



**Strathmore**  
UNIVERSITY

**STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES**

**BBS ACTUARIAL SCIENCE**

**BSA 3218: Actuarial Modelling II**

**END OF SEMESTER EXAMINATION**

**Monday, 25<sup>th</sup> July, 2022**

**Time: 2 Hours**

**Question One.**

- a. Given any two ages  $x$  and  $y$  ( $y > x$ ), describe the difference between the following assumptions about mortality between the two ages explaining the shape of the survival function between ages  $x$  and  $y$  under each of the two assumptions. (4 marks)
- b. An investigation is carried out into the mortality rates of married male accountants. A group of 10,000 married male accountants is selected at random on 1 January 2016. Each member of the sample group supplies detailed personal information as at 1 January 2016 including name, address and date of birth. The same information is collected as at each 1 January in the years 2017, 2018, 2019 and 2020. The investigation closes in 2020. Describe the ways in which the available data for this investigation may be censored. (5 marks)
- c. Carry out the smoothness test on the following set of graduated rates: (4 marks)

$X$	$\mu_x^0$
55	0.00429
56	0.00478
57	0.00535
58	0.00596
59	0.00667
60	0.00754
61	0.00867

- d. Briefly discuss the three methods of graduation (6 marks)
- e. In a certain population, 5000 individuals are currently aged exactly 65 years. The force of mortality for this population is given by:

	$\mu_x$
$60 < x < 70$	0.01
$70 < x < 80$	0.015
$x > 80$	0.025

Calculate the expected number of individuals currently age 65 who will die between exact ages 80 and 83. (5 marks)

### Question Two

- Mortality of a group of lives is assumed to follow Gompertz' law. Calculate  $\mu_x$  for a 30-year old and a 70-year old, given that  $\mu_x$  is 0.003 for a 50-year old and 0.01 for a 60-year old. (5 marks)
- You want to use a Cox regression model to estimate the force of mortality for a group of endowment assurance policyholders. You propose using a model that takes account of duration (*ie* the time that has elapsed since the policy was issued) and the age and sex of the policyholder. You start by investigating the model:

$$\mu(x, z_1, z_2) = \mu_0(x)e^{\beta_1 z_1 + \beta_2 z_2}$$

Where:

$x$  denotes the age of the policy holder,

$$Z_1 = \begin{cases} 0 & \text{if the duration is less than 1 year} \\ 1 & \text{if the duration is at least 1 year} \end{cases}$$

$$Z_2 = \begin{cases} 0 & \text{for males} \\ 1 & \text{for females} \end{cases}$$

You have estimated the values of the parameters  $Z_1$  and  $Z_2$ , and have obtained the following results:

Covariate	Parameter	Standard error
Duration	0.416	0.067
Sex	-0.030	0.017

- State the class of policyholders to which the baseline hazard refers. (2 marks)
- Explain whether the duration covariate is significant in determining mortality. (3 marks)
- Compare the force of mortality for a new female policyholder to that of a male policyholder of the same age, who took out a policy 2 years ago. (6 marks)

- Give an example of a situation in which the hazard function may be expected to follow each of the following distributions: (4 marks)

- i. exponential
- ii. decreasing Weibull
- iii. Gompertz-Makeham
- iv. log-logistic

**Question Three**

- a. The following data relate to 12 patients who had an operation that was intended to correct a life-threatening condition, where time 0 is the start of the period of the investigation:

Patient number	Time of operation (in weeks)	Time observation ended (in weeks)	Reason observation ended
1	0	120	Censored
2	0	68	Death
3	0	40	Death
4	4	120	Censored
5	5	35	Censored
6	10	40	Death
7	20	120	Censored
8	44	115	Death
9	50	90	Death
10	63	98	Death
11	70	120	Death
12	80	110	Death

You can assume that censoring was non-informative with regard to the survival of any individual patient.

- i. Compute the Nelson-Aalen estimate of the cumulative hazard function,  $\Lambda(t)$ , where  $t$  is the time since having the operation. (12 Marks)
  - ii. Using the results of part (i), deduce an estimate of the survival function for patients who have had this operation. (2 Marks)
  - iii. Estimate the probability of a patient surviving for at least 70 weeks after undergoing the operation. (2 Marks)
- b. Show that the Kaplan-Meier and the Nelson-Aalen estimates are equal (4 marks)

**Question Four**

- a. A group of six lives was observed over a period of time as part of a mortality investigation. Each of the lives was under observation at all ages from age 55 until they died or were censored. The table below shows the sex, age at exit and reason for exit from the investigation.

Life	Sex	Age at Exit	Reason for Exit
1	M	56	Death
2	F	62	Censored
3	F	63	Death
4	M	66	Death
5	M	67	Censored
6	M	67	Censored

The following model has been suggested for the force of mortality:

$$\mu(x|Z = z) = \mu_0(x)e^{\beta z}$$

Where:

- $X$  denotes age
  - $\mu_0(x)e^{\beta z}$  is the baseline hazard
  - $z = 0$  for females and 1 otherwise
- i. Write down the partial likelihood for these observations using the model above. (6 marks)
  - ii. Construct the 95% CI for  $\beta$  explaining what can be inferred from this. Use  $Z = 1.96$  (6 marks)
  - iii. Suppose the following additional data is obtained:

Life	Sex	Age at Exit	Reason for Exit
7	M	56	Censored
8	F	62	Censored

Explain how these extra data values affect the contribution to the partial likelihood from the first death. (3 marks)

- b.  $T_x$  is the future lifetime random variable of an individual aged  $x$ . Show that the pdf of this random variable is:

$$f_x(t) = {}_t p_x \mu_{x+t} \quad (0 \leq t < \omega - x) \quad (5 \text{ marks})$$

### Question Five

- a. The disreputable insurance company Honest Sid's Mutual had mixed fortunes in the year 2018. At both the start and the end of the year 547 policies were in force in respect of policyholders aged 40 last birthday, but these figures do not tell the whole story. There was adverse publicity early in the year linking the company's investment managers with a gambling syndicate. As a result, many policyholders 'took their money elsewhere'. Following a successful marketing campaign offering a free toaster to all applicants, the number of policyholders aged 40 last birthday rose from 325 at 1 June 2018 to 613 at 1 September 2018. Calculate an approximate value for the central exposed to risk at age 40 last birthday for the calendar year 2018. (6 marks)

- b. Survival of certain patients largely depends on the gender of the patient. The following are results from the survey with Sex = 1 for males and 2 for female. The following are results from a Cox-PH model fitted in R. Clearly explain your results (8 marks)

```
all:
coxph(formula = Surv(time, status) ~ sex, data = lung)
n= 228, number of events= 165
      coef exp(coef) se(coef)      z Pr(>|z|)
sex(2) -0.5310    0.5880   0.1672  -3.176 0.00149 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
exp(coef) exp(-coef) lower .95 upper .95
sex      0.588      1.701    0.4237    0.816
Concordance= 0.579 (se = 0.022 )
Rsquare= 0.046 (max possible= 0.999 )
Likelihood ratio test= 10.63 on 1 df,  p=0.001111
Wald test               = 10.09 on 1 df,  p=0.001491
Score (logrank) test = 10.33 on 1 df,  p=0.001312
```

- c. The table below is data on 5 ages from a certain research in 2019. Assuming  $\log(e^{\mu_x} - 1)$  can be modelled as  $\beta_0 + \beta_1 x$ , obtain the graduated rates  $q_x^0$ . (6 marks)

$x$	$E_x^c$	$d_x$	$\hat{\mu}_x$
30	70,000	39	0.000557
31	66,672	43	0.000645
32	68,375	34	0.000497
33	65,420	31	0.000474
34	61,779	23	0.000372