_2 P_d + \gamma_3 I + \gamma_4 I_d}{N} \right].$$

3.5 Model Assumptions

1. The time between diagnosis and treatment is too short such that those diagnosed are put immediately on treatment and so they don't get diagnosed for depression.
2. When an individual goes for deprtession screening and treatment, they are also tested for HIV.
3. Individuals on ART and Virally suppressed are less infectious. The R_0 is calculated using classes P, P_d, I and I_d.
4. It is assumed that the time dependent force of infection for HIV is given by
5. If you do not adhere to ART you remain in the same class/Compartment.
6. The relative infectiousness of γ_i for i=1,2,3,4 are compared to those in I. Such that:

$$\beta_H = \beta \left[\frac{I + \gamma_1 P + \gamma_2 P_d + \gamma_3 I_d}{N} \right].$$

3.6 Model Equations

The resulting nonlinear system of differential equations are:

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\mu + \beta_H)S \\
\frac{dP}{dt} &= \beta_H S - (\mu + \alpha_1 + \theta_1)P \\
\frac{dP_d}{dt} &= \theta_1 P - (\mu + \alpha_2)P_d \\
\frac{dI}{dt} &= \alpha_1 P - (\lambda_1 + \mu + \delta)I \\
\frac{dI_d}{dt} &= \alpha_2 P_d - (\lambda_2 + \mu + \delta)I_d \\
\frac{dT}{dt} &= \lambda_1 I + \omega_1 T_d - (\psi_1 + \theta_2 + \mu + \delta)T \\
\frac{dT_d}{dt} &= \lambda_2 I_d + \theta_2 T - (\omega_1 + \psi_2 + \mu + \delta)T_d \\
\frac{dV}{dt} &= \psi_1 T + \omega_2 V_d - \mu V \\
\frac{dV_d}{dt} &= \psi_2 T_d - (\omega_2 + \mu)V_d
\end{aligned}
\tag{3.1}
$$

With initial conditions

$S(0) > 0, P(0) \geq 0, P_d(0) \geq 0, I(0) \geq 0, I_d(0) \geq 0, T(0) \geq 0, T_d(0) \geq 0, V(0) \geq 0$ and $V_d(0) \geq 0$.

3.7 Mathematical Analysis of the HIV-Depression Model

3.7.1 Basic Properties: Boundedness and Positivity

In this subsection, the main aim was to investigate the feasibility and positivity of the solution of the depression and HIV care cascade model.

3.7.2 Dynamics of the Model

Let a total population $N(t) = S(t) + P(t) + P_d(t) + I(t) + I_d(t) + T(t) + T_d(t) + V_d(t) + V(t)$ and taking the time derivative of $N(t)$ along solutions of model the equation (3.1), we obtain:

The sum of all the differential equations given above is:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dP}{dt} + \frac{dP_d}{dt} + \frac{dI}{dt} + \frac{dI_d}{dt} + \frac{dT}{dt} + \frac{dT_d}{dt} + \frac{dV_d}{dt} + \frac{dV}{dt}. \tag{3.2}$$

$$\begin{aligned} \frac{dN}{dt} =&\Lambda - (\mu + \beta_H)S + \beta_H S - (\mu + \theta_1 + \alpha_1)P \\ &+ \theta_1 P - (\mu + \alpha_2) + \alpha_1 P - (\lambda_1 + \mu + \delta)I \\ &+ \alpha_2 P_d - (\lambda_2 + \mu + \delta)I_d + \lambda_1 I + \omega_1 T_d - (\psi_1 + \theta_2 + \mu + \delta)T \\ &+ \lambda_2 I_d - \theta_2 T(\omega_1 + \psi_2 + \mu + \delta)T_d \\ &+ \psi_1 T + \omega_2 V_d - \mu V + \psi_2 T_d - (\omega_2 + \mu)V_d \end{aligned} \tag{3.3}$$

All parameters for the model system are assumed to be non-negative for all time $t > 0$.

3.7.3 Boundedness

The feasibility of the model describes the region in which the solution of the system of equation (3.1) is biologically meaningful.

Theorem 3.1. Suppose equation (3.3) holds, every solution of the model in system of equation (3.1) with initial conditions in $\Re^9_+$ approaches and stays in the compact set (Ω) as $t \to \infty$. Then, the feasible solution which is a positively invariant set of the model is given by:

$$\Omega = \left\{ (S, P, P_d, I, I_d, T, T_d, V_d, V) \in \Re^9_+ : N(t) \leq \frac{\Lambda}{\mu} \right\}$$

Proof. From the equation (3.3) where changes of N leads to change of all variables in the population (i.e $N = S + P + P_d + I + I_d + T + T_d + V_d + V$) we have:

$$
\begin{aligned}
\frac{dN}{dt} =& \Lambda - \mu S - \mu P - \mu P_d - (\mu+\delta) I_d \\
& - (\mu+\delta) T - (\mu+\delta) T_d \\
& - \theta V + \theta V_d - \mu V_d - \mu V
\end{aligned} \tag{3.4}
$$

In the absence of disease ($P = P_d = I = I_d = T = T_d = V_d = V = 0$), the equation (3.4) reduces to

$$\frac{dN}{dt} = \Lambda - \mu N(t) \tag{3.5}$$

Where $S = N$

From the equation (3.5) we observe that,

$$\frac{dN}{dt} \le 0 \quad \text{if} \quad N(t) \ge \frac{\Lambda}{\mu}.$$

Therefore,

$$\frac{dN}{dt} \le \Lambda - \mu N(t) \tag{3.6}$$

Applying Birkhoff and Rota's theorem (Birkhoff and Rota, 1978) on differential inequalities and method of integrating factor (IF) on the inequality (3.6) we will have:

$$\frac{d}{dt}\left[\mathrm{e}^{\mu t} N(t)\right] \le \mathrm{e}^{\mu t}\Lambda. \tag{3.7}$$

Integrating the inequality (3.7) on both sides along with the initial condition $t = 0$ we obtain

$$N(t) \leq N(0)\mathrm{e}^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - \mathrm{e}^{-\mu t}\right) \tag{3.8}$$

Hence, at $\lim t \to \infty$ of equation (3.8) becomes

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu}$$

we then have:

N(t) $\leq \frac{\Lambda}{\mu}$

which implies that $0 \leq N \leq \frac{\Lambda}{\mu}$, then trajectories of the model equation (3.1) are bounded in the region Ω. This completes the proof.

Hence, the feasible solution which is given by

$$\Omega = \left\{ (S, I_1, I_{1d}, I_2, I_{2d}, T, T_d, V_d, V) \in \Re^9_+ : N(t) \leq \frac{\Lambda}{\mu} \right\},$$

is a compact forward invariant set for the system in the equation (3.1). This implies that, Ω is positively invariant. The solution of the system of equation (3.1) remains in Ω for all $t > 0$ and thus the model is biologically meaningful and epidemiologically well posed in the domain Ω.

3.7.4 Positivity of the Solutions

The positivity of the solution describes the non-negativity of the solutions of model equation (3.1). For model in equation (3.1) to be epidemiologically meaningful, it is important to prove that all its state variables are non negative for all time t. We considered the lemma below.

Lemma 3.1. Let the initial value of the system in equation (1) be
$\{(S(0), P(0), P_d(0), I(0), I_d(0), T(0), T_d(0), V_d(0), V(0)) \geq 0\} \in \Omega$. Then, the solution set

$\{S(t), P(t), P_d(t), I(t), I_d(t), T(t), T_d(t), V_d(t), V(t))\}$ of equation (3.1) is positive for all $t > 0$.

The ***Proof*** follows in the subsequent Sub-sections.

3.7.4.1 Positivity of $S(t)$

From the first equation in system of equation (3.1), it is assumed that

$$\frac{dS}{dt} = \Lambda - (\mu + \beta_H)S \geq -(\beta_H + \mu)S, \text{ for } \beta \in [0, 1) \text{ and } \beta_H \leq \frac{\Lambda}{SI}$$

$$\frac{dS}{dt} \geq -(\beta_H + \mu)S. \tag{3.9}$$

Integrating inequality (3.9) by separating variables gives

$$S(t) \geq S(0)\mathrm{e}^{-(\beta_H+\mu)t}, \text{ since } (\beta_H + \mu) > 0. \tag{3.10}$$

So, $S(0) \geq 0$ and

$$\mathrm{e}^{-(\beta_H+\mu)t} \geq 0$$

The inequality (3.10) show that the variable S is positive for all $t > 0$.

3.7.4.2 Positivity of $P(t)$

The solution of second equation in the system of equation (3.1) is obtained as

$$\frac{dP}{dt} = \beta_H S - (\mu + \alpha_1 + \theta_1)P \geq -(\mu + \alpha_1 + \theta_1)P. \tag{3.11}$$

The solution of the inequality (3.11) is

$$P(t) \geq P(0)\mathrm{e}^{-(\mu+\alpha_1+\theta_1)t} \geq 0, \text{ since } (\mu + \alpha_1 + \theta_1) > 0 \tag{3.12}$$

The inequality (3.12) show that the variable P is positive for all $t > 0$.

3.7.4.3 Positivity of $P_d(t)$

The solution of third equation in the system of equation (3.1) is obtained as

$$\frac{dP_d}{dt} = \theta_1 P - (\mu + \alpha_2)P_d \geq -(\mu + \alpha_2)P_d. \tag{3.13}$$

The solution of the inequality (3.13) is

$$P(t) \geq P(0)\mathrm{e}^{-(\mu+\alpha_2)t} \geq 0, \text{ since } (\mu + \alpha_2) > 0 \tag{3.14}$$

The inequality (3.14) show that the variable $\mathrm{P_d}$ is positive for all $t > 0$.

3.7.4.4 Positivity of $I(t)$

Further, the solution of fourth equation in the system of equation (3.1) is obtained as

$$\frac{dI}{dt} = \alpha_1 P - (\lambda_1 + \mu + \delta)I \geq -(\lambda_1 + \mu + \delta)I. \tag{3.15}$$

The solution of the inequality (3.15) is

$$I(t) \geq I(0)\mathrm{e}^{-(\lambda_1+\mu+\delta)t} \geq 0, \text{ since } (\lambda_1 + \mu + \delta) > 0 \tag{3.16}$$

The inequality (3.16) show that the variable I is positive for all $t > 0$.

3.7.4.5 Positivity of $I_d(t)$

The solution of fifth equation in the system of equation (3.1) is obtained as

$$\frac{dI_d}{dt} = \alpha_2 P_d - (\lambda_2 + \mu + \delta)I_d \geq -(\lambda_2 + \mu + \delta)I_d. \tag{3.17}$$

The solution of the inequality (3.17) is

$$I_d(t) \geq I_d(0)\mathrm{e}^{-(\lambda_2+\mu+\delta)t} \geq 0, \text{ since } (\lambda_2 + \mu + \delta) > 0 \tag{3.18}$$

The inequality (3.18) show that the variable $\mathrm{I_d}$ is positive for all $t > 0$.

3.7.4.6 Positivity of $T(t)$

The solution of sixth equation in the system of equation (3.1) is obtained as

$$\frac{dT}{dt} = \lambda_1 I + \omega_1 T_d - (\psi_1 + \theta_2 + \mu + \delta)T \geq -(\psi_1 + \theta_2 + \mu + \delta)T. \tag{3.19}$$

The solution of the inequality (3.19) is

$$T(t) \geq T(0)\mathrm{e}^{-(\psi_1+\theta_2+\mu+\delta)t} \geq 0, \text{ since } (\psi_1 + \theta_2 + \mu + \delta) > 0. \tag{3.20}$$

The inequality (3.20) show that the variable T is positive for all $t > 0$.

3.7.4.7 Positivity of $T_d(t)$

In addition, the solution of seventh equation in the system of equation (3.1) is obtained as

$$\frac{dT_d}{dt} = \lambda_2 I_d + \theta_2 T - (\omega_1 + \psi_2 + \mu + \delta)T_d \geq -(\omega_1 + \psi_2 + \mu + \delta)T_d. \tag{3.21}$$

The solution of the inequality (3.21) is

$$T_d(t) \geq T_d(0)\mathrm{e}^{-(\omega_1+\psi_2+\mu+\delta)t} \geq 0, \text{ since } (\omega_1 + \psi_2 + \mu + \delta) > 0 \tag{3.22}$$

The inequality (3.22) show that the variable $\mathrm{T_d}$ is positive for all $t > 0$.

3.7.4.8 Positivity of $V(t)$

The solution of eighth equation in the system of equation (3.1) is obtained as

$$\frac{dV}{t} = \psi_1 T + \omega_2 V_d - \mu V \geq -\mu V_d \tag{3.23}$$

The solution of the inequality (3.23) is

$$V(t) \geq V(0)\mathrm{e}^{-(\mu)t} \geq 0, \text{ since } (\mu) > 0 \tag{3.24}$$

The inequality (3.24) show that the variable V is positive for all $t > 0$.

3.7.4.9 Positivity of $V_d(t)$

Finally, the solution of ninth equation in the system of equation (3.1) is obtained as

$$\frac{dV_d}{dt} = \psi_2 T_d - (\omega_2 + \mu)V_d \geq -(\omega_2 + \mu)V_d. \tag{3.25}$$

The solution of the inequality (3.25) is

$$V_d(t) \geq V_d(0)\mathrm{e}^{-(\omega_2+\mu)t} \geq 0, \text{ since } (\omega_2 + \mu) > 0 \tag{3.26}$$

The inequality (3.26) show that the variable $\mathrm{V_d}$ is positive for all $t > 0$.

Thus, the inequalities in (3.10), (3.12), (3.14), (3.16), (3.18), (3.20), (3.22) and (3.24) show that the state variables $\{S(t), P(t), I(t), I_d(t), T(t), T_d(t), V_d(t), V(t))\}$ are positive for all $t > 0$.

3.8 Equilibrium States

3.8.1 Disease-Free Equilibrium Point of the Model

The disease-free equilibrium is defined as the point at which no disease is present in the population. In this research, the compartments S is the only compartment without the disease. Disease-free equilibrium point of the HIV infection model (3.1) is obtained by making its right-hand side as zero and setting the infected groups to zero as P=P_d = I = I_d = T = T_d = V = V_d = 0 we have got:

$$\frac{dS}{dt} = \Lambda - \mu S - \beta S$$

This becomes:

$$\Lambda - \mu S - \beta_H S = 0$$

Making S the subject:

$$S^0 = \frac{\Lambda}{(\mu + \beta)}$$

but $\beta = 0$ is the force of infection.

$$S^0 = \frac{\Lambda}{\mu}$$

The model (3.1) possesses a disease-free equilibrium, ξ_0, given by

$$\xi_0 = \{S, P, P_d, I, I_d, T, T_d, V_d, V\} = \left\{ \left(\frac{\Lambda}{\mu} \right), 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

3.9 Reproductive Number

3.9.1 Basic Reproduction Number of the Model

The basic reproduction number R_0 is used to measure the transmission potential of a disease. It is the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible. Following Van den Driessche and Watmough (2002), the HIV reproductive number of system (1), R_0, is given by the spectral radius of the matrix FV^{-1} where the matrices F and V are give the results below.

The matrix of class of new infections, F was: $\begin{pmatrix} \beta\gamma_1 & \beta\gamma_2 & \beta\gamma_3 & \beta\gamma_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

The matrix of transfer out V was:

$$\begin{pmatrix} \theta_1 + \mu + \alpha_1 & 0 & 0 & 0 \\ -\theta_1 & \mu + \alpha_2 & 0 & 0 \\ -\alpha_1 & 0 & \delta + \mu + \lambda_1 & 0 \\ 0 & -\alpha_2 & 0 & \delta + \mu + \lambda_2 \end{pmatrix}$$

The eigenvalues was given by:

$\left\{0, 0, 0, \frac{Q1+Q2}{Q3}\right\}$

where:

$\mathrm{Q1} = \delta\mu\alpha_1\beta\gamma_3 + \mu^2\alpha_1\beta\gamma_3 + \delta\alpha_1\alpha_2\beta\gamma_3 + \mu\alpha_1\alpha_2\beta\gamma_3 + \theta_1\delta\alpha_2\beta\gamma_4$

$\mathrm{Q2} = \theta_1\mu\alpha_2\beta\gamma_4 + \theta_1\alpha_2\beta\gamma_4\lambda_1 + \mu\alpha_1\beta\gamma_3\lambda_2 + \alpha_1\alpha_2\beta\gamma_3\lambda_2 + (\mu + \alpha_2)\beta\gamma_1(\delta + \mu + \lambda_1)(\delta + \mu + \lambda_2) + \theta_1\beta\gamma_2(\delta + \mu + \lambda_1)(\delta + \mu + \lambda_2)$

$\mathrm{Q3} = (\theta_1 + \mu + \alpha_1)(\mu + \alpha_2)(\delta + \mu + \lambda_1)(\delta + \mu + \lambda_2)$

The reproduction number is:

$R_0 = \frac{Q1+Q2}{Q3}$

This number R_0 is a threshold such that if $R_0 < 1$ the disease clears from the population. If $R_0 > 1$ the steady state ξ_0 becomes unstable and the disease establishes itself into the population. This number is comprehensively analyzed further in Section below to reveal the impact of depression treatment.

3.10 Stability Analysis

3.10.1 Local Stability

Theorem 3.1: *The disease-free equilibrium D^0 of the system in ODEs is locally asymptotically stable in Ω if $R_0 < 1$ and unstable if $R_0 > 1$ for all parameters: are all positive.*

Proof: The local stability of the HIV-Depression model (3.1) at the disease-free equilibrium point

$$D_k^0 = \left(S^0, P^0, P_d^0, I^0, I_d^0, T^0, T_d^0, V^0, V_d^0\right)$$

The stability can be analysed using Routh-Hurwitz local stability criteria stated in Theorem 4 (Teklu and Terefe, 2022). The Jacobian matrix of the HIV-Depression dynamical system (3.1) at the given disease-free equilibrium point is given by:

$$\begin{pmatrix} -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_1 & Z6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & Z2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & Z3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & Z4 & \omega_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & \theta_2 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 & -\mu & \omega_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & Z7 \end{pmatrix}$$

where: Z1 $= -\theta_1 - \mu - \alpha_1$

Z2 $= -\delta - \mu - \lambda_1$

Z3 $= -\delta - \mu - \lambda_2$

Z4 $= -\delta - \theta_2 - \mu - \psi_1$

Z5 $= -\delta - \mu - \psi_2 - \omega_1$

Z6 $= -\mu - \alpha_2$

Z7 $= -\mu - \omega_2$

Then the characteristic equation of the Jacobian matrix above is given by

$$\begin{pmatrix} -1-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+Z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_1 & -1+Z6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & -1+Z2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & -1+Z3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -1+Z4 & \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & \theta_2 & -1+Z5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 & -1+\mu & \omega_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & -1+Z7 \end{pmatrix}$$

The eigenvalues of the characteristics polynomials of Jacobian matrix are:

$\{-1-\mu, -1-\mu, -1-\theta_1-\mu-\alpha_1, -1-\mu-\alpha_2, -1-\delta-\mu-\lambda_1,$

$1-\delta-\mu-\lambda_2, -1-\delta-\theta_2-\mu-\psi_1, -1-\delta-\mu-\psi_2-\omega_1, -1-\mu-\omega_2\}$

Hence, since all the eigenvalues of the characteristics polynomials of Jacobian matrix for the HIV-Depression model given in Equation (3.1) are negative if $\mathcal{R}_0 < 1$ the DFE is locally asymptotically stable. Here the biological implication of Theorem 3.1 is that the HIV-Depression diseases can be eradicated from the population (when the threshold quantity $\mathcal{R}_0 < 1$) if the initial sizes of the population of the HIV-Depression infection are in the basin of attraction of the disease-free equilibrium (E_C^0). Therefore, a small change of HIV-Depression infected individuals into the population will not generate large outbreaks of the diseases, and the diseases will eradicate from the community over time.

3.10.2 Global Stability of Disease Free Equilibrium Point

Theorem 3.2*: The disease-free equilibrium $\mathcal{D}_0$ of the system (1) is globally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable otherwise. The disease-free equilibrium $\mathcal{E}_0$ is the only equilibrium when $\mathcal{R}_0 \leq 1$ (Huo and Feng, 2012).*

3.10.3 Endemic Equilibrium Point of the Model

Theorem 3.3. If $\mathcal{R}_0 > 1$, the system (3.1) has a unique Endemic equilibrium given:

$$\mathcal{E}_1 = [S^*, P^*, P_d^*, I^*, I_d^*, T^*, T_d^*, V^*, V_d^*]$$

Proof. To compute the endemic equilibrium points of the system (3.1), we equate the right-hand side of the system to zero and solve in terms of force of infection. Thus, we have: $S^* = \frac{\Lambda}{\beta_H+\mu}$, $P^* = \frac{\beta_H\Lambda}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)}$, $P_d^* = \frac{\beta_H\Lambda\theta_1}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)}$,
$I^* = \frac{\beta_H\Lambda\alpha_1}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\lambda_1+\mu+\delta)}$,
$I_d^* = \frac{\beta_H\Lambda\theta_1\alpha_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)}$,
$T^* = (\frac{\lambda_1\beta_H\Lambda\alpha_1}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\lambda_1+\mu+\delta)} + \frac{\omega_1\beta_H\Lambda\theta_1\alpha_2\lambda_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)(\omega_1+\psi_2+\mu+\delta)})\frac{1}{(\psi_1+\theta_2+\mu+\delta)}$,
$T_d^* = \frac{\beta_H\Lambda\theta_1\alpha_2\lambda_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)(\omega_1+\psi_2+\mu+\delta)}$,
$V^* = (\frac{\lambda_1\beta_H\Lambda\alpha_1}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\lambda_1+\mu+\delta)} + \frac{\omega_1\beta_H\Lambda\theta_1\alpha_2\lambda_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)(\omega_1+\psi_2+\mu+\delta)})\frac{\psi_1}{(\psi_1+\mu+\delta)(\mu)} +$
$\frac{\omega_2\beta_H\Lambda\theta_1\alpha_2\lambda_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)(\omega_1+\psi_2+\mu+\delta)(\mu)}$,
$V_d^* = \frac{\beta_H\Lambda\theta_1\alpha_2\lambda_2\psi_2}{(\mu+\alpha_1+\theta_1)(\beta_H+\mu)(\alpha_2+\mu)(\lambda_2+\mu+\delta)(\omega_1+\psi_2+\mu+\delta)(\omega_2+\mu)}$

Substituting the expressions for the state variables P^*, P_d^*, I^* and I_d^* into β_H and simplifying, we obtain the following polynomial

$$B_1\beta_H^* + B_0 = 0 \tag{3.27}$$

We note that $B_0 > 0$ if $\mathcal{R}_0 < 1$ and expression (3.27) does not have a positive solution implying that there exists a unique disease-persistent equilibrium if and only if $\mathcal{R}_0 > 1$.

3.10.4 Local Stability for Endemic equilibrium Point of the Model

Theorem 3.4. *The disease-persistent equilibrium $\mathcal{E}_1$ is globally asymptotically stable whenever $\mathcal{R}_0$ is greater than unity (Ma et al., 2013).*

Chapter 4: Sensitivity Analysis and Numerical simulations

In this section using parameter values given in Table 3.2, both the sensitivity analysis using forward sensitivity index and numerical simulations using R-Studio was carried out to justify the analytical results in the previous sub-sections. Also, the numerical simulations of the HIV-Depression model (3.1) was carried out to assess the impact of of depression treatment on HIV care Cascade Dynamic.

4.1 Sensitivity Analysis

Compartmental models have several parameters that govern the dynamics of the system. Sensitivity analysis is an important tool for evaluating how changes in these parameters affect the behavior of the model. Sensitivity analysis can help identify which parameters have the most significant impact on the model's output and can also help to identify areas of uncertainty or model limitations.

The normalized forward sensitivity index of a variable effective reproduction number $\mathcal{R}_0$ depends differentially on a parameter ζ is defined as SI $(p) = \frac{\partial \mathcal{R}_0}{\partial \zeta} * \frac{\zeta}{\mathcal{R}_0}$.

The sensitivity indices in this study allowed us to justify the relative importance of various parameters in the HIV and depression incidence and prevalence. The most sensitive parameter has the magnitude of the sensitivity index greater than all other parameter's sensitivity indices. In this study, we computed the sensitivity index value in terms of $\mathcal{R}_0$.

Using parameter values in Table 3.2, we have computed the sensitivity indices given in Table 4.1 . Moreover, sensitivity analysis given in Table 4.1 explains that the human population recruitment rate Δ and HIV transmission rate β_H are highly affecting the HIV effective reproduction $\mathcal{R}_0$. The results shows that β is the most sensitive parameter but it is not varied in the simulations because the focus is on the depression treatment. Similarly, the parameters $\gamma_2, \gamma_4, \psi_1, \psi_2, \lambda_2 \text{and} \mu$ are not considered in the simulations because they have no association with depression treatment.

Sensitivity Analysis of Model Parameters

Parameter	Sensitivity Index
β	1.00000
θ_1	0.02720
θ_2	0.00000
γ_1	0.10680
γ_2	0.10680
γ_3	0.85963
γ_4	0.01373
α_1	-0.07718
α_2	-0.41545
λ_1	-0.01177
λ_2	-0.01648
ψ_1	0.00000
ψ_2	0.00000
ω_1	-0.00266
ω_2	-0.50366
δ	0.00000
μ	0.00000

Table 4.1: Sensitivity Analysis of Parameters.

4.2 Numerical Simulations

Parameter values used in the numerical simulations of model system (3.1) are shown in Table 4.1. HIV/depression co-infection study is still in its infancy and so a number of numeric values for the parameters shown in Table 4.1 are reasonable estimates. One of the parameter was estimated by modifying baseline values from published literature and it is denoted with an asterisk in Table 4.1. In this study, the population of Kenya is assumed to be 47,564,296 as per the 2019 Kenya Population and Housing Census Results (KNBS, 2019).

In this study, numerical simulations was performed using the R programming environment. The following data were input as initial conditions (Data was done per 100,000 persons in the population):

$$N_0 = [S, I, I_d, P, P_d, T, T_d, V, V_d] = [460, 14, 5, 9, 4, 8, 3, 7, 2]$$

The HIV/depression model (3.1) was examined numerically using the parameter values given in Table 4.1, to assess the impact of the model solutions and the possible effects of parameters change on the depression treatment in HIV care cascade dynamics throughout the community. From the parameters shown in Table 4.1, the basic reproduction number was, $R_0 = 1.8956$ which implies that the disease will persist in the population. The figures below shows the different outputs obtained from the simulation.

4.3 Impact of Depression Screening and Treatment on the Basic Reproduction Number

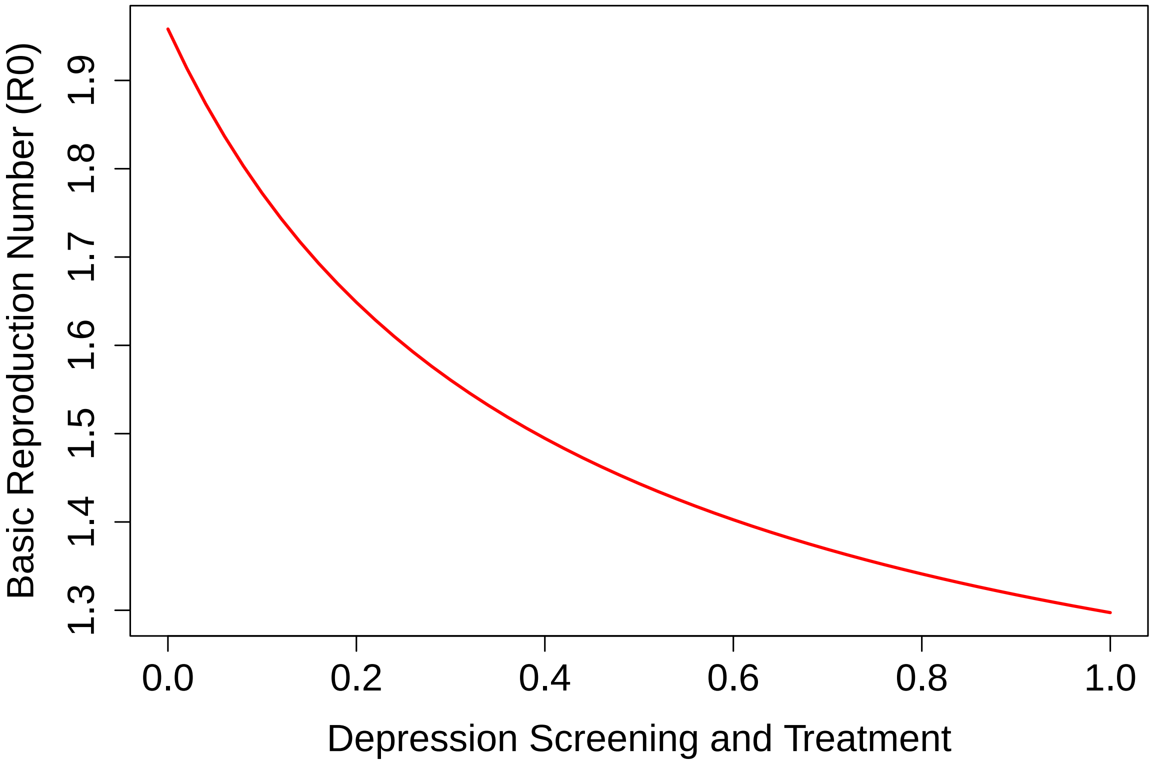

Figure 4.1: A graph showing the impact of depression screening and Treatment on the Basic Reproduction Number

Fig 4.1 presented above illustrates the influence of depression screening and treatment on the HIV/AIDS Basic Reproduction Number (R0). The data depicted in the graph shows that a relationship between depression treatment and a decrease in R0, indicating that addressing depression among people living with HIV/AIDS (PLWHA) can have a positive impact on reducing the transmission of the virus.

From the graph, it shows that initially, without depression screening and treatment, the R0 remains relatively high. This suggests that in the absence of addressing depression among PLWHA, the transmission dynamics of HIV/AIDS may persist or even worsen, potentially leading to a higher R0.

However, as the graph progresses, the R0 line associated with depression screening and treatment begins to show a decline or a slower increase compared to the R0 without treatment. This indicates that implementing depression treatment interventions can potentially contribute to a decrease in the rate of HIV transmission.

The observed reduction in R0 can be attributed to several factors associated with depression treatment. Firstly, depression treatment may enhance adherence to antiretroviral therapy (ART), leading to better viral suppression and reduced infectiousness. Improved medication adherence among PLWHA can contribute to a decrease in the number of secondary infections and subsequently lower the R0. Depression screening and treatment interventions often address risky behaviors, such as unsafe sexual practices or substance abuse, which can increase the risk of HIV transmission. By targeting these behaviors and providing appropriate support, depression screening and treatment can help mitigate the associated transmission risks, indirectly influencing the R0. Also, depression screening and treatment can have positive effects on mental health, social support, and overall well-being among PLWHA. By improving mental health outcomes, individuals may be better equipped to engage in self-care, adhere to HIV treatment regimens, and maintain healthier behaviors, all of which can contribute to a reduction in transmission risks and subsequently impact the R0.

4.3.1 Epidemic Curves for HIV/Depression Model

4.3.1.1 Impact of Depression Screening on HIV Prevalence

Individuals who are living with HIV may be at an increased risk for depression due to the stress and stigma associated with the disease. Depression, in turn, can affect the immune system and lead to decreased adherence to HIV medication, which could potentially increase the risk of transmitting HIV to others. If no one is screened for depression, the risk factors associated with depression, such as substance abuse, risky sexual behaviours and injection drug use, may go unaddressed, which could potentially increase the risk of HIV transmission. This results in higher rates of HIV transmission as shown by the blue line in Figure 4.2. If partial screening for depression is employed, where some individuals

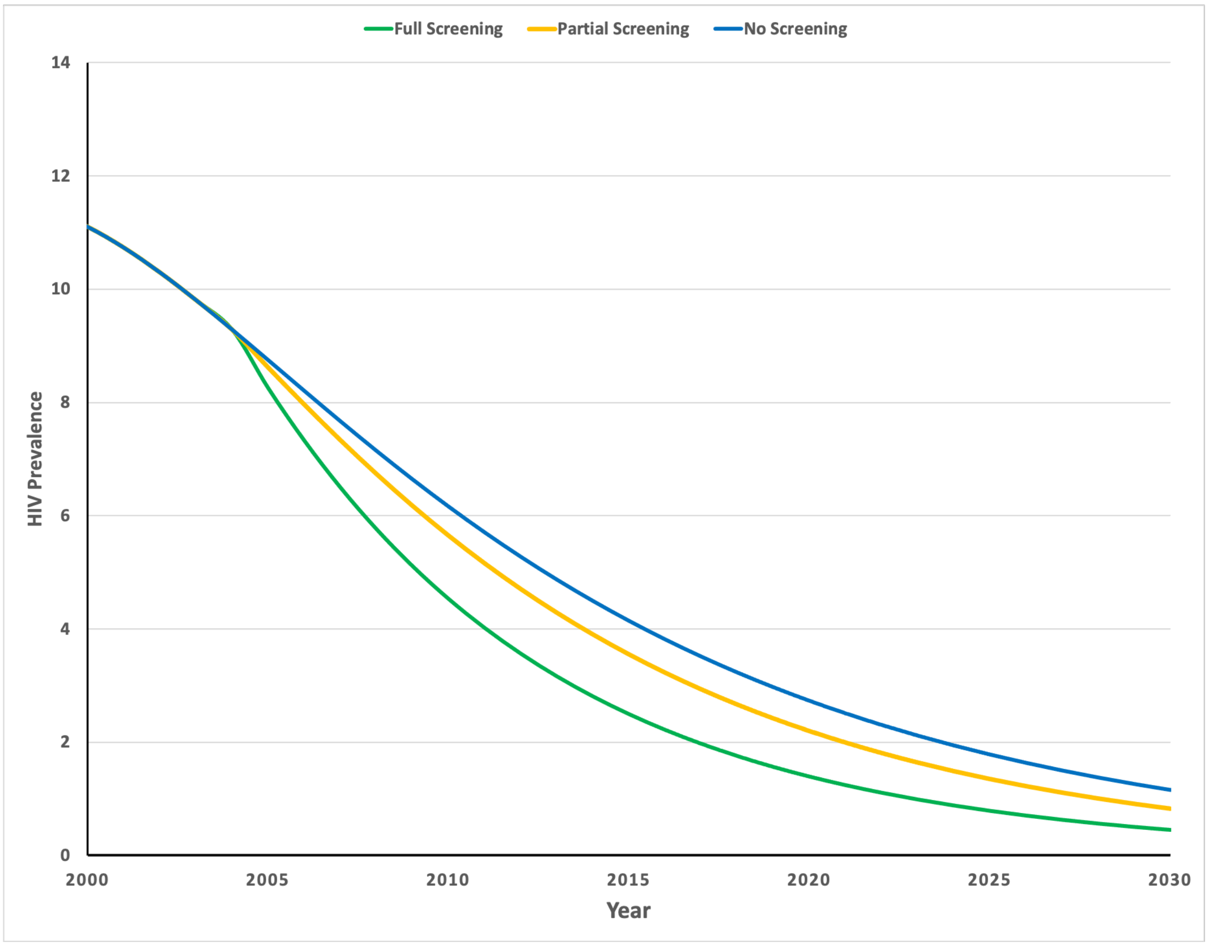

Figure 4.2: A graph showing the impact of depression screening on HIV prevalence

who need mental health care are identified and connected to appropriate services, this would have a some impact on HIV prevalence as shown by the red line in Figure 4.2. But, if full depression screening is considered for PLWHA this would lead to improved mental health outcomes and better engagement in HIV prevention and care behaviors, resulting in lower rates of HIV transmission hence lower HIV prevalence as shown by the green line in Figure 4.2.

4.3.1.2 Impact of Depression Treatment on ART Treatment

Depression is a common comorbidity among people living with HIV, and it can have negative effects on ART treatment outcomes. Depression can reduce adherence to ART medication, increase viral load, and decrease CD4 counts. Therefore, effective treatment of depression can be important in improving ART outcomes for people living with HIV. In the absence of depression treatment, the number of co-infections on ART exceeds the number of those with just HIV on ART this is due to the negative effects of untreated

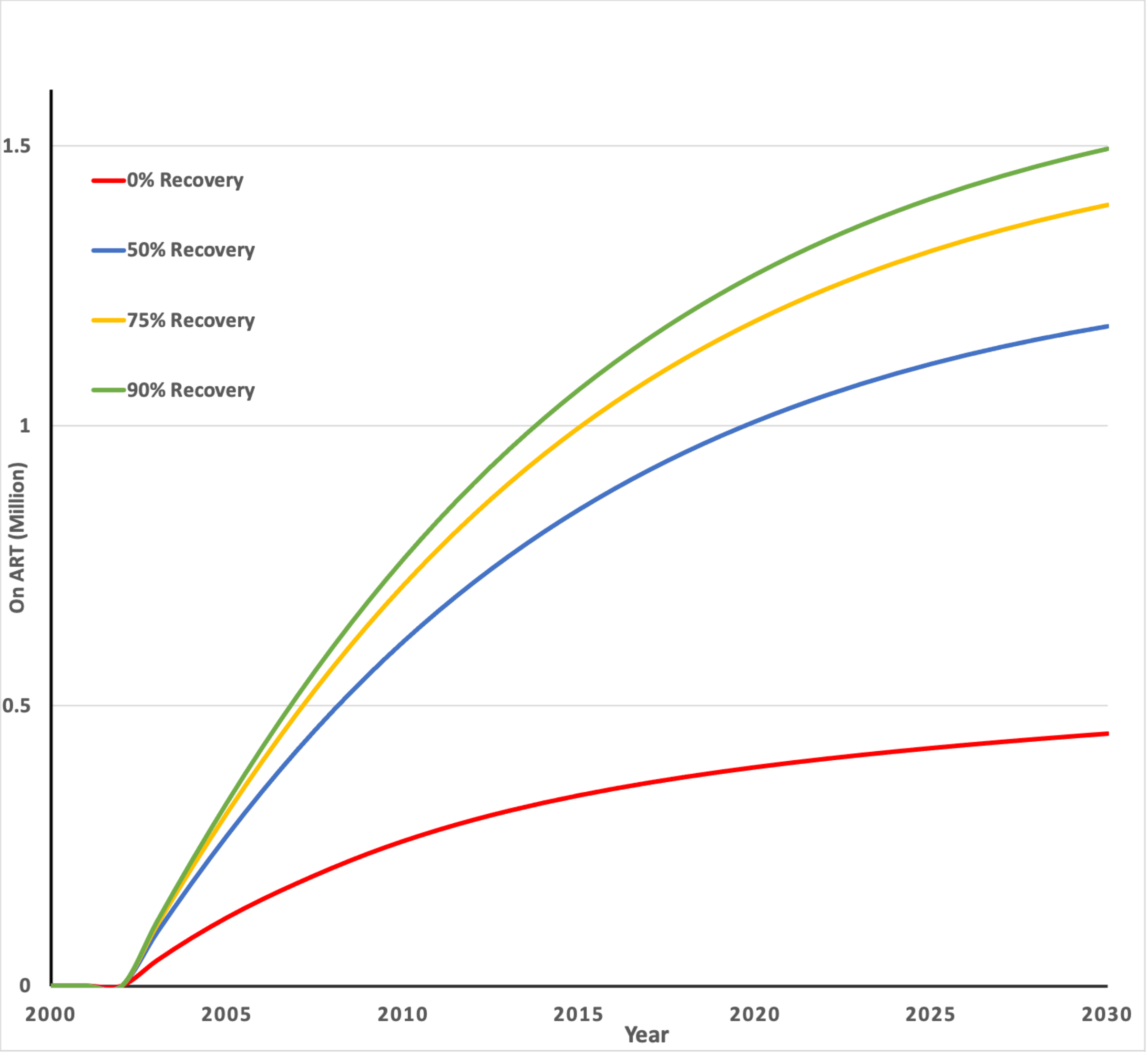

Figure 4.3: A graph showing the impact of depression treatment on ART treatment

depression. If 50% of the population recover from depression after being provided with depression treatment, there is a moderate impact on ART adherence as seen in fig:tdx. By identifying and treating depression in half of the population, the risk factors associated with depression and HIV transmission are reduced, and adherence to ART improves for those living with HIV/AIDS who receive treatment. However, non-adherence to ART may still remain high among those who are not screened or treated for depression. If 75% of the population recovers from depression after treatment, there is a more substantial and significant impact on HIV prevalence. By identifying and treating depression in a larger proportion of the population, the risk factors associated with depression and HIV transmission are further reduced, and adherence to ART improves for a larger number of individuals living with HIV/AIDS. similarly, 90% recovery from depression gave a slight increament in persons On ART. The highest increase among persons On ART is seen in the first 50% recovery from depression, as seen in Figure 4.3 above this is because the drug efficacy lies at 75%. The ability of the depression treatment to produce the desired effect is at 75%. Previous research have also shown that treatment for depression can make a significant difference in the physical and emotional well-being of individuals living with HIV (Whiteley et al., 2021; El-Halabi et al., 2022). Other studies indicate that depression is associated with viral suppression among PLWHA which has also been seen in this study as depression treatment increases people who are On ART. Depression treatment has also been linked to improved linkage to care, engagement to care and ART adherence. Turan et al. (2019) also showed that decreasing depression improves ART adherence among PLWHA and a lot of people will achieve viral load suppression.

4.3.1.3 Impact of Depression Treatment on Viral Load Suppression

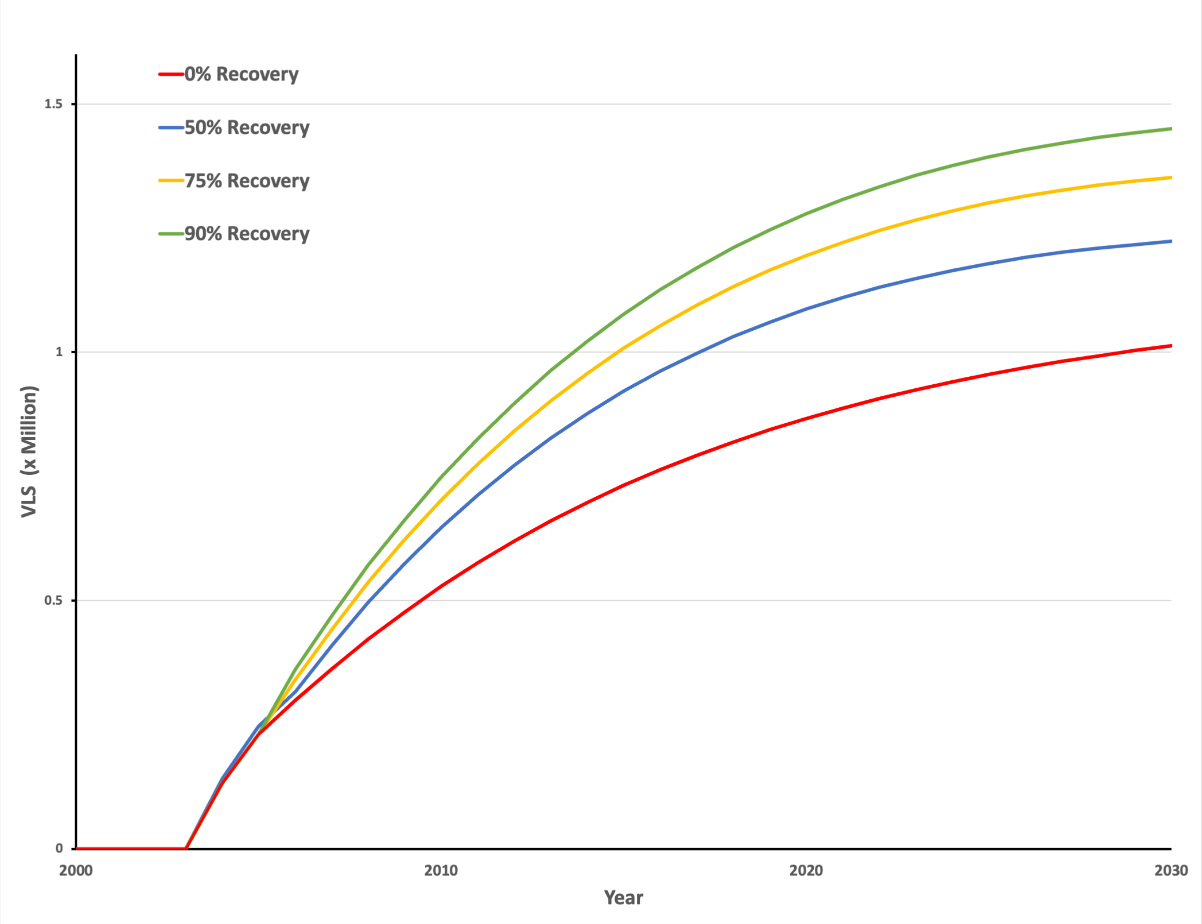

Figure 4.4: A graph showing the impact of depression treatment on Viral Load Suppression

Depression treatment have a positive impact on viral load suppression in individuals with HIV/AIDS by improving immune function and overall health outcomes. It is important for individuals with HIV/AIDS who also experience symptoms of depression to seek treatment from a healthcare provider. Figures 4.4 above shows the impact of depression treatment among then co-infections. With 0% treatment for depression, a lot of individuals who are virally suppressed are affected by depression but with just 50% depression treatment, the co-infections reduces the depressed individuals drastically, this is shown in Figure 4.4 . Similarly 75% depression treatment among the virally suppressed co-infections reduces depression among the virally suppressed and a lot of persons become virally suppressed not affected by depression. 90% depression treatment gave a slight decrease in viral load co-infections. Therefore, this research shows that depression medication among HIV-infected people enables them to attain viral load suppression, this is also seen in a study by Whiteley et al. (2021). Depression treatment increases ART adherence and hence a lot of people achieve viral suppression (Concepcion et al., 2022).

The results of this graph highlight the importance of addressing depression in individuals with HIV/AIDS. Effective treatment of depression can lead to improved immune function and overall health outcomes, which may contribute to better viral load suppression. Additionally, through depression treatment among PLWHA, Kenya will be able to attain the 95:95:95 UNAIDS target.

4.3.1.4 Impact of Depression Treatment on UNAIDS 95-95-95 Cascade

In the year 2023, the 95:95:95 target in Kenya is shown in Figure 4.3 above with the impact of depression treatment on the UNAIDS 2025 target. The percentage of people diagnosed is taken as a constant percentage of 79.5% from KENPHIA (2018) report. The results shows that If no depression treatment is given to PLWHA then of the 79.5% who are diagnosed, 42% are put on ART treatment without depression and 34.64% of the persons on ART without depression attains viral load suppression, this is seen in 4.5. Similarly, with 50% depression recovery rate, there is a rise in people on ART to 78.49% and the virally suppressed are 75.61%. Still in 2023, if depression treatment of 75% is given to persons On ART then 91.65% of people are on ART and 90.54% attain viral load suppression but with 90% depression then the country attains the 95% target. This shows that one of the barriers to attaining the 95:95:95 target in Kenya is depression. provision of depression treatment on every viral load suppression check-up can aid in attaining the

Figure 4.5: A graph showing the impact of depression treatment on UNAIDS 95-95-95 cascade

UNAIDS target. A study by Kulisewa et al. (2019) also agrees with this study results as it shows that greater integration of such mental health care task-shifting into HIV programs will be critical to realizing the 90–90–90 goals and ending the HIV epidemic. Similarly, Remien et al. (2019) showed that barriers to ending the HIV epidemic includes depression and treating depression can help curb this problem.

Chapter 5: Conclusion and Recommendations

In this research an HIV/Depression compartmental was developed. The aim of the model was to assess the impact of depression treatment on HIV care Cascade Dynamics in Kenya. The well-posedness of the model was tested by checking for the positivity and the boundedness of the solutions. The basic reproduction number, R_0 was derived using the Next generation matrix and it was evident that the model was locally stable and the simulated results (Global and local stability of Disease Free Equilibrium, DFE) from the model emphasized the importance of maintaining R_0 below one. That is, the Lyapunov function was used to check for global stability of DFE, the analysis showed that the DFE of the model is globally asymptotically stable whenever $\mathcal{R}_0$ is less than unity.

Depression treatment play a very crucial role in the system of equations as far as co-infection control is concerned. It is evident from the numerical results that depression treatment among HIV/Depression co-infections helps reduce depression among PLWHA. The results suggest that integrating depression treatment into HIV care cascade can lead to substantial improvements in patient outcomes, including reduced mortality and increased quality of life. This is because treatment for depression can make a significant difference in the physical and emotional well-being of individuals living with HIV. It improves linkage to care, retention in care and ART adherence among PLWHA.

The findings of this study also suggest that depression screening and treatment should be integrated into routine HIV care. Integration of depression screening and care into all HIV testing and treatment settings would not only strengthen HIV prevention and care outcomes, but it would additionally improve global access to mental healthcare. Depression screening and treatment programs helps to identify and treat depression among PLWHA, which may lead to improved HIV outcomes. The effectiveness of depression treatment on ART adherence and viral load suppression highlights the need for mental health services as a vital component of HIV care. The findings also suggest that depression treatment can reduce the risk of mortality among PLWHA. Addressing depression will reduce the barriers to accessing adequate and sustained healthcare, and are among the most significant barriers to achieving the 95–95–95 UNAIDS targets. In addition, the medical practitioners and the government should initiate depression treatment programs for HIV programs and management polices that will lead to having $R_0 < 1$.

5.1 Recommendations

Despite the significant challenges that depression presents to HIV prevention and treatment, there are many important and unmet opportunities to integrate depression treatment with HIV care. The Government of Kenya should, in particular, ensure that they expand HIV care, and the concomitant strengthening of their healthcare systems to offer substantial benefits to wider healthcare delivery. This will go along way in assisting the Government to meet the UNAIDS goals that, by 2025, 95% of all people living with HIV will know their HIV status, 95% of all people with diagnosed HIV infection will receive sustained antiretroviral therapy and 95% of all people receiving antiretroviral therapy will have viral suppression. The government should also be aggressive in educating the citizens on the need of taking and adhering to the ARTs. Further integration of depression screening and care into the HIV healthcare systems would not only strengthen HIV prevention and care outcomes, but it would additionally improve access to depression care. Seizing these opportunities will be crucial if we are to further 'bend the curve' of the HIV epidemic and eventually find an end to AIDS. On a very fundamental and basic level, there can be no health, without mental health.

5.2 Future Research

There are several areas for future research that can be explored based on the results of our study on the impact of depression screening and treatment on HIV care continuum. Some of these areas include:

1. Comparison of different treatment modalities: More research is needed to compare the effectiveness of different depression treatment modalities (e.g., psychotherapy, pharmacotherapy) on improving HIV outcomes.

2. Cultural factors: Cultural factors play an important role in depression screening and treatment, as well as HIV care. Future research could investigate how cultural factors influence the impact of depression treatment on HIV outcomes.

3. Cost-effectiveness: While depression treatment can improve HIV outcomes, it can also be costly. Future research could investigate the cost-effectiveness of depression screening and treatment in HIV care.

Overall, further research in these areas could help to refine understanding of the relationship between depression and HIV outcomes and inform the development of effective interventions to improve the mental and physical health of people living with HIV/AIDS.

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Appendix A: R Codes

```
###########Screening
#load libraries
library(deSolve)
library(ggplot2)
library(plotly)

# Model the dynamics of an SEIR with demography and
# disease induced mortality
seir <- function(time, state, parameters) {
  with(as.list(c(state, parameters)), {

    N <- S + P + Pd + I + Id + TT + Td + V + Vd

    betaH = beta*(gamma1*P + gamma2*Pd + gamma3*I +
                  gamma4*Id)

    dS  <- Lambda -  (mu +betaH) * S
    dP  <- betaH * S -  (mu +alpha1 + gamma ) * P
    dPd <- gamma * P -  (mu + alpha2 ) * Pd
    dI  <- alpha1 * P -  ( lambda1 + mu+delta) *I
    dId <- alpha2 * Pd -  (lambda2 + mu + delta) *Id
    dTT  <- lambda1 * I + omega1*Td -  (psi1 + mu+delta) *TT
    dTd <- lambda2 * Id -  (omega1 + psi2 +mu+delta) * Td
    dV  <- psi1 * TT + omega2 * Vd -  mu * V
    dVd <- psi2 * Td -  (omega2 + mu) * Vd
    return(list(c(dS, dP, dPd, dI,dId,dTT,dTd,dV,dVd)))
  })
}
```

```
#model parameters
## 0% Screening
parameters <- c(Lambda = 0.028,
                beta = 0.037,
                gamma1=1.5,
                gamma2=1.6
                gamma3=1.7,
                gamma4=1.8,
                theta1 = 0.4725,
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.0,
                lambda1 = 0.92,
                lambda2 = 0.138,
                psi1 = 0.906,
                psi2 = 0.52548,
                omega1 = 0.0, # star
                omega2 = 0.0, # star
                delta = 0.0157142857142857,
                mu = 0.007

)

# fit ode
results <- ode(y=c(S = 460, P = 14, Pd = 5, I = 9, Id = 4,
                   TT = 8 , Td = 3, V = 7, Vd = 2 ),
                   times=seq(0,70*1,by=1),
                   func=seir, parms=parameters)

df<-as.data.frame(results)
write_xlsx(df,"D:/Work/CHEMU/MASTERS_THESIS/Project/Thesis/
Graphs/Round_4/Infection7.xlsx")
```

```
#plot dynamics - Treatment
popscale <- 340000
plot_ly(df, x = ~df[1]+1980, y = ~df[,7]*popscale, name = 'T',
    type = 'scatter', line=list(width=3),mode = 'lines',
    color=I("blue")) %> add_trace(y = ~df[,8]*popscale, name = 'Td',
   mode = 'lines',
  line=list(width=3),color=I("red))␣\%>\%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)',range=c(1985,2030,by␣=␣10)),
␣␣yaxis␣=␣list(title␣=␣'Fraction␣of␣People',␣range␣=␣c(0,3.6*1.0E+6)))

library("writexl")
write_xlsx(df,"D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/Graphs/
Round 4/ART90.xlsx")

##␣Partial␣Screening␣Screening
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.50,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.40,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.40,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007
)
```

```
#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣func=seir,␣parms=parameters)

df<-as.data.frame(results)
write_xlsx(df,"D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/Graphs/
Round 4/Infection7.xlsx")


#plot␣dynamics␣-␣Treatment
popscale␣<-␣340000
plot_ly(df,␣x␣=␣df[,1]+1980,␣y␣=␣~df[,7]*popscale,␣name␣=␣'T',
type␣=␣'scatter',␣line=list(width=3),mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,8]*popscale,␣name␣=␣'Td',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)range=c(1985,2030,by␣=␣10)),
␣␣yaxis␣=␣list(title␣=␣'Fraction␣of␣People',␣range␣=␣c(0,3.6*1.0E+6)))

library("writexl")
write_xlsx(df,"D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/
Graphs/Round 4/ART90.xlsx")


##␣99.9999%␣Screening
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
```

```
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.99,
                lambda1 = 0.92,
                lambda2 = 0.138,
                psi1 = 0.906,
                psi2 = 0.52548,
                omega1 = 0.90, # star
                omega2 = 0.90, # star
                delta = 0.0157142857142857,
               mu = 0.007

)

# fit ode
results <- ode(y=c(S = 460, P = 14, Pd = 5, I = 9, Id = 4,
                  TT = 8 , Td = 3, V = 7, Vd = 2 ),
                  times=seq(0,70*1,by=1),
                  func=seir , parms=parameters)

df<-as.data.frame(results)
write_xlsx(df,"D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/Graphs/
Round 4/Infection7.xlsx")



#plot dynamics - Treatment
popscale <- 340000
plot_ly(df, x = ~df[,1]+1980, y = ~df[7]*popscale, name ='T',
type ='scatter',
line=list(width=3),mode = 'lines', color=I("blue"))
  add_trace(y = ~df[,8]*popscale, name = 'Td', mode = 'lines',
  line=list(width3),color=I("red"))
  layout(xaxis = list(title = 'Time (Years)',range=c(1985,2030,by = 10)),
  yaxis = list(title = 'Fraction of People', range = c(0,3.6*1.0E+6)))
```

```
library(”writexl”)
write_xlsx(df,”D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/Graphs/
Round 4/ART90.xlsx”)

#plot␣dynamics␣–␣VLS

plot_ly(df,␣x␣=␣~df[,1]+1980,␣y␣=␣~df[,9]*popscale,␣name␣=␣’V’,
type␣=␣’scatter’,
line=list(width=3),mode␣=␣’lines’,␣color=I(”blue”))␣%>%
␣␣add_trace(y␣=␣~df[,10]*popscale␣name␣=␣’Vd’,␣mode␣=␣’lines’,
␣␣line=list(width=3),
␣␣color=I(”red”))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣’Time␣(Years)’,range=c(1985,2030,by␣=␣10)),
␣␣yaxis␣=␣list(title␣=␣’Fraction␣of␣People’,␣range␣=␣c(0,1.2*1.0E+6)))

write_xlsx(df,”D:/Work/CHEMU/MASTERS THESIS/Project/Thesis/Graphs/
Round 4/VLS90.xlsx”)



#load␣libraries
library(deSolve)
library(ggplot2)
library(plotly)

#Model␣the␣dynamics␣of␣an␣SEIR␣with␣demography␣and
#disease␣induced␣mortality
seir␣<-␣function(time,␣state,␣parameters)␣{
␣␣with(as.list(c(state,␣parameters)),␣{

␣␣␣␣N␣<-␣S␣+␣P␣+␣Pd␣+␣I␣+␣Id␣+␣TT␣+␣Td␣+␣V␣+␣Vd

␣␣␣␣betaH␣=␣beta*(gamma1*P␣+␣gamma2*Pd␣+␣gamma3*I␣+␣gamma4*Id)/N
```

```
␣␣␣␣dS␣␣<-␣Lambda␣-␣␣(mu␣+betaH)␣*␣S
␣␣␣␣dP␣␣<-␣betaH␣*␣S␣-␣␣(mu␣+alpha1␣+␣gamma␣)␣*␣P
␣␣␣␣dPd␣<-␣gamma␣*␣P␣-␣␣(mu␣+␣alpha2␣)␣*␣Pd
␣␣␣␣dI␣␣<-␣alpha1␣*␣P␣-␣␣(␣lambda1␣+␣mu+delta)␣*I
␣␣␣␣dId␣<-␣alpha2␣*␣Pd␣-␣␣(lambda2␣+␣mu␣+␣delta)␣*Id
␣␣␣␣dTT␣␣<-␣lambda1␣*␣I␣+␣omega1*Td␣-␣␣(psi1␣+␣mu+delta)␣*TT
␣␣␣␣dTd␣<-␣lambda2␣*␣Id␣-␣␣(omega1␣+␣psi2␣+mu+delta)␣*␣Td
␣␣␣␣dV␣␣<-␣psi1␣*␣TT␣+␣omega2␣*␣Vd␣-␣␣mu␣*␣V
␣␣␣␣dVd␣<-␣psi2␣*␣Td␣-␣␣(omega2␣+␣mu)␣*␣Vd
␣␣␣␣return(list(c(dS,␣dP,␣dPd,␣dI,dId,dTT,dTd,dV,dVd)))
␣␣})
}

#model␣parameters
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007


)
```

```
#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣func=seir,␣parms=parameters)

df<-as.data.frame(results)

#plot␣dynamics␣-␣All
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,2],␣name␣=␣'S',
type␣=␣'scatter',
line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,3],␣name␣=␣'P'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),␣color=I("orange"))␣%>%
␣␣add_trace(y␣=␣~df[,4],␣name␣=␣'Pd',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣add_trace(y␣=␣~df[,5],␣name␣=␣'I'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("green"))␣%>%
␣␣add_trace(y␣=␣~df[,6],␣name␣=␣'Id',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("purple"))␣%>%
␣␣add_trace(y␣=␣~df[,7],␣name␣=␣'T'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("pink"))␣%>%
␣␣add_trace(y␣=␣~df[8],␣name␣=␣'Td',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("cyan"))␣%>%
␣␣add_trace(y␣=␣~df[,9],␣name␣=␣'V'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("violet"))␣%>%
␣␣add_trace(y␣=␣~df[,10],name␣=␣'Vd',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("brown"))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)'),yaxis␣=
␣␣list(title␣=␣'Fraction␣of␣People'))
```

```
#plot␣dynamics␣—␣P,I,T
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,4],␣name␣=␣'Pd',
type␣=␣'scatter',␣line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,3],␣name␣=␣'P'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),␣color=I("orange"))␣%>%
␣␣add_trace(y␣=␣~df[,5],␣name␣=␣'I'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("green"))␣%>%
␣␣add_trace(y␣=␣~df[,6],␣name␣=␣'Id',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("purple"))␣%>%
␣␣add_trace(y␣=␣~df[,7],␣name␣=␣'T'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣add_trace(y␣=␣~df[,8],␣name␣=␣'Td',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("cyan"))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)'),
␣␣yaxis␣=␣list(title␣=␣'Fraction␣of␣People'))


#plot␣dynamics␣—␣All
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,3],␣name␣=␣'P',
type␣=␣'scatter',
line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,4],␣name␣=␣'Pd',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))

#plot␣dynamics␣—␣Infections
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,6],␣name␣=␣'Id',
type␣=␣'scatter',
line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,5],␣name␣=␣'I'␣,␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
```

```
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)'),yaxis␣=
␣␣list(title␣=␣'Fraction␣of␣People'))


#plot␣dynamics␣-␣Treatment
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,7],␣name␣=␣'T',
type␣=␣'scatter',
line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,8],␣name␣=␣'Td',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)'),
␣␣yaxis␣=␣list(title␣=␣'Fraction␣of␣People'))


#plot␣dynamics␣-␣VLS
plot_ly(df,␣x␣=␣~df[,1],␣y␣=␣~df[,9],␣name␣=␣'V',
type␣=␣'scatter',
line=list(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,10],␣name␣=␣'Vd',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣layout(xaxis␣=␣list(title␣=␣'Time␣(Years)'),
␣␣yaxis␣=␣list(title␣=␣'Fraction␣of␣People'))


###␣Checking␣Depression␣treatment␣on␣ART
#load␣libraries
library(deSolve)
library(ggplot2)
library(plotly)

#Model␣the␣dynamics␣of␣an␣SEIR␣with␣demography␣and␣disease
#induced␣mortality
```

```
seir␣<-␣function(time,␣state,␣parameters)␣{
␣␣with(as.list(c(state,␣parameters)),␣{

␣␣␣␣N␣<-␣S␣+␣P␣+␣Pd␣+␣I␣+␣Id␣+␣TT␣+␣Td␣+␣V␣+␣Vd

␣␣␣␣betaH␣=␣beta*(gamma1*P␣+␣gamma2*Pd␣+␣gamma3*I␣+␣gamma4*Id)/N

␣␣␣␣dS␣␣<-␣Lambda␣-␣␣(mu␣+betaH)␣*␣S
␣␣␣␣dP␣␣<-␣betaH␣*␣S␣-␣␣(mu␣+alpha1␣+␣gamma␣)␣*␣P
␣␣␣␣dPd␣<-␣gamma␣*␣P␣-␣␣(mu␣+␣alpha2␣)␣*␣Pd
␣␣␣␣dI␣␣<-␣alpha1␣*␣P␣-␣␣(␣lambda1␣+␣mu+delta)␣*I
␣␣␣␣dId␣<-␣alpha2␣*␣Pd␣-␣␣(lambda2␣+␣mu␣+␣delta)␣*Id
␣␣␣␣dTT␣␣<-␣lambda1␣*␣I␣+␣omega1*Td␣-␣␣(psi1␣+␣mu+delta)␣*TT
␣␣␣␣dTd␣<-␣lambda2␣*␣Id␣-␣␣(omega1␣+␣psi2␣+mu+delta)␣*␣Td
␣␣␣␣dV␣␣<-␣psi1␣*␣TT␣+␣omega2␣*␣Vd␣-␣␣mu␣*␣V
␣␣␣␣dVd␣<-␣psi2␣*␣Td␣-␣␣(omega2␣+␣mu)␣*␣Vd
␣␣␣␣return(list(c(dS,␣dP,␣dPd,␣dI,dId,dTT,dTd,dV,dVd)))
␣␣})
}

####################################################
#␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣0%␣DTX␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣#
####################################################

#model␣parameters
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
```

```
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007

)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣func=seir,␣parms=parameters)

df<-as.data.frame(results)



#plot␣dynamics␣-␣VLS
popscale␣<-␣340000
q␣<-␣plot_ly(df,␣x␣=␣~df[,1]+1980,␣y␣=␣~df[7]*popscale,
name␣=␣'ART',␣type␣=␣'scatter',␣line=list(width=3),mode␣=␣'lines',
color=I("blue")␣%>%
␣␣add_trace(y␣=␣~df[,8]popscale,␣name␣=␣'ART␣D␣+␣DTX␣(0%)',
␣␣mode␣=␣'lines',␣line=list(width=3),color=I("red"))␣%>%
c#␣Step␣3:␣Run␣dev.off()␣to␣create␣the␣file!
q
#export(q,␣file␣=␣"ART00.png")
```

```
##########################################################
#                     50% DTX                        #
##########################################################

#model parameters
parameters <- c(Lambda = 0.028,
                beta = 0.037,
                gamma1=1.5,
                gamma2=1.6,
                gamma3=1.7,
                gamma4=1.8,
                theta1 = 0.4725,
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.028,
                lambda1 = 0.92,
                lambda2 = 0.138,
                psi1 = 0.906,
                psi2 = 0.52548,
                omega1 = 0.50, # star
                omega2 = 0.0, # star
                delta = 0.0157142857142857,
                mu = 0.007



# fit ode
results <- ode(y=c(S = 460, P = 14, Pd = 5, I = 9, Id = 4,
                   TT = 8 , Td = 3, V = 7, Vd = 2 ),
                   times=seq(0,70*1,by=1),
                   func=seir, parms=parameters)

df<-as.data.frame(results)
```

```
#plot dynamics - ART
popscale <- 340000
q <- plot_ly(df, x = ~df[,1]+1980, y = ~df[,7]*popscale,
name = 'ART', type = 'scatter', line=list(width=3),mode = 'lines',
color=I("blue")) %>%
  add_trace(y = ~df[,8]*popscale, name = 'ART D + DTX (20%)',
  mode = 'lines',
  line=list(width=3),color=("red")) %>%
  layout(title='ART Uptake', xaxis = list(title = 'Time (Years)',
  range=c(1985,2030)),
  yaxis = list(title = 'People', range = c(0,3.6*1.0E+6)))
# Step 3: Run dev.off() to create the file!
export(q, file = "ART20.png")




###########################################################
#                   90% DTX                      #
###########################################################

#model parameters
parameters <- c(Lambda = 0.028,
                beta = 0.037,
                gamma1=1.5,
                gamma2=1.6,
                gamma3=1.7,
                gamma4=1.8,
                theta1 = 0.4725,
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.028,
```

```
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.90,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007

)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),␣func=seir␣,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣parms=parameters)

df<-as.data.frame(results)



#plot␣dynamics␣-␣ART
popscale␣<-␣340000
q␣<-␣plot_ly(df,␣x␣=␣~df[,1]+1980,␣y␣=␣~df[,7]*popscale,
name␣=␣'ART',␣type␣=␣'scatter',␣=list(width=3),mode␣'lines',
color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,8]*popscale,
␣␣name␣=␣'ART␣D␣+␣DTX␣(40%)',␣mode␣=␣'lines',
␣␣line=list(width=3),color=I("red"))␣%>%
␣␣layout(title='ART␣Uptake',xaxis␣=␣list(title␣=␣'Time␣(Years)',
␣␣range=c(1985,2030)),yaxis␣=␣list(title␣=␣'People',
␣␣range␣=␣c(0,3.6*1.0E+6)))
#␣Step␣3:␣Run␣dev.off␣to␣create␣the␣file!
export(q,␣file␣=␣"ART90.png")
```

```
############################################################
#␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣90%␣DTX␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣#
############################################################

#model␣parameters
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.90,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007
)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),␣func=seir ,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣parms=parameters)

df<-as.data.frame(results)
```

```
#plot dynamics - ART
popscale <- 340000
q <- plot_ly(df, x = ~df[,1]+1980, y = ~df[,7]*popscale,
name ART', type = 'scatter', line=list(width=3),mode = 'lines',
color=I("blue")) %>%
  add_trace(y = ~df[,8] popscale, name = 'ART D + DTX (60%)',
  mode = 'lines', line=list(width=3),color=I("red")) %>%
  layout(title='ART Uptake', xaxis = list(title = 'Time (Years)',
  range=c(1985,2030)), yaxis = list(title = 'People',
  range = c(0,3.6*1.0E+6)))
# Step 3: Run dev.off to create the file!
export(q, file = "ART90.png")


### Checking Depression treatment on VLS
#load libraries
library(deSolve)
library(ggplot2)
library(plotly)

#Model the dynamics of an SEIR with demography
#and disease induced mortality

seir <- function(time, state, parameters) {
  with(as.list(c(state, parameters)), {

    N <- S + P + Pd + I + Id + TT + Td + V + Vd

    betaH = beta*(gamma1*P + gamma2*Pd + gamma3*I +
    gamma4*Id)/N

    dS  <- Lambda -  (mu +betaH) * S
    dP  <- betaH * S -  (mu +alpha1 + gamma ) * P
    dPd <- gamma * P -  (mu + alpha2 ) * Pd
```

```
␣␣␣␣dI␣␣<-␣alpha1␣*␣P␣-␣␣(␣lambda1␣+␣mu+delta)␣*I
␣␣␣␣dId␣<-␣alpha2␣*␣Pd␣-␣␣(lambda2␣+␣mu␣+␣delta)␣*Id
␣␣␣␣dTT␣␣<-␣lambda1␣*␣I␣+␣omega1*Td␣-␣␣(psi1␣+␣mu+delta)␣*TT
␣␣␣␣dTd␣<-␣lambda2␣*␣Id␣-␣␣(omega1␣+␣psi2␣+mu+delta)␣*␣Td
␣␣␣␣dV␣␣<-␣psi1␣*␣TT␣+␣omega2␣*␣Vd␣-␣␣mu␣*␣V
␣␣␣␣dVd␣<-␣psi2␣*␣Td␣-␣␣(omega2␣+␣mu)␣*␣Vd
␣␣␣␣return(list(c(dS,␣dP,␣dPd,␣dI,dId,dTT,dTd,dV,dVd)))
␣␣})
}

##################################################
#␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣0%␣DTX␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣#
##################################################

#model␣parameters
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007
```

```
)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),␣func=seir,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣parms=parameters)

df<-as.data.frame(results)



#plot␣dynamics␣-␣VLS
popscale␣<-␣4250
p<-␣plot_ly(df,␣x␣=␣~df[,1]+1985,␣y␣=␣~df[9]*popscale,␣name␣=␣'VLS',
type␣=␣'scatter',␣line=lst(width=3),mode␣=␣'lines',
color=I("blue"))␣%>%
add_trace(y␣=␣~df[,10]*popscale,␣name␣=␣'VLS␣D␣+␣DTX␣(0%)',
mode␣=␣'lines',␣line=list(width=3),colr=I("red"))␣%>%
layout(title='Viral␣Load␣Supression'␣=␣list(title␣=␣'Time␣(Years)',
range=c(1985,2030)),yaxis␣=␣list(title␣=␣'People',
range␣=␣c(0,1.2*1.0E+6)))

#␣Step␣3:␣Run␣dev.off()␣to␣create␣the␣file!
export(p,␣file␣=␣"VLS00.png")

#Model␣the␣dynamics␣of␣an␣SEIR␣with␣demography␣and
#disease␣induced␣mortality
```

```
#######################################################
#␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣50%␣DTX␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣#
#######################################################

#model␣parameters
parameters␣<-␣c(Lambda␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣beta␣=␣0.037,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma1=1.5,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma2=1.6,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma3=1.7,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣gamma4=1.8,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.4725,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣theta1␣=␣0.39,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha1␣=␣0.056,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣alpha2␣=␣0.028,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda1␣=␣0.92,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣lambda2␣=␣0.138,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi1␣=␣0.906,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣psi2␣=␣0.52548,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega1␣=␣0.0,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.50,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007

)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),␣func=seir ,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣parms=parameters)

df<-as.data.frame(results)
```

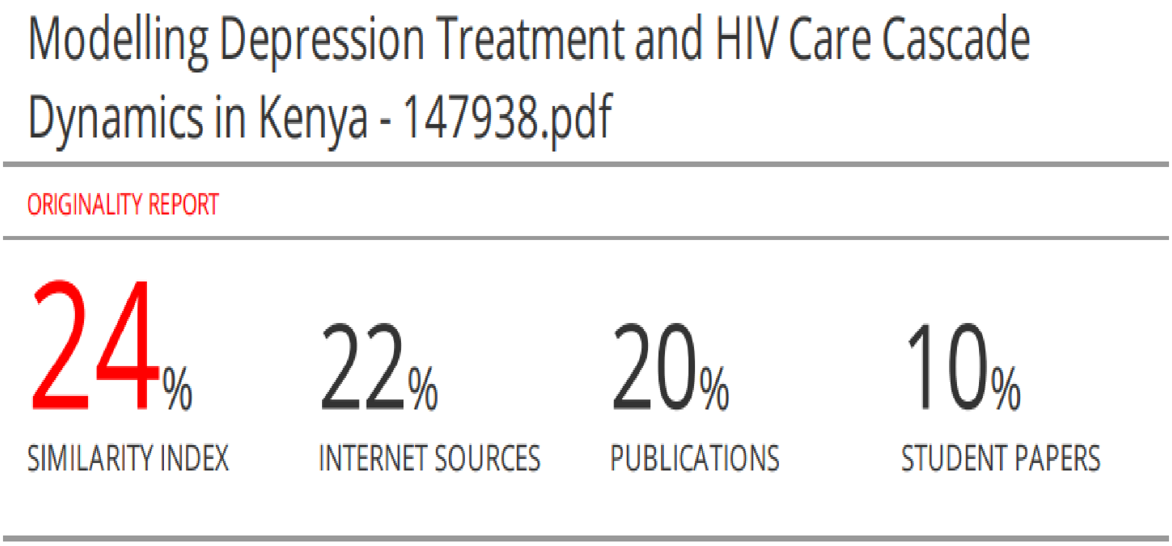
```
#plot dynamics - VLS
popscale <- 4250
p<- plot_ly(df, x = ~df[,1]+1985, y = ~df[,9]*popscale,
name = 'VLS', type = 'scatter', line=list(width=3),
mode = 'lines', color=I("blue")) %>%
  add_trace(y = ~df[,10]*popscale, name = 'VLS D + DTX (20%)',
  mode = 'lines', line=list(width=3),color=I("red")) %>%
  layout(title='Viral Load Supression', xaxis = list(title =
  'Time (Years)',range=c(1985,2030)),yaxis = list(title = 'People',
  range = c(0,1.2*1.0E+6)))

export(p, file = "VLS50.png")


###################################################
#                   75% DTX                       #
###################################################

#model parameters
parameters <- c(Lambda = 0.028,
                beta = 0.037,
                gamma1=1.5,
                gamma2=1.6,
                gamma3=1.7,
                gamma4=1.8,
                theta1 = 0.4725,
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.028,
                lambda1 = 0.92,
                lambda2 = 0.138,
                psi1 = 0.906,
                psi2 = 0.52548,
                omega1 = 0.0, # star
```

```
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣omega2␣=␣0.75,␣#␣star
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣delta␣=␣0.0157142857142857,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣mu␣=␣0.007

)

#␣fit␣ode
results␣<-␣ode(y=c(S␣=␣460,␣P␣=␣14,␣Pd␣=␣5,␣I␣=␣9,␣Id␣=␣4,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣TT␣=␣8␣,␣Td␣=␣3,␣V␣=␣7,␣Vd␣=␣2␣),
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣times=seq(0,70*1,by=1),␣func=seir ,
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣parms=parameters)

df<-as.data.frame(results)

#plot␣dynamics␣-␣VLS
popscale␣<-␣4250
p<-␣plot_ly(df,␣x␣=␣~df[,1]+1985,␣y␣//=␣~df[,9]*popscale,
name␣=␣'VLS',␣type␣=␣'scatter',␣line=list(width3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,10]*popscale,␣name␣=
␣␣'VLS␣D␣+␣DTX␣(4%)',␣mode␣=␣'lines',␣line=list(width=3),
␣␣color=I("red"))␣%>%
␣␣layout(title='Viral␣Load␣Supression',xaxis␣=
␣␣list(title␣=␣'Time␣(Years)',range=c(1985,2030)),
␣␣yaxis␣=␣list(title␣=␣'People',␣range␣=␣c(0,1.2*1.0E+6)))

export(p,␣file␣=␣"VLS75.png")
```

```
########################################################
#                   90% DTX                        #
########################################################

#model parameters
parameters <- c(Lambda = 0.028,
                beta = 0.037,
                gamma1=1.5,
                gamma2=1.6,
                gamma3=1.7,
                gamma4=1.8,
                theta1 = 0.4725,
                theta1 = 0.39,
                alpha1 = 0.056,
                alpha2 = 0.028,
                lambda1 = 0.92,
                lambda2 = 0.138,
                psi1 = 0.906,
                psi2 = 0.52548,
                omega1 = 0.0, # star
                omega2 = 0.90, # star
                delta = 0.0157142857142857,
                mu = 0.007

)

# fit ode
results <- ode(y=c(S = 460, P = 14, Pd = 5, I = 9, Id = 4,
                  TT = 8 , Td = 3, V = 7, Vd = 2 ),
                  times=seq(0,70*1,by=1),
                  func=seir , parms=parameters)
```

```
df<-as.data.frame(results)

#plot␣dynamics␣-.-␣VLS
popscale␣<-␣4250
p<-␣plot_ly(df,␣x␣=␣~df[,1]+1985,␣y␣=␣~df[,9]*popscale,
name␣=␣'VLS',␣type␣=␣'scatter',␣line=l..ist(width=3),
mode␣=␣'lines',␣color=I("blue"))␣%>%
␣␣add_trace(y␣=␣~df[,10]*popscale,␣name␣=␣'VLS␣D+␣DTX␣(60%)',
␣␣mode␣=␣'lines',␣line=list(width=3),color=I("red"))␣%>%
␣␣layout(title='Viral␣Load␣Supression',xaxis␣=
␣␣list(title␣=␣'Time␣(Years)',range=c(1985,2030)),
␣␣yaxis␣=␣list(title␣=␣'People',␣range␣=␣c(0,1.2*1.0E+6)

export(p,␣file␣=␣"VLS90.png")
```

Appendix B: Similarity Report

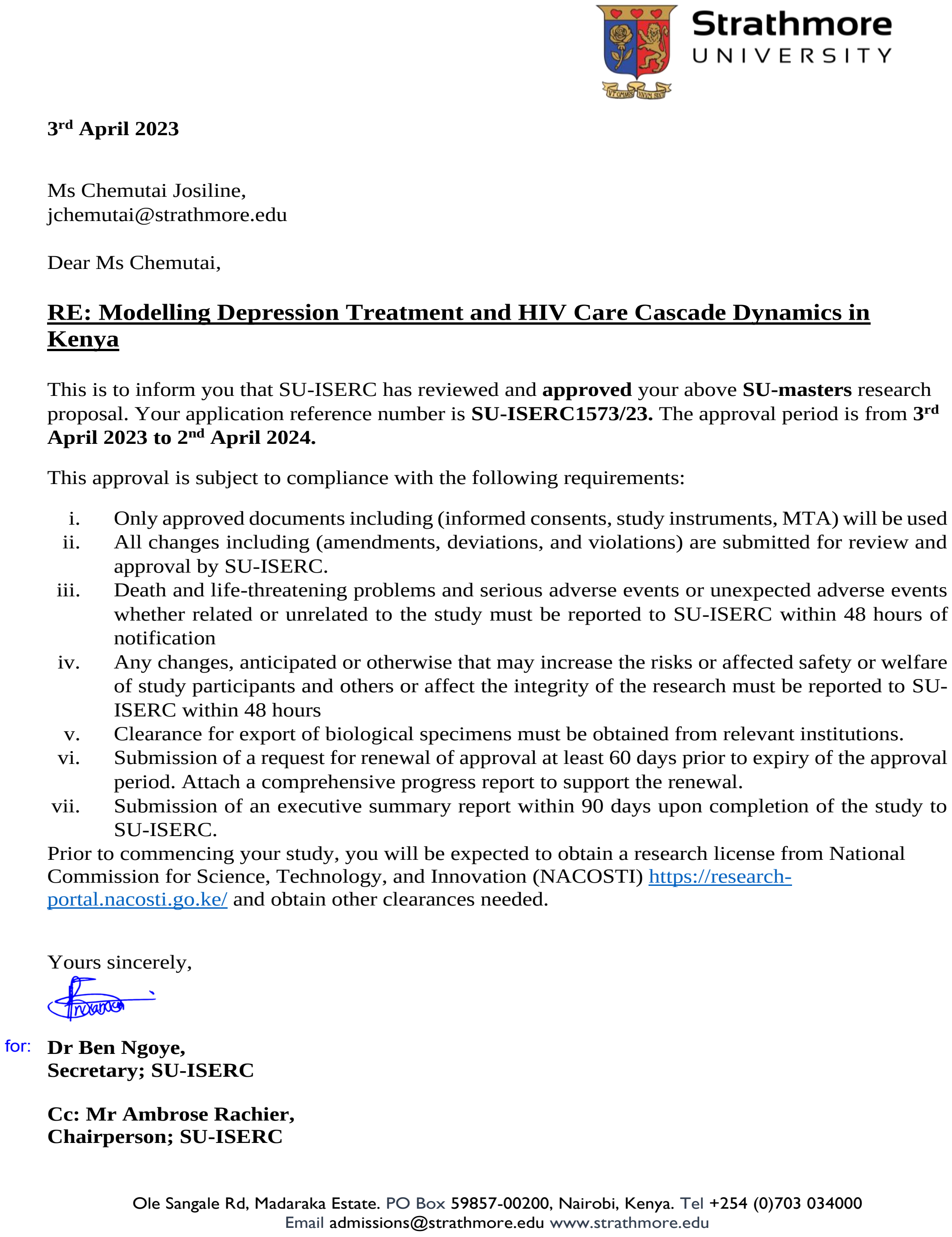


Modelling Depression Treatment and HIV Care Cascade Dynamics in Kenya - 147938.pdf

ORIGINALITY REPORT

24%	22%	20%	10%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

Appendix C: Ethical Clearance Confirmation



Strathmore
UNIVERSITY

3rd April 2023

Ms Chemutai Josiline,
jchemutai@strathmore.edu

Dear Ms Chemutai,

RE: Modelling Depression Treatment and HIV Care Cascade Dynamics in Kenya

This is to inform you that SU-ISERC has reviewed and **approved** your above **SU-masters** research proposal. Your application reference number is **SU-ISERC1573/23.** The approval period is from **3rd April 2023 to 2nd April 2024.**

This approval is subject to compliance with the following requirements:

i. Only approved documents including (informed consents, study instruments, MTA) will be used
ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-ISERC.
iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-ISERC within 48 hours of notification
iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-ISERC within 48 hours
v. Clearance for export of biological specimens must be obtained from relevant institutions.
vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.
vii. Submission of an executive summary report within 90 days upon completion of the study to SU-ISERC.

Prior to commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology, and Innovation (NACOSTI) https://research-portal.nacosti.go.ke/ and obtain other clearances needed.

Yours sincerely,

for: **Dr Ben Ngoye,**
Secretary; SU-ISERC

Cc: Mr Ambrose Rachier,
Chairperson; SU-ISERC

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