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FORECASTING FOREIGN EXCHANGE RATES IN KENYA USING STOCHASTIC MODELS

(Geometric Brownian motion and Merton jump process,)

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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ABSTRACT

The shift of exchange rate regime from fixed to floating has prompted the need for forecasting exchange rates in Kenya. Portfolio managers and corporate finance managers whose forms' and clients' cash flows are affected by exchange rate movements would benefit from an estimate as this would provide a basis for decision making. The paper examines the accuracy of the two stochastic models, Geometric Brownian motion and Merton-diffusion jump using an error statistic; Mean Absolute Percentage Error (MAPE) of the USD/KES forecasts under consideration. A lower value of the MAPE indicated the inclusion of the Merton jump process in the Geometric Brownian motion was superior as a smaller error was obtained.

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND AND THE RATIONALE OF THE STUDY

1.1.1 BRIEF HISTORY OF THE KENYAN EXCHANGE RATE

The Kenyan currency has been subjected to different shifts in exchange rate policy which were mostly triggered by economic events, especially the Balance of Payment crises in the 1990's Ndung'u (2000). The historical policy regime shifts of the exchange rates can be divided into two phases; fixed exchange before 1982 and floating rate after 1982 Ndung'u (2000).

The East African shilling was predominant in the British colonies from 1919-1966. The currency was supplied by the East African Currency Board. The Board's main function was to ensure the shilling was pegged to the Sterling pound and adequately backed by Sterling securities. After the break-up of the East African Currency Board in 1974, discrete devaluations were conducted by the regulators and it was pegged to the US Dollar[1].The board was eventually replaced by the independent Central Banks of the British colonies (Kenya, Uganda and Tanzania).

Financial globalization marked the end of the fixed exchange rate regime and welcomed the fixed exchange rate regime. In 1990, the dual exchange rate was adopted where the fixed exchange rate was applied on the essential segments such as imports and exports while the other sectors implemented the floating regime Ndung'u (2000). Meanwhile, the price of the capital account is determined by a market driven exchange rate which is necessary for maintaining the foreign reserves in the country. In 1993, the official rate was merged with the market rate after a series of discrete devaluations and Kenyan currency adopted the current dirty float. The pegged exchange rate economy was costly due to the high level of reserves that the Central Bank was required to maintain Ndung'u (2000).

The Central Bank in determining the crawl (floating) regime took consideration of the value of the parallel markets but not entirely Ndung'u (2000). The Central Bank used the indexation as a means of controlling inflation in order to maintain purchasing power from a macroeconomic

[1] *https://en.wikipedia.org/wiki/East_African_Currency_Board*

standpoint. However, at this point in time, the 'black market' was not active due to the high level of regulations financial assets such as shares were subjected to.

The Central Bank of Kenya participates in the foreign exchange markets to ensure that the currency reserves attain a set minimum requirement[2]. In the case of any short term fluctuations, financial instruments such as Inter-bank rate provide liquidity insurance that may arise from the speculative behavior of exchange rate market participants.

1.1.2 THE TREND OF THE USD/KES FOR THE PAST DECADE

The trend of the USD/KES exchange rate has exhibited instability for the last 10 years. In January 1993, the currency was going for 36.23. Hardly a decade passed when it hit an all-time low in October 2011 when it was going for 105.75[3] to the US dollar. The non-stationarity for in the USD/KES exchange rate follows an assumption that currency rates follow a random walk or a particular stochastic process and accordingly the estimates have been derived by assuming some of the distributions available in literature for randomness involved in the currency rates movement Kumar (2014). This was a clear indication that currency like its peers in the asset markets, exchange rate is grossly affected by powers of demand and supply.

1https://www.tradingview.com/chart/?symbol=FX_IDC:USDKES

Figure 2.1 The trend Of The USD/KES for the past decade

[2] *https://www.centralbank.go.ke/*

1.1.3 FACTORS THAT CAUSE CURRENCY FLUCTUATION

There are many variables that shape the value of a currency through their influence on the demand and supply conditions in the market for the currency. The leading determinants are as follows:

i. Trade Balance

The Central Bank of Kenya recently recorded a deficit of 67156 Million Kenyan Shillings as of March 2015. This is backed by data from the Kenya National Bureau of Statistics that indicates that the country has been importing more machinery since September last year. In addition, imports reached an all-time high of 159936 Million Kenya Shillings in September 2014[4].

A high imports to exports ratio increases demand for the foreign currency. This result in an increased supply of the domestic currency and consequently, its price, relative to the foreign currency, would fall.

ii. Gross Domestic Product

Kenya's GDP has been steadily increasing over the years. As of 2014, the GDP stood at $53.4 Billion[5]. At Ceteris Paribus, an increase in the Gross Domestic Product will lead to an increase in the demand for that local currency. This will reflect directly on the interest rates as the real money demand exceeds the real money supply.

iii. Interest rates

The interest rate in Kenya has averaged 14.37% relative to the benchmark of 11.5%.[6]. The interest rate hit an all-time high of 86.47% in July 1993. As mentioned earlier, the value of the currency is shaped by the factors of demand and supply. Whenever there is an increase in the prevailing interest rates, relative to foreign interest rates the incentive for local and foreign investors to buy domestic financial assets increase. This is due to the fact that foreign deposits and investment attract relatively higher interest during that period.

[4]*www.**tradingeconomics**.com/**kenya/balance**-of-**trade***

[5]*www.forbes.com/.../**2014**/.../**kenya**-joins-africas-top-10-economies*

[6]http://www.tradingeconomics.com/kenya/interest-rate

This increases the demand increases for the local currency. Conversely, if the domestic country lowers her interest rates, investors tend to take a short position on the domestic country's assets as their returns become less attractive and at Ceteris Paribus, the domestic currency depreciates.

iv. Political climate

Political instability erodes expected future cash flows of assets in an economy and since those assets support the domestic currency, fundamental analysis dictates that the value of the currency declines

Market sentiment has great influence on the Foreign exchange markets. In Kenya, the 2006/2007 post election violence resulted in capital flight which led to the depreciation of the Kenyan shilling during that period due to decreased demand for the domestic currency.

1.1.3 RATIONALE OF THE STUDY

This study uses the US Dollar because 20.5% of foreign current financial transactions in Kenya take place using the US Dollar currency (with the exclusion of remittances). In addition, since Kenya adopted the current dirty float after 1982, bilateral USD/KES will be the focus of the study.

The Kenya Electronic Payment Settlement System (KEPSS) is Kenya's Real Time Gross Settlement System (RTGS), it was established and is wholly owned and managed by the Central Bank of Kenya. The RTGS system is a tool used in the mitigation of credit risk whereby the transactions are considered final and irrevocable because they take place in the books of the Central Bank[7].

The KEPSS processes transactions to the extent that a Commercial Bank is permitted by the Kenya Bankers Association. The Kenya Bankers Association clears the operations of a Commercial Banks settlement account in the Central Bank of Kenya. The current transactions

[7]https://www.centralbank.go.ke/index.php/2012-09-21-11-44-41/kepss

are processed in two main currencies, the US Dollar and the Sterling Pound. More currencies are bound to be used in the anticipated implementation of the East Africa Border Payment System[8].

Few studies have proposed the short-term forecasting of Foreign exchange in emerging economies. Most studies concentrate on long-term forecasting for monetary policy making purposes i.e. to ensure the domestic reserves are less susceptible to macroeconomic shocks. Meese and Rogoff (1983), (Cheung, 2005) and Gözgör et al (2010)did a comparison of the various models that are used to forecast the exchange rate and found that models based on Stochastic processes provide more accurate estimates than the time series models (ARIMA and VAR) and the macroeconomic model (Uncovered interest parity).

The study intends to use the two stochastic models; Merton jump process and Geometric Brownian motion on to forecast Foreign Exchange Rates, particularly the US Dollar versus the Kenyan Shilling. The research will use in-sample data (historical Foreign Exchange) to estimate the parameters (μ and σ) of the two models, and then use them to simulate the future foreign exchange, compare the simulated values with the actual rates.

The intention behind this is to determine if the models can be used to forecast the foreign exchange, at least at a certain level of confidence. Further, there will be an accuracy test for both models to compare which of the two provide a better fit for the Currency exchange forecast.

1.2 RESEARCH OBJECTIVE

To use an error statistic; mean absolute percentage Error (MAPE) in determining the better model for forecasting Foreign Exchange between Geometric Brownian motion and Merton Diffusion jump for different time horizons.

1.3 RESEARCH QUESTIONS

1. Between the Merton jump processes, the Geometric Brownian motion and Merton-diffusion jump, which one provides more Kenyan Shilling exchange rate forecast over a short time horizon?

[8]https://www.centralbank.go.ke/index.php/bank-services

2. Which stochastic model between Geometric Brownian and Merton-Diffusion jump accurately at different time horizons?

1.4 SIGNIFICANCE OF THE STUDY

Exchange rate and monetary policy are key tools in economic management and in the stabilization and adjustment policies in emerging economies Ndung'u (2000). According to Rashid (2006), the Foreign Exchange market is the largest financial market in terms of size. This highlights the importance of studies in the Foreign Exchange market (Rashid, 2006). Reforms such as trade liberalization warrant participants such as countries or even multinational companies to forecast foreign Exchange rates (Rashid, 2006). The forecasting is for a variety of reasons such as capital Budgeting and investing. This is the case where the nature of the cash flows from these activities is dependent on the future exchange rate.

Kumar (2014) highlights the need for the estimation of currencies because most countries have moved from fixed regime to the floating regime there is a need for foreign exchange forecasting. Gözgör et al (2010)on his part argues that because most countries have assumed huge positions in foreign exchange contracts, it is prudent to forecast foreign exchange rates as risk management strategy against fluctuations.

McNichols and Rizzo (2012)state that organizations such as Insurance companies and fiduciary corporations require credible methods of forecasting foreign exchange in order to mitigate and risk exposures in case of any event.

The study aims to test the accuracy of the Stochastic models (Wiener process and the Geometric Brownian motion) over different time horizons. This will be beneficial to current and potential investors in foreign Exchange market as well as portfolio managers and corporate finance managers whose forms' and clients' cash flows are affected by exchange rate movements.

1.5 PROBLEM STATEMENT

In the past, countries would get into a fixed currency agreement (regime) Gözgör et al (2010).The nature of these agreements would be that regardless of the prevailing economic conditions, the KES against the US Dollar as in this case would remain fixed. During this period there was no need in foreign Exchange currency forecasting the shift from the fixed to the floating rate regime indicates the need for forecasting.

The shift was triggered by two main factors; financial crisis and because most developing countries wanted liberalizing of their financial system in a bid to be integrated into the global markets Gözgör (2010). This shift shows there is need for forecasting especially among speculators and fiduciaries in the financial markets to hedge against foreign exchange risk. Therefore, there is need for studies which focus on forecasting the currency exchange rates Gözgör (2010)

A model that is provides relatively accurate estimates for both short and long-term horizons would be beneficial to both domestic and foreign investors. This study will use two stochastic models; Merton jump process and geometric Brownian motion. Most studies that have been carried out on using Geometric Brownian motion have focused on share price Modeling and very few on the foreign Exchange.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter is organized as follows; the first section of this chapter reviews other models and the second section provides a review of findings of similar studies.

