INVESTIGATING THE EFFECTIVENESS OF VALUE AT RISK (VaR) MODELS IN PORTFOLIO RISK MANAGEMENT

FOR DIVERSIFIED PORTFOLIOS

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BRIAN MUTISO MUTUNGA

Submitted in partial fulfilment of the requirements for the Degree of Financial Engineering at Strathmore University

Strathmore Institute of Mathematical Sciences

Strathmore University

Nairobi, Kenya

JANUARY 2025

DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

..…………….. [*Name of Candidate*]
…………….. [*Signature*]
……………... [*Date*]

This Research Project has been submitted for examination with my approval as the Supervisor.

..……Dr BRIAN W. MUGANDA............. [*Name of Supervisor*]
………………...... .... [*Signature*]
………17th February 2025, Nairobi............... [*Date*]

Strathmore Institute of Mathematical Sciences
Strathmore University

TABLE OF CONTENT

ABSTRACT

Value at Risk (VaR) has become a widely accepted risk management tool in the financial industry, providing a single metric that represents the maximum potential loss of a portfolio over a specific time horizon and confidence level (Kenton, 2024). However, the reliability and effectiveness of VaR models have been the subject to an ongoing debate, particularly when applied to diversified investment portfolios. The study aims to address the research gap by conducting a comprehensive evaluation of three prominent VaR models – historical simulation, variance-covariance, and Monte Carlo Simulations- and their suitability for diversified portfolios (Harper, 2024) The analysis focuses on highlighting the theoretical foundations, methodologies, strengths and limitations of each VaR model, as well as evaluating their effectiveness in predicting portfolio losses for diversified portfolios with varying asset class compositions. Using back-testing techniques, the accuracy and reliability of the VaR models are assessed.

The analysis revealed that the Historical Simulation method provided the most reliable and adaptable estimates of VaR for the portfolio. The Variance-Covariance and Monte Carlo methods proved effective alternatives when normality assumptions were reasonably met. The findings provide valuable insights for portfolio managers and risk analysts on the most appropriate VaR models to use in managing the risk of diversified investment portfolios.

Keywords: Value at Risk (VaR), portfolio risk management, diversified portfolios, asset allocation, back-testin

CHAPTER 1: INTRODUCTION

1.1 Background Information

1.1.1 Value at Risk (VaR)

Modern financial markets encounter various types of risk, including credit risk, operational risk, liquidity risk, and market risk. Recently, there has been growing interest among researchers and financial professionals in using Value at Risk (VaR) to assess market risk. Market risk refers to the possibility of an investment losing value due to macroeconomic factors that influence the entire market. VaR aims to quantify the risk associated with unforeseen price fluctuations or changes in the log-return rate over a specified time frame (Li, 2016). This method is favoured for its simplicity in providing a single value that summarizes the market risk a company faces. Value at Risk is not only useful for examining market risk but also plays a role in managing other forms of risk. The system is primarily intended for risk management and regulatory purposes and is widely utilized by financial institutions, including commercial banks and investment banks, to estimate the maximum potential loss of their portfolio under specific market conditions within a set period. (Li, 2016)

VaR is defined as the maximum potential loss that a portfolio could incur over a specific time horizon, given a certain confidence level, under normal market conditions. Three main components characterize Value at Risk. The first is the time period under consideration, which determines the duration over which potential losses are assessed. Second is the confidence level, which reflects the degree of certainty with which the estimated loss is expected not to exceed. Finally, the dollar value of the VaR, which represents the amount of potential loss in dollar terms (Li, 2016).

1.1.2 Financial Portfolios

A portfolio refers to a collection of financial assets such as stocks, bonds, commodities, cash, and cash equivalents, which may include closed-end funds, and exchange treaded funds (ETFs), all held and managed by an individual or an organization. Typically, investors view stocks and bonds as the fundamental components of a portfolio, though it can also include a variety of other assets like real estate, gold, artwork, and other collectables (Tardi, 2024)

A central concept in portfolio management is diversification. Simply put, diversification is the practise of ‘not putting all your eggs in one basket’. It aims to minimize risk by spreading investments across various financial instruments, industries, and other categories thus maximizing returns by investing in different sectors that would each respond differently to the same event. No matter what the asset mix is, a portfolio should maintain a certain level of diversification, which reflects the investor’s risk tolerance, return objectives, time horizons, tax considerations, and other individual circumstances.

Portfolios can take on many forms, each designed to reflect a specific investment strategy or scenario and structured to meet different needs. Examples of portfolio types include a hybrid portfolio, which diversifies across various asset classes; an aggressive equity-focused portfolio, which typically takes on more risk in pursuit of higher returns; a defensive equity-focused portfolio, which tends to invest in consumer staples that are resilient during economic downturns; an income-focused equity portfolio, which generates income through dividend-paying stocks or other distributions; and a speculative equity-focused portfolio, which is suitable for investors with a high tolerance for risk. (Tardi, 2024)

The process of building a portfolio involves an investor first identifying their goals, time horizon, and level of risk tolerance. Afterwards, they research and select assets, such as stocks or other investments, that fit the established criteria. Regular monitoring and updating of the portfolio is essential, particularly for rebalancing purposes. Rebalancing involves selling some assets and buying others to ensure that the asset allocation consistently aligns with the investor’s strategy and desired level of return.

1.1.3 Portfolio Risk Management

Risk refers to any potential threat or unforeseen event that could lead to failure. In the context of investments, portfolio risk describes the possibility of a decline in the value or performance of an investment portfolio due to various factors such as market volatility, credit defaults, interest rate fluctuations, and currency changes. Portfolio risk management, therefore, involves the process of identifying and assessing the risks associated with an investment portfolio (Tamplin, 2023)

Investors need to recognize and mitigate different types of portfolio risks to manage their investments effectively. Some of these risks include credit risk, which is the possibility of a decline in investment value due to a bond issuer or other debt instruments defaulting; liquidity risk, which arises when an investment cannot be sold quickly enough to prevent losses;

inflation risk, which refers to the potential loss of value due to rising general prices; interest rate risk, which occurs when investment value decrease due to changes in interest rates; currency risk, which is the risk of losing value in investments denominated in foreign currency due to exchange rates changes; political risks where investment value diminishes because of government policy changes or instability in certain regions; and reinvestment risk, which involves the likelihood that future cash flows from an investment will be reinvested at a lower rate of return.

Various measures have been developed to provide investors with a comprehensive understanding of the risks associated with their portfolios, aiding in more informed investment decisions. Some of these measures include standard deviation, which indicates the volatility of a portfolio by measuring dispersion of returns around the mean; beta, which assesses an investment's sensitivity to overall market changes, with a beta greater than 1 indicating higher volatility than the market; drawdown, which measures the decline in portfolio value from its peak to it's lowest point, useful for those concerned with capital loss; the Sharpe ratio, which evaluates a portfolio's excess return over the risk-free rate relative to its volatility; (insights, 2023) and Value at Risk, which shall be the focus of this study.

Once risks are measured, investors can implement strategies like diversification, asset allocation (distributing a portfolio across different asset classes), and hedging (using financial instruments such as options or futures and forwards contracts). These strategies help mitigate potential losses and allow investors to maintain a portfolio aligned with their objectives and risk tolerance (Tamplin, 2023).

1.2 Problem Statement

Effective risk management is at the centre of successful portfolio management, guiding the decisions that ultimately shape investment outcomes and market stability. Among the tools available for this is Value at Risk (VaR). VaR models stand out for their ability to estimate the potential losses a portfolio might face within a certain time frame and at a given confidence level (team, n.d.). These models have become popular because they provide a clear, probabilistic measure of risk, which helps portfolio managers navigate the uncertainties of the financial markets. However, the effectiveness of these VaR models is still a matter of debate among experts (George E & Apostolos S, 2019). The reason for the ongoing discussion lies in the differences between the various approaches to estimating VaR, such as Historical Simulations, the Variance-Covariance approach, and the Monte Carlo simulations, and the unique challenges each method presents (Pritsker, 2006)

Each of these methods brings something different to the table. For instance, Historical Simulations is often valued for its straightforward data-driven approach but has limitations because it relies on past data that might not predict future risks accurately (Pritsker, 2006). The Variance-Covariance approach assumes a normal distribution which can lead to an underestimation of the risk during turbulent market periods. Meanwhile, Monte Carlo Simulations offer he flexibility to model more complex scenarios, but they can be computationally intense and require careful setup to be effective (Dowd, 2005)

In this study, the aim is to shed light on this debate by taking a close look at how these three widely used VaR models perform when applied to diversified investment portfolios. The theoretical underpinnings of each model will be explored as well as the methodologies, and pros and cons, aiming to understand how well they predict portfolio losses in different market conditions. The accuracy and reliability of these models will be determined using back-testing techniques. The insights gained from this study valuable for portfolio managers and risk analysts, helping them choose the most appropriate model for managing the risk associated with diversified portfolios. In a world where financial markets are more complex and interconnected, it is essential to have the right tools to manage risk. (Jorion, 2007; Alexander, 2008)

1.3 Research Objectives

This study aims to achieve several key objectives:

1. To review the various approaches and methodologies for determining Value at Risk (VaR), understanding their underlying assumptions, methodologies, merit and demerits.
2. To determine if VaR is a suitable tool for managing risk in managing risks in diversified portfolios.
3. To offer insights and recommendations to portfolio managers regarding the application of the VaR in portfolio risk management.

1.4 Research Questions

Several questions have been formulated to guide this study

1. How do different VaR calculation methods perform when applied diversified portfolios?
2. Do the assumptions underlying VaR estimation methods hold true for diversified portfolios?
3. How do the correlations and diversification benefits among assets influence the accuracy of VaR estimates for diversified portfolios?
4. Can VaR be used as a single risk metric for diversified portfolios or if there are limitations to its applicability?

1.5 Significance of this Research

This research provides valuable insights into the application of Value at Risk (VaR) as a risk management tool for diversified portfolios. By examining the performance of various VaR models, namely Historical Simulation, Variance-Covariance, and Monte-Carlo Simulation, the study will shed light on their strengths and limitations when applied to portfolios containing multiple asset classes.

A deeper understanding of these models will enhance their practical applicability. This will allow portfolio managers to make more informed decisions when assessing potential losses under varying market conditions. The findings of this research could also contribute to the development of more robust and efficient risk management strategies tailored to diversified portfolios, ultimately improving their resilience to adverse market conditions.

Additionally, the study's insights could inspire future research on alternative risk measures or model enhancements, advancing the field of financial risk management and equip portfolio managers with actionable recommendations on selecting and implementing VaR models based on portfolio characteristics and market environments.

CHAPTER TWO: LITERATURE REVIEW

Introduction

Value at Risk (VaR) is a crucial tool in finance, particularly in risk management. It offers a quantifiable estimate of the potential losses a portfolio might incur. This chapter will look into the development of VaR, review the various models used to estimate it, and identify gaps in existing research regarding their effectiveness in managing portfolio risk.

2.1 THEORETICAL LITERATURE REVIEW

2.1.1 History and Evolution of Value at Risk

Value at Risk (VaR) gained widespread use for measuring market risk in trading portfolios during the 1990s, but its roots date back to 1922, when the New York Stock Exchange implemented capital requirements for member firms. VaR is also linked to portfolio theory, with an early version appearing in 1945.

The beginning of portfolio theory is traced back to non-mathematical discussions on portfolio construction, with authors Hardy (1923) and Hicks (1935) emphasizing the benefits of diversification. In 1945, Leavens provided a quantitative example, potentially introducing the first VaR measure. He analysed a portfolio of ten bonds over a specific period, where each bond had a 50/50 chance of either maturity at USD 1,000 or become worthless, leading to a binomial distribution of the portfolio's value. Although Leavens didn't explicitly identify a VaR metric, he referred to the "spread between probable losses and gain" suggesting his consideration of the portfolio's market value standard deviation.

In 1952, Markowitz and Roy independently introduced similar VaR measures for optimizing portfolio reward against risk, incorporating covariances between risk factors for hedging and diversification. Markowitz used variance for simple return metric, while Roy employed a shortfall risk metric to represent the probability that a portfolio's return would fall below a specified catastrophic level. However, neither addressed how to specify probabilistic assumptions directly. Roy's approach required a mean vector and covariance matrix for risk factors estimated from past data, whereas Markowitz's method only needed a covariance matrix for risk factors. Markowitz proposed a Bayesian approach for this. Both measures aimed to optimize portfolios in practice.

Markowitz's 1959 book explained his approach to a broader audience, but computational limitations persisted until the 1970s. To address this, he suggested a simpler VaR measure with a diagonal covariance matrix, which was further described by William Sharpe in his Ph.D. thesis and later in his 1963 paper, motivating Shaper's (1964) Capital Asset Pricing Model (CAPM).

In the early stages, VaR measures were mostly theoretical and discussed within the framework of emerging portfolio theory. Works by Tobin (1958), Treynor (1961), Litner (1965), and Mossin (1966) laid the foundation for CAPM, which is closely related to VaR through concepts of risk and return, portfolio theory, market efficiency, risk metrics, and risk management. These early measures were best suited for equity portfolios since alternative assets, such as real estate, posed significant modelling challenges. The infrequent marking-to-market of real estate made VaR impractical, while debt instruments and futures contracts required complex modelling of term structures and credit spreads, with early measures by Schrock (1971) and Dusak (1972) inadequately addressing these challenges.

Lietaer (1971) described a practical VaR measure for foreign exchange risk during the decline of fixed exchange rates. Corporations, facing frequent currency devaluations and government secrecy, needed continuous hedging. Lietaer proposed an advanced method for optimizing hedges using a VaR measure that assumed normally distributed devaluations, simplified through a modification of Sharpe's (1963) measure. This work possibly marked the first use of the Monte Carlo method in a VaR measure.

As market volatility increased in the 1970s and 1980s, firms became leveraged, amplifying the need for financial risk measures like VaR. Despite the growing resources for implementing VaR, it remained primarily a theoretical tool within portfolio theory. Firms required a method to measure market risk across various asset categories but had yet to recognize VaR's potential. During this time, U.S. regulators began laying the groundwork for the broader adoption of VaR.

Kenneth Garbade, working in the Bankers Trust Cross Market Research group during the 1980s developed advanced VaR measures for assessing internal capital requirements in the U.S debt market. His 1986 work modelled each bond's price sensitivity to yield changes, known as the 'value at basis point'. Assuming normally distributed portfolio market values and using a covariance matrix for yields at various maturities, he calculated VaR metrics, including the standard deviation of loss and the .99-quantile of loss. Garbade extended his work in 1987 by introducing a bucketing scheme for mapping large bond portfolios to smaller representative sets and techniques to disaggregate and allocate portfolios risk across multiple profit centres.

In the late 1980s, JP Morgan developed a firm-wide VaR system that modelled several hundred risk factors, updating a covariance matrix quarterly from historical data. Each day, trading units reported their positions delta through email, which were aggregated to express the portfolio's value as a linear polynomial of the risk factors. The standard deviation of portfolio value was then calculated using various VaR metrics, including a one-day 95% USD VaR based on the assumption of normally distributed portfolio value. This system allowed JP Morgan to replace its complex system of notional market risk limits with a simpler VaR limit system. By 1990, VaR numbers were integrated with P&L reports for daily 4:15 pm Treasury meetings in New York, which were then forwarded to Chairman Weatherstone.

Till Guldimann, a key figure in the development and promotion of JP Morgan's VaR system, played a significant role in its internal adoption. With a background in asset/liability analysis and later as chairman of the firm's market risk committee, Guldimann drove the firm's risk management efforts. In 1990, he took responsibility for global research, managing activities to support marketing to institutional clients. Guldimann organized an annual research conference, and in 1993, with risk management as the theme, he gave the keynote address and demonstrated JP Morgan's VaR system. This demonstration sparked significant interest, with clients inquiring about purchasing or leasing the system. As JP Morgan was not a software vendor, they decided to provide clients with the means to generate their own systems. Guldimann proposed publishing a methodology, distributing the necessary covariance matrix, and encouraging software vendors to develop compatible software.

In 1993, Thomas Wilson, working as a project manager for McKinsey & Co., published a sophisticated VaR measure aimed at incorporating stochastic covariance matrices into risk capital calculations using simple assumptions. Wilson suggested using the t-distribution, with fatter tails than the normal distribution, to reflect limited prior information about the covariance matrix as a random variable. Wilson's work was the first published attempt to address leptokurtosis and heteroskedasticity in practical VaR measures used on trading floors since Garbade's 1987 work. His casual assumption that readers were familiar with VaR measures indicated their widespread adoption by that time. Wilson also touched on a philosophical issue by suggesting that the covariance matrix for risk factors might exist but be unknown to the user, challenging Markowitz's subjective approach that the covariance matrix is constructed based on the user's perceptions.

JP Morgan formed a small team to develop the RiskMetrics service, which included a detailed technical document and a covariance matrix for several hundred key factors, updated daily and distributed for free over the internet. Launched with significant fanfare in October 1994, RiskMetrics benefited from a public relations campaign and a multi-city promotional tour by JP Morgan representatives. The timing was perfect, amid global concerns about derivatives and leverage. While RiskMetrics was not a technical breakthrough, its linear VaR measure was arguably less sophisticated than those by Garbade (1986) or Wilson (1993), but its key contribution was in publicizing VaR to a wide audience, significantly raising awareness and adoption across the financial industry.

The Basel Committee on Banking Supervision recommended VaR models as regulatory market risk capital requirements in 1996. In 1997, the U.S. Securities and Exchange Commission required public corporations to disclose quantitative information about their derivatives activity, with major banks and dealers choosing to include VaR information in the notes to their financial statements. By the late 1990s and early 2000s, VaR had become a standard risk management tool, with various methodologies, including historical simulation, variance-covariance approaches, and Monte Carlo simulations, developed by both academics and practitioners.

2.1.2 Measuring Value at Risk

In this section, we outline and contrast the fundamental methods for calculating Value at Risk (VaR). There are three main approaches, each with its variations. VaR can be computed analytically by assuming specific return distributions for market risks and considering the variances and covariances among these risks. Alternatively, it can be estimated by applying historical data to hypothetical portfolios or through Monte Carlo Simulations.

2.1.2.1 Variance-Covariance Method

This approach simplifies the Value at Risk (VaR) calculation by creating a probability distribution of possible asset values. While this method is straightforward, accurately determining probability distributions can be challenging (Damodaran).

Overview

For a single asset with normally distributed potential values, calculating VaR is relatively easy. However, for portfolios, it becomes more complicated due to the interdependencies between assets. The variance of a portfolio depends on the covariances between asset pairs, making it difficult to calculate for large portfolios with changing positions. To simplify this, the risk associated with individual investments is linked to general market risks, allowing VaR to be estimated based on these exposures (Damodaran, A. (n.d.)).

This process generally involves four steps. The first step involves mapping assets to standardized instruments. Each portfolio asset is mapped to simpler, standardized instruments. For instance, a ten-year coupon bond can be divided into ten zero-coupon bonds with corresponding cash flows. This process is more complex for assets like stocks and options but follows the same principle. Mapping reduces the number of variances and covariances to estimate, focusing only on the common market risk instruments that these assets are exposed to. The next step is stating positions in standardized instruments. Each asset is represented as a set of positions in these standardized market instruments. While this is straightforward for simple assets like a bond, it becomes more complicated for complex assets such as convertible bonds, stocks, or derivatives (Damodaran, A. (n.d.)).

The third step involves estimating variances and covariances. The variances of each standardized instrument and the covariances among them are estimated using historical data. When calculating VaR these estimates are crucial. The last step is calculating Value at Risk. The VaR for the portfolio is determined using the weights of the standardized instruments from step 2 and the variances and covariances from step 3. This step combines the mapped positions with the statistical estimates to calculate the portfolio's risk (Damodaran, A. (n.d.)).

Assumptions in VaR Calculation

The calculation of VaR in step 4 relies on assumptions on the distribution of returns on standardized risk measures. The most convenient assumption is that returns follow a normal distribution, as this simplifies both the calculation and the estimation of probabilities. If each market risk factor is assumed to have returns that are normally distributed, then the returns on any portfolio exposed to multiple risk factors will also be normally distributed. Even Value at Risk methods that allow for non-normal return distributions for individual risk factors often ensure that the final portfolio values follow a normal distribution (Damodaran).

Evaluation

The main advantage of the Variance-Covariance method is its simplicity. Once the return distribution is assumed and the means, variances, and covariances are determined, calculating VaR is straightforward.

However, there are three main weaknesses in this estimation process: The first is **the** Distributional Assumption. If actual returns do not follow a normal distribution, the calculated VaR will underestimate the true risk. Specifically, if the actual return distribution has more outliers than a normal distribution would suggest, the true VaR will be higher than the calculated VaR (Duffie, D. