SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION
MAT 1102: APPLIED MATHEMATICS I

Date: 30th October, 2024 TIME: 2 Hours

INSTRUCTIONS

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) A packet contains 100 washers, 24 of which are brass, 36 copper and the remainder steel. One washer is taken at random, retained, and a second washer similarly drawn. Determine the probability that the first is brass and the second copper. [2 Marks]

(b) The lengths of 50 copper plugs gave the following frequency distribution:

Length, x (mm)	14.0 - 14.2	14.3-14.5	14.6-14.8	14.9-15.1
Frequency, f	3	4	8	15

Length, x (mm)	15.2 - 15.4	15.5-15.7	15.8-16.0
Frequency, f	10	7	3

(i) Calculate the mean and the standard deviation. [3 Marks]

(ii) For a full batch of 2400 plugs, calculate the number of plugs with lengths greater than 15.09 mm. [3 Marks]

(c) Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be two vectors in $\mathbb{R}^3$, and let θ be the angle the two vectors form when their feet are place together. Show that the cross product of $\mathbf{u}$ and $\mathbf{v}$ is given as follows: [3 Marks]

$$\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle.$$

(d) A family that owns two automobiles is selected at random.
Let A_1={the older car is American} and A_2={the newer car is American}.
If $P(A_1) = 0.7$, $P(A_2) = 0.5$ and $P(A_1 \cap A_2) = 0.4$, compute:

(i) The probability that at least one car is American. [2 Marks]

(ii) The probability that neither car is American. [2 Marks]

(e) The following information about 5 observations of x is shown below.

$$\sum_{i=1}^{5}\left(\frac{x_i-255}{2}\right)=50 \quad \text{and} \quad \sum_{i=1}^{5}\left(\frac{x_i-255}{2}\right)^2=1650.$$

Calculate the mean and standard deviation of x. [4 Marks]

(f) Relative to a fixed origin O, the point A has coordinates $(6,-4,1)$. The point B is such that $\overrightarrow{BA}=\mathbf{i}-\mathbf{j}+3\mathbf{k}$. If the point M is the midpoint of OB, show that $|\overrightarrow{AM}|=k\sqrt{10}$, where k is a rational constant to be found. [5 Marks]

(g) Let $P(A)=0.4$ and $P(A \text{ or } B)=0.7$. For what value of $P(B)$ are events A and B mutually exclusive? [2 Marks]

(h) The points A and B have position vectors $\begin{bmatrix}-5\\-2\\8\end{bmatrix}$ and $\begin{bmatrix}11\\6\\20\end{bmatrix}$, respectively. The point M lies on AB so that $|AM|:|MB|=3:1$.

The point P has position vector $\begin{bmatrix}10\\8\\19\end{bmatrix}$. Determine the position vector of the point Q, if M is the midpoint of PQ. [4 Marks]

QUESTION TWO (15 MARKS)

(a) The points $A(-1,4)$, $B(2,3)$ and $C(8,1)$ lie on the x-y plane, where O is the origin.

(i) Show that A, B and C are collinear. [2 Marks]

The point D lies on BC so that $\overrightarrow{BD}:\overrightarrow{BC}=2:3$.

(ii) Find the coordinates of D. [2 Marks]

The straight line OB is extended to the point P, so that $\overrightarrow{AP}$ is parallel to $\overrightarrow{OC}$.

(iii) Determine the coordinates of P. [3 Marks]

(b) The points $A(2,-1,5)$, $B(5,2,10)$ and $D(-1,1,4)$, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB}=\overrightarrow{DC}$. Find the position vector of C. [4 Marks]

(c) If A is the point $(1,-1,2)$, B is the point $(-1,2,2)$ and C is the point $(4,3,0)$, find the direction cosines of $\overrightarrow{BA}$ and $\overrightarrow{BC}$, and show that the angle $ABC=69^0 14'$. [4 Marks]

QUESTION THREE (15 MARKS)

(a) Describe the empirical rule of a normally distributed data. [3 Marks]

(b) A sample of size 40 yields the following sorted data. Note that I have x-ed out $x_{(39)}$ (the second largest number). This fact will NOT prevent you from answering the questions below.