The foreign exchange forecasting models fall into three categories; Time series, macroeconomic and the stochastic models. There two models under the time series, namely; Autoregressive Integrated Moving Average (ARIMA) and Vector Auto Regression (VAR). The Uncovered interest parity falls under macroeconomic model category while the stochastic models are Merton jump process and the Geometric Brownian Motion.

2.2 REVIEW OF TRADITIONAL MODELS

2.2.1 TIME SERIES MODELS

AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA),

Popularly known as the Box Jenkins methodology, the ARIMA is an integrated autoregressive process whose characteristic equation has a root on the unit circle (Brooks, 2008). The model is generally applied to processes that are non-stationary Kumar (2014).

Conventionally the ARIMA is denoted by $ARIMA\ (a, b, c), where\ a$ indicates the lag, b denotes the differencing to be done if not stationary c denotes the Moving Average of the commodity of interest.

The parameters for the ARIMA are based on the underlying currency data in this case will be:

$$\text{LN}\, S_t \;=\; \alpha \;+ \beta_1 \,\text{LN}\, S_{t-1} + \;\beta_2 \,\text{LN}\, S_{t-2} \;+\; \ldots\ldots\ldots\ldots\ldots\ldots .. + e_t \quad \ldots\ldots eqn\ 2.1$$

The assumption would be the model to be univariate, where in this case the future foreign exchange rate would only be dependent on the behavior of the past exchange rate. In addition, it is simple to use since the parameters that need to be estimated are few Kumar (2014).

However, according to Brooks (2008) the ARIMA model has some limitations for instance commodities such as Foreign exchange rates prefer non-linear models because they follow a random walk process. Secondly the model relaxes the impact of variables such as change in policies and interest rates. This provides an assumption that is not entirely realistic. And again although the univariate model can be extended to the Multivariate models, it is considered inadequate in comparison to other models (Brooks, 2008).

VECTOR AUTOREGRESSIVE MODELS (VAR)

Unlike the ARIMA, as the name suggests the VAR model deals with one or more independent variables which have an influence on the dependent variable. Moreover element of auto regression comes in due to the appearance of lagged value of the dependent variable on the Right-hand side as seen below Gujarati (2008).

The simplest case of VAR is bivariate, where there is the assumption of only 2 variables y_{t11}andy_{t22}. This is to accommodate as many causal decision variables and error terms into the estimate as seen below (Kumar P. P., 2014)

$$y_{t11} = \beta_{10} + \beta_{11}y_{t11-1} + \alpha_{11}y_{2t-1} + \mu_{t1} \text{....} eqn\ 2.2$$

$$y_{t21} = \beta_{20} + \beta_{21}y_{t22-2} + \alpha_{21}y_{1t-1} + \mu_{t2} \ldots eqn\ 2.3$$

Where μ_{ti} is a white noise disturbance term.

However, according to Brooks (2008) the VAR suffers from some shortfalls for example the estimate obtained heavily relies on the skills and experience of the developer of the model hence is highly prone to error. Secondly it is difficult to determine the number of lags that are appropriate to a specific model and the number of parameters is also difficult to determine since it is entirely subjective to the participant.

2.2.2 MACROECONOMIC MODELS

UNCOVERED INTEREST PARITY

Forecasting based on this exchange rate model is considered the most parsimonious to use. This is because it only considers the interest rate differential. The Macroeconomic approach explains that given that change in the exchange rate is proportional to the differential of the domestic and foreign interest rates only and accordingly based on the interest rates prevailing in the market, uncovered interest rate parity can forecast currency rates especially in the long term Kumar (2014).

However, this approach suffers from some limitations Cheung (2005). For example it is best suited for long term horizons; hence the model is not efficient for short term horizon forecasts. Secondly, it assumes no arbitrage opportunities in the currency and interest rate markets and this is never the case because of the huge transaction costs involved in such positions. Thirdly the model relaxes the assumption that the currency market is affected in the short term this is an unrealistic view that the currency markets are perfect (Cheung, 2005). Lastly the model makes an assumption that there are no major frictions in the currency market as a result of rational investors and perfect markets Kumar (2014).

SUMMARY OF THE REVIEW OF TRADITIONAL MODELS

However, Meese and Rogoff (1983), (Cheung, 2005) and Gözgör et al (2010)did a comparison of the various models that are used to forecast the exchange rate and found out that models based on Stochastic processes provide more accurate estimates than the time series models (ARIMA and VAR) and the macroeconomic model (Uncovered interest parity). Lucrezia Reichlin and Kenneth West (2010)adds that the current exchange rate is often a better predictor of future exchange rates than a linear combination of macroeconomic fundamentals. There exists strong nonlinear serial dependence for exchange rate changes, which cannot be solely attributed to the well-known volatility clustering, Hong.YandLeeT.H (2003).

Kumar (2014) added that the Geometric Brownian motion (GBM) provides better estimates than some models such as the Constant Elasticity in periods of low volatility. The Stochastic models (Geometric Brownian Motion and Merton Jump) will be explained in detail in the next chapter.

2.3REVIEW OF FINDINGS OF THE EMPIRICAL LITERATURE

Rashid (2006)finds no need in forecasting of foreign exchange rate. He reckons that the presence of a random walk in currency exchange rates implies that today's exchange rate is the best predictor of future exchange rates. The Foreign Exchange market participants do not gain any advantage positions from using data from past interest rate movements (Rashid, 2006). The Foreign exchange market is said to be efficient because the exchange rates must fully reflect all the information in the market independent of all available information (Rashid, 2006). Exchange rate increments have zero correlation that is why it is difficult and necessary to predict exchange them.

According to (Cheung, 2005), no single model can estimate the Foreign exchange rate unless it is subject to some customization. Meese and Rogoff (1983) had a slightly different approach using rolling regressions based on standard macroeconomic models. Meese and Rogoff (1983) found the relationship between nominal exchange rates and macroeconomic fundamentals is unstable. This was seconded by (West, 2010) there exists an unstable relationship between the exchange rates and the fundamentals. This instability leads to parameter estimation errors (West, 2010).

Meese and Rogoff (1983) pioneered ex-ante forecasting power. It was found that random walk was significant when coming up with an estimate of the Foreign exchange market. Due to the shift of regime from Fixed to floating regimes in the early 90's especially in developing countries, Gözgör et al (2010) states that the significance of studies related to exchange rates increased especially amongst individual and institutional investors.

In the present era of globalization, prominent players such as Multi-National companies and countries across the globe have taken up huge positions in various currencies over a period of time Kumar (2014). Therefore, Kumar (2014) asserts there is the need for estimation for the forecasting of the exchange rates. Nichols and Rizzo (2012) adds that the motivation behind the forecasting is to mitigate the risk exposure brought about by market fluctuations

Clearly from the above discussion, researchers conflict as to whether or not there is need for forecasting foreign exchange. The study uses mean Absolute percentage Error (MAPE) to determine the better model for forecasting Foreign Exchange between Merton jump process and Geometric Brownian motion for different time horizons. By doing so it will be determined whether or not there is a need for forecasting foreign exchange. This will reconcile the conflicting findings above. Moreover, most of the above studies have been in the developed markets and little in the developing markets. However, the developed and developing markets operate in different social, economic and political environments.

CHAPTER 3: METHODOLOGY AND DATA

3.1 INTRODUCTION

This chapter is divided into two parts. The first part covers the mathematical construction of the Merton jump process and the Geometric Brownian motion and also of the accuracy tests. The second part of this chapter will outline and discuss the most appropriate methods of design, sample selection, data collection and analysis.

3.2 RESEARCH DESIGN

The design takes on a case study approach. This is because it is useful for testing theoretical models in real life situations. Case studies are also helpful when doing an in depth study on a specific situation.

3.3 NATURE OF STUDY

The study has follows a quantitative approach due to the nature of the findings for the analysis. Using the simulation approach, the dynamic process allows the generation of data and information with the aid of an artificial environment under controlled conditions Kothari (2004). The simulation approach will allow us to see the trend of the forecast over a specific time period or periods given certain parameters; drift term and the random term.

3.4 POPULATION AND SAMPLING

3.4.1 POPULATION

The study will focus on the Foreign Exchange Markets in Kenya under controlled economic conditions. Specifically, the study will focus on the US/Kenyan Shilling exchange rate. The population of interest is bilateral daily USD/KES exchange rates for the period since Kenya adopted the floating exchange rate regime.

3.4.2 SAMPLING

The daily exchange data of 314 trading days will be sampled based on the most recent complete calendar year (2013-2014). The seminal work of Gözgör et al. (2010) and (Meese, 1983) performed similar tests. Gözgör et al. (2010) used a sample size of year

3.5 DATA COLLECTION AND PROCEDURE

The data required for the study is the US Dollar/Kenya Shilling exchange rates from 2013-2014. The data will be sourced from (http://www.dailyfx.com/forex_rates)

The mean and volatility parameters will be obtained from the historical data available in the Foreign Exchange market through computation. There will be a consideration of at least 1000 simulations each day for the given time period.

The prediction of the exchange rate given the parameters will be followed by involving various error statistics (Mean Absolute Percentage Error) to test on the accuracy given a given time horizon .MAPE was selected because it obtains the drift error in absolute percentage terms.

3.6 MODEL SPECIFICATION

Under this section, the models under consideration (Merton jump process and the Geometric Brownian) will be discretized in order to estimate the model parameters and to calculate the confidence intervals.

The derivation of the mathematical constructions of the Merton jump process and the Geometric Brownian (see appendix)

3.6.1 GEOMETRIC BROWNIAN MOTION

According to Yang (2010) Geometric Brownian motion is controlled by the "trend. This means that after a number of Geometric Brownian simulations, the graph will be heading towards a specific direction subject to some standard deviations.

Under this model the US Dollar rate, D_t is assumed to follow a stochastic process with the following differential equation:

$$dD_t = \mu D_t dt + \sigma D_t dB_t$$

According to this model, the right hand side term (deterministic component) controls the trend $\mu D_t dt$ and the random part, $\sigma D_t dB_t$ controls the random noise (Yang, 2010). In this case the drift parameter(μD_t) and the volatility parameter (σD_t) are functions of the current exchange rate.

Where μ the rate of is return and σ is the volatility and B_t is a standard Brownian motion.