14.1	46.0	49.3	53.0	54.2	54.7	54.7
54.7	54.8	55.4	57.6	58.2	58.3	58.7
58.9	60.8	60.9	61.0	61.1	63.0	64.3
65.6	66.3	66.6	67.0	67.9	70.1	70.3
72.1	72.4	72.9	73.5	74.2	75.3	75.4
75.9	76.5	77.0	x	88.9		

(i) Calculate range and median of these data. [2 Marks]

(ii) Given that the mean of these data is 63.50 (exactly) and the standard deviation is 12.33, what proportion of the data lie within one standard deviation of the mean? [3 Marks]

(iii) How does your answer to (ii) compare to the empirical rule approximation? [2 Marks]

(c) It is given that $\overrightarrow{AP} + 4\overrightarrow{BP} + 3\overrightarrow{PC} = \overrightarrow{0}$. Show that $\overrightarrow{AP} = \frac{1}{2}\left[\overrightarrow{AB} - 3\overrightarrow{BC}\right]$. [3 Marks]

(d) Differentiate between the following terminologies.

(i) Population and Sample [1 Marks]

(ii) Mutually exclusive events and independent events [1 Marks]

QUESTION FOUR (15 MARKS)

(a) State and explain 3 approaches of assigning probabilities [3 Marks]

(b) State the multiplicative law for non-independent events. [2 Marks]

(c) A chain of video stores sells three different brands of videocasette recorders (VCRs). Fifty percent of its VCR sales are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a one-year warranty on parts and labour. It is known that 25% of brand 1's VCRs require warranty repair work, whereas the corresponding percentages for brand 2 and 3 are 20% and 10%, respectively.

(i) What is the probability that a randomly selected purchaser has bought a brand 1 VCR that will need repair while under warranty? [2 Marks]

(ii) What is the probability that a randomly selected purchaser has a VCR that will need repair while under warranty? [2 Marks]

(iii) If a customer returns to the store with a VCR that needs warranty repair work, what is the probability that it is a brand 1 VCR? A brand 2 VCR? A brand 3 VCR? [6 Marks]

QUESTION FIVE (15 MARKS)

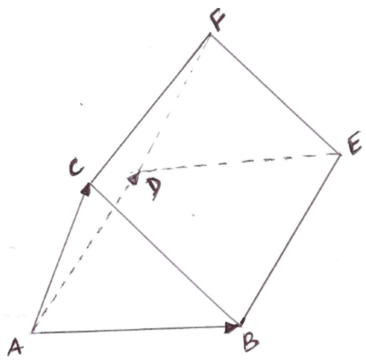
(a) The lengths, in millimetres, of 40 bearings were determined with the following results:

16.6	18.7	13.6	17.3	15.3	16.4	18.3	16.6	16.3	19.0
17.2	15.3	14.2	15.8	18.0	16.4	16.7	18.4	15.8	17.3
17.3	15.1	19.3	16.9	18.2	17.0	16.8	14.7	15.6	18.9
17.7	16.2	14.9	18.3	16.8	17.4	17.2	15.9	17.9	15.6

(i) Group the data into six equal width classes between 13.5 and 19.4 mm and hence the frequency distribution. [3 Marks]

(ii) Calculate the **mode** and the **median** lengths. [4 Marks]

(b) A triangular prism has vertices at $A(3,3,3)$, $B(1,3,t)$, $C(5,1,5)$ and $F(8,0,10)$, where t is a constant.



The face ABC is parallel to the face DEF and the lines AD, BE and CF are parallel to each other.

(i) Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$, in terms of t. [3 Marks]

(i) Find the value of $\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AD}$, in terms of t. [2 Marks]

The value of t is taken to be 6.

(iii) Determine the volume of the prism for this value of t. [2 Marks]

(iv) Explain the geometrical significance if $t = -1$. [1 Marks]

END OF PAPER