The discretized version of this model will be:

$$\Delta D_T = \mu D_T \Delta T + \Sigma D_T \epsilon \sqrt{\Delta T} \ldots \textit{eqn 3.1}$$

The solution for this stochastic differential equation can be obtained using Ito's Lemma as:

$$D_t = D_0 e^{\left[\mu - \frac{1}{2}\sigma^2\right]\Delta t + \sigma\epsilon\sqrt{\Delta t}}$$

$$D_t \sim \log Normal\left(\left[\log D_0 + \mu - \frac{1}{2}\sigma^2\right] t, \sigma^2 t\right)$$

3.6.2 MERTON DIFFUSSION JUMP PROCESS

Merton (1976) added a jump to the geometric Brownian motion in order to approximate the movements of the stock prices subject to the occasional discontinuous breaks. The model hence becomes:

dD_t =Geometric Brownian motion has the volatility parameter, σand the drift parameter μ. Given the closing rates over a period of time ($D_0, D_1 D_2 \ldots \ldots \ldots \ldots D_n$) these parameters can be estimated as follows:

$$\mu_M D_t dt + \sigma_M D_t dB_t + Y_t \ldots \textit{eqn 3.2}$$

The solution for this using Ito's calculus is:

$$D_t = D_0 e^{[\mu_M - 1/2\sigma_M{}^2]t + \sigma B_t + \sum_{j=1}^{N(t)} Y_j}$$

The Y_t represents the size of the jump and is normally distributed with mean μ_J and variance$\sigma_J{}^2$. The jump is a rare event hence is assumed to follow a Poisson distribution with parameter λ. That is:

$$Y_t \sim N(\mu_J, \sigma_J{}^2)$$

$$P(J = j) = \frac{e^{-\lambda}\lambda^j}{j!}$$

ESTIMATION OF THE GEOMETRIC BROWNIAN MOTION PARAMETERS

Geometric Brownian motion has the volatility parameter, σand the drift parameter μ. Given the closing rates over a period of time $(D_0, D_1 D_2 \ldots\ldots\ldots\ldots D_n)$ these parameters can be estimated as follows:

1) Get the daily log returns,$log\left(\frac{D_{t+1}}{D_t}\right)$,of closing dollar rates μ_t
2) Get the mean of log returns, $log\left(\frac{D_{t+1}}{D_t}\right)$
3) Get the variance of the log returns, $log\left(\frac{D_{t+1}}{D_t}\right)$
4) Get the standard deviation of the log returns, $log\left(\frac{D_{t+1}}{D_t}\right)$
5) To getthe drift parameter μ, the mean of the log returns in (2) above is multiplied by the number of the trading days (252). i.e.

$$\mu = 252 \times \sum_{t=1}^{n} \mu_t$$

6) To get the volatility parameter, σ, the standard deviation of the log returns in (4) above is multiplied by square root of the number of the trading days (252). i.e.

$$\Sigma = \sqrt{\left(\frac{1}{n-1}\Sigma_1^n(\mu_t - \bar{\mu}_t)^2\right)} \times \sqrt{252} \ldots eqn\ 3.3$$

CONFIDENCE INTERVAL FOR THE GEOMETRIC BROWNIAN MOTION

The study will use the discretized solution to simulate foreign exchange rates in excel, calculate the 90%,95% and 99% confidence intervals, according to the geometric Brownian motion model and compare with the actual rates if the actual rates fall within the range.

For the case of 95%,

$$A < D_t < B$$

$$P[D_t \leq A] = P\left\{Z \leq \left[\frac{\text{LN}\,A-(\text{LOG}\,D_0+\mu-\frac{1}{2}\sigma^2)t}{\sqrt{\sigma^2 t}}\right]\right\} = 0.025..... \textit{ eqn 3.4}$$

$$P[D_t \leq A] = P\left\{Z \leq \left[\frac{\ln B-(\log D_0+\mu-\frac{1}{2}\sigma^2)t}{\sqrt{\sigma^2 t}}\right]\right\} = 0.975..... \textit{ eqn 3.5}$$

MEAN ABSOLUTE PERCENTAGE ERROR (MAPE)

This is an error statistics that the study will use to determine the accuracy of each of the two models (Merton Jump and the Geometric Brownian motion). Under this the study will be able to calculate the error in the simulated value as a percentage of the actual historical value. This can be done by the following formula:

$$MAPE = \frac{1}{n}\sum_t^n \left|\frac{A_t-F_t}{A_t}\right| ...eqn\ 3.6$$

Where F_t the forecast is exchange rate and A_t is the actual exchange rate.

CHAPTER 4: DATA ANALYSIS

The study is based on quarterly time periods (monthly, 3-monthly, 6-monthly, 9-monthly and 12-monthly forecasts). At this point, 314 trading days are used. The in-sample forecasts obtained from the 2 different models (Geometric Brownian Motion and the Merton diffusion jump process) will be compared using an error statistic (Mean Absolute Percentage Error)

$$MAPE = \frac{1}{n}\sum_{t}^{n} \left|\frac{A_t - F_t}{A_t}\right|$$

$Where\ F_t\ the\ forecast\ is\ exchange\ rate\ and\ A_t\ is\ the\ actual\ exchange\ rate.$

A lower value of the MAPE will indicate the superior model given the expectation of forecast errors. Furthermore, there will be a judgment of accuracy of the forecasts aside from the different confidence intervals that will be used.

Table 4.1 : Judgment of forecast accuracy

MAPE	Judgment of Forecast Accuracy
<10%	Highly accurate
11% to 20%	Good accurate
21% to 50%	Reasonable forecast
>51%	Inaccurate forecast

4.1 GEOMETRIC BROWNIAN MOTION

4.1.1 THE RESULTS OF GEOMETRIC MOTION PARAMETERS

The parameters obtained from the discretized Geometric Brownian Motion μ_B and$\sigma^2{}_B$. This was done using excel. The discretized GBM function below is employed later during simulation using the Monte Carlo approach:

$$\Delta D_t = \mu D_t \Delta t + \sigma D_t \epsilon \sqrt{\Delta t}$$

Where μ denotes mean σ denotes volatility and

The results are as follows:

Table 4.2 Parameters of discretized Geometric Brownian Motion model

Annual Drift $\boldsymbol{\mu_B}$	0.00232679
Annual volatility $\boldsymbol{\sigma^2{}_B}$	0.033980285

4.1.2 THE GEOMETRIC BROWNIAN MOTION SIMULATED RESULTS

Using the parameters 1000 simulations for each of the time horizons (1-month, 3-months, 6-months, 9-months and 12-months) was performed. These were the results. These were compared with the actual exchange rates that were observed. The confidence intervals for various periods were also calculated and these are tabulated as well.

Table 4.3 Confidence Intervals for the GBM estimates

Time period	Actual value	90% CI		95% CI		99% CI	
		Lower	Upper	Lower	Upper	Lower	Upper
1-month	86.5	86.3217	86.4086	86.3134	86.4169	86.2971	86.4332
3-months	**86.36**	**86.33685**	**86.4873**	**86.3224**	**86.5017**	**86.2943**	**86.5300**
6-months	87.74	86.3691	86.5819	86.3487	86.6023	86.3089	86.6421
9-months	89.36	86.40553	86.6662	86.3806	86.6911	86.3318	86.7399
12 months	90.62	86.44413	86.7451	86.4153	86.7739	86.3590	86.8302

The actual value fell in between the 90% CI, 95% CI and 99% CI at 3 months. Time horizon plays a key role as accuracy decreases as time increases. From 6-12 months the actual value drifts further away from the simulated value.

4.1.3 THE GEOMETRIC MAPE RESULTS

The table below shows the MAPE results for Geometric Brownian.

Table 4.4 MAPE results for GBM estimates

Time Horizon	Date	Simulated value GBM	Actual value	MAPE
1-month	1-Feb	86.3255	86.5	0.20%
3-months	**1-Apr**	**86.3435**	**86.36**	**0.02%**
6-months	1-Jul	86.3785	87.74	1.55%
9-months	1-Oct	86.4171	89.36	3.29%
12 months	31-Dec	86.4574	90.62	4.59%

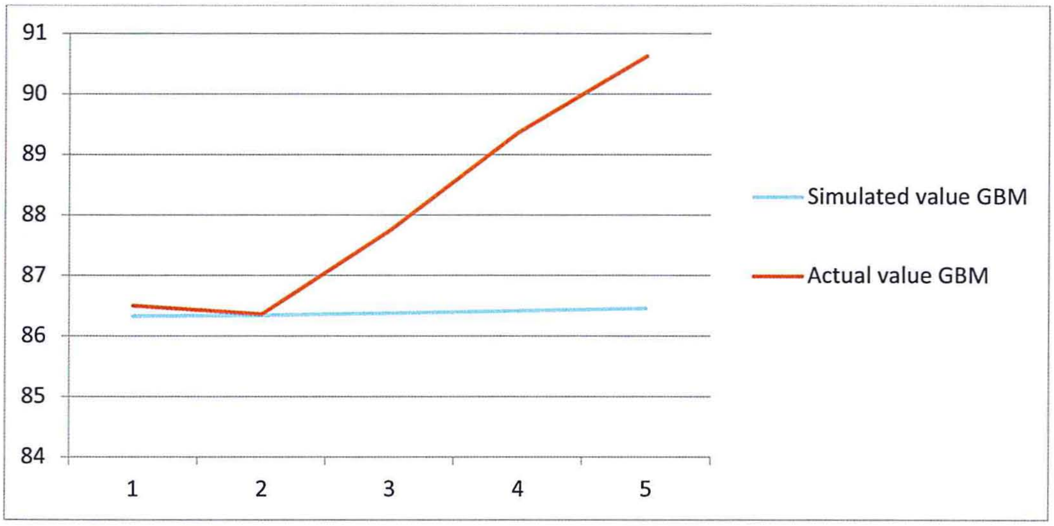

Figure 4.1 Actual Vs the Simulated Values with increase in time horizon

As seen from Table 4.4, the simulated value at 3 months produced an error of 0.02%. The model can be considered optimal at 3 months however, it concurs with (Cheung, 2005) that no single model can approximate the exchange rate of all currencies at different time horizons. This is characterized by increase of errors from 0.02% to 2% as time horizon increased

4.2 THE MERTON JUMP DIFFUSION PROCESS

4.2.1 RESULTS OF MERTON JUMP PARAMETERS

The parameters obtained for the Merton jump diffusion model$\mu_M \sigma_M, \mu_J, \sigma_J$ and λ. This was done by maximum likelihood estimation using the R-optim procedure. The function below was used:

$$l(\theta, y) = \sum_{t=0}^{T} \ln\left[\sum_{j=0}^{Q} \frac{e^{-\lambda}\lambda^j}{j!} \frac{1}{\sqrt{2\pi({\sigma^2}_M + j\sigma_J)}} exp\left(\frac{-(\ln\left(\frac{S_{t+1}}{S_t}\right) - \mu_M - j\mu_J)^2}{2({\sigma^2}_M + {\sigma_J}^2)}\right)\right]$$

These were the Merton jump parameters obtained:

Table 4.5 Parameters of discretized Merton Jump model

Annual Merton Drift μ_M	0.00232679
Annual Merton volatility σ_M	0.184337
Jump Mean, μ_J	0.01
Jump volatility, σ_J	0.002
Jump Intensity, λ	0.01

4.2.2 THE MERTON JUMP SIMULATED RESULTS

When obtaining the exchange rate at time$t$, D_t, the below solution for the Merton jump diffusion model was used:

$$\log D_t = \log D_0 + \left(\mu_M - 1/2{\sigma_M}^2\right) + \sigma B_t + \sum_{j=1}^{N(t)} Y_j$$

Using the parameters and the equation above 1000 simulations for each of the time horizons (1-month, 3-months, 6-months, 9-months and 12-months) was performed. These were the results. These were compared with the actual exchange rates that were observed. The confidence intervals for various periods were also calculated and these are tabulated as well.

Table 4.6 Confidence Intervals for the Merton Jump estimates

Time period	Actual value	90% CI		95% CI		99% CI	
		Lower	Upper	Lower	Upper	Lower	Upper
1-month	85.94	86.3149	86.3640	86.3102	86.3687	86.3011	86.3779
3-months	**86.53**	**86.3165**	**86.3997**	**86.3085**	**86.4076**	**86.2929**	**86.4232**
6-months	87.42	86.3371	86.4542	86.3259	86.4654	86.3040	86.4874
9-months	88.55	86.3651	86.5083	86.3514	86.5221	86.3260	86.5489
12 months	90.27	86.3967	86.5621	86.3809	86.5779	86.3499	86.6089

Similarly, the actual value fell in between the 90% CI, 95% CI and 99% CI at 3 months. The accuracy reduced with increase in time period in this case as well.

4.2.3 THE MERTON MAPE RESULTS

The table below shows the MAPE results for Merton Jump diffusion.

Table 5.7 MAPE Results for Merton Jump estimates

Time Horizon	Date	Simulated value Merton	Actual value	MAPE
1-month	1-Feb	86.3255	86.5	0.18%
3-months	**1-Apr**	**86.3435**	**86.36**	**0.01%**
6-months	1-Jul	86.3785	87.74	1.51%
9-months	1-Oct	86.4171	89.36	3.25%
12 months	31-Dec	86.4574	90.62	4.54%

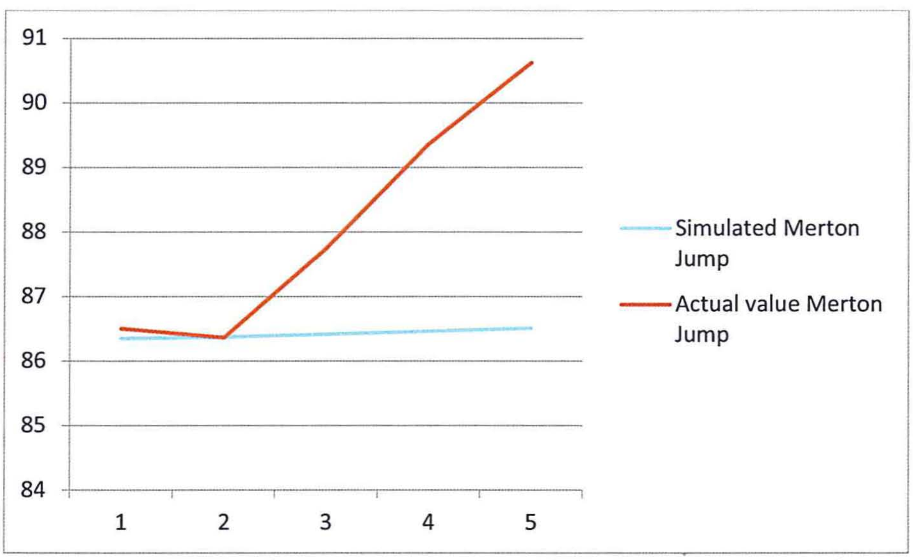

Figure4.3 Actual Vs the Merton Simulated Values with increase in time horizon

The MAPE provided a smaller error statistic for the Merton in comparison to the GBM. The error was reduced from 0.02% to 0.01% indicates the jump process is a superior model. It would be insufficient to forecast the change of the asset at the prevailing rate (given a D_0) without the inclusion of the jump.

CHAPTER 5: CONCLUSIONS AND FURTHER RESEARCH

5.1 CONCLUSIONS

Gözgör et al (2010) argues that because most countries have assumed huge positions in foreign exchange contracts, it is prudent to forecast foreign exchange rates as risk management strategy against fluctuations. McNichols and Rizzo (2012) state that organizations such as Insurance companies and fiduciary corporations require credible methods of forecasting foreign exchange in order to mitigate and risk exposures in case of any event.

High frequency foreign exchange data is characterized by potential jumps, volatility clustering and fat tailed distributions. The aim is to find a parsimoniously parameterized model that captures the essential features in the data.

In this paper we have compared the in-sample USD/KES forecasts of the Geometric Brownian Motion versus Merton jump against the actual value using MAPE. Based on the above data it can be concluded that stochastic models provide less accurate estimates as the time horizon increases which agrees with the seminal work of Gözgör et al. (2010) .The co-authors used various approaches and deduced that the GBM provided better estimates for the USD across the quarterly time horizons although it was contrary in the Euro where no conclusion for the best model.

It can be deduced that time horizon plays a critical role in forecasting. Moreover, it is observed that the use of fractional Brownian motion with a Merton jump process provides an estimate closer to the actual value. A specification that allows for volatility clustering and jumps is crucial because geometric Brownian motion cannot match the higher sample moments in the data.

5.2 AREAS OF FURTHER STUDY

Daily data might be problematic because of noise. I would suggest weekly data and use of 2 years instead of a year. Authors who have conducted similar tests used a 1 year data sample size however with weekly data , the sample size will need to be increased.

The Stochastic approach has not been fully exhausted in an emerging economy. Option pricing theorists added jumps and stochastic volatility to the standard geometric Brownian motion process. The additional stochastic differential equation a second state variable in volatility gives a tractable system of differential equations for pricing options Craine et al. (1999) The Stochastic Volatility Jump Diffusion could be added for comparison to obtain comprehensive results although time can be a constraint.

Ex-ante studies are encouraged using a comparison of various approaches the traditional VAR to the state-of-the-art microstructure approach. A new strand of literature of microstructure variables (e.g., order flows) needs to be appreciated. Comparison to the other approaches (Stochastic models, Micro-structure, Macro-economic and Time series) that lean on the predictive ability of the microstructure variables would provide better out-of-sample estimates of the return of different classes of financial returns.

In addition, various bilateral exchange rate currencies should be included as this would be beneficial not only academically but also to individual and institutional investors in different spheres of the world.

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APPENDIX

1. Geometric Brownian Motion

The solution for the Geometric Brownian Motion

The solution for this stochastic differential equation can be solved as follows:

$$\frac{dD_t}{D_t} = dlog\, D_t = \mu dt + \sigma dB_t$$

$$Taking\ the\ function\ f = log\, D_t\,(D_t, t)$$

$$df = d\,log\, D_t = \left[\frac{\partial f}{\partial D_t} D_t \mu + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial D^2{}_t}\sigma^2 {D_t}^2\right] dt + \frac{\partial f}{\partial D_t} D_t \sigma dB_t$$

Differentiating the function with respect to each of the two variables

$$\frac{\partial f}{\partial D_t} = \frac{1}{D_t}\frac{\partial F}{\partial t} = 0 - \frac{\partial^2 F}{\partial D^2{}_t} = \frac{-1}{D^2{}_t} = -D^{-2}{}_t$$

Replacing these in the ito's formula

$$df = \left[\frac{1}{D_t} S_t \mu + 0 + \frac{1}{2}\frac{-1}{D^2{}_t}\sigma^2 {D_t}^2\right] dt + \frac{1}{D_t} D_t \sigma dB_t = d\,log\, D_t = \left[\mu - \frac{1}{2}\sigma^2\right] dt + \sigma dB_t$$

$$Integrating\ both\ sides$$

$$\int_0^t d\,log\, D_t = \int_0^t \left[\mu - \frac{1}{2}\sigma^2\right] dt + \sigma \int_0^t dB_t = log\, D_t - log\, D_0 = \left[\mu - \frac{1}{2}\sigma^2\right] t + \sigma(B_t - B_0)$$

Exponentiating both sides and using the fact that $B_0 = 0$

$$D_t = D_0 e^{\left[\mu - \frac{1}{2}\sigma^2\right] t + \sigma B_t}$$

The discretized version of this model will be:

$$\Delta D_t = \mu D_t \Delta t + \sigma D_t \epsilon \sqrt{\Delta t}$$

$$D_t = D_0 e^{\left[\mu - \frac{1}{2}\sigma^2\right]\Delta t + \sigma \epsilon \sqrt{\Delta t}}$$

2. Merton Jump process

The solution for the Merton jump process

The solution for this stochastic differential equation can be solved as follows:

$$dD_t = \mu dt + \sigma dB_t$$

Integrating both sides

$$\int_0^t dD_u = \int_0^t \mu du + \int_0^t \sigma dB_t$$

$$D_t = D_0 + \mu t + \sigma B_t$$

The statistical distribution for the Merton jump process

$$B_t \textit{has a normal distribution}$$

Multiplying a normally distributed random variable by a constant leads to another normal distribution and adding a constant to a normally distributed random variable also leads to another normal distribution

$$E[D_t] = D_0 + \mu t$$

$$E[D_t] = \sigma^2 t$$

$$E[D_0 + \mu t + \sigma B_t] = D_0 + \mu t$$

$$Var[D_0 + \mu t + \sigma B_t] = 0 + \sigma^2 var(B_t) = \sigma^2 t$$

$$Hence\ D_t \sim Normal(D_0 + \mu t, \sigma^2 t)$$

The statistical distribution for the Geometric brownian motion

$$= \log D_t - \log D_0 = \left[\mu - \frac{1}{2}\sigma^2\right] t + \sigma B_t$$

The RHS represents the solution of Brownian motion with a drift (generalized Weiner process) hence is normally distributed, with the mean and variance determined as follows:

$$E\left[\mu - \frac{1}{2}\sigma^2\right]t + \sigma E[B_t] = \left[\mu - \frac{1}{2}\sigma^2\right]t, \qquad E[B_t] = 0$$

$$Variance\left[\mu - \frac{1}{2}\sigma^2\right]t + \sigma E[B_t] = \sigma^2 t, \qquad Var[B_t] = t$$

$$= \log D_t - \log D_0 \sim (\left[\mu - \frac{1}{2}\sigma^2\right]t, \sigma^2 t)$$

$$= \log D_t \sim Normal(\left[\log D_0 + \mu - \frac{1}{2}\sigma^2\right]t, \sigma^2 t)$$

$if \log X\ is\ normally\ distributed\ with\ (\mu, \sigma^2) then\ X\ is \log normally\ \ distributed\ with\ (\mu, \sigma^2$

This implies that:

$$D_t \sim \log Normal\ (\left[\log D_0 + \mu - \frac{1}{2}\sigma^2\right]t, \sigma^2 t)$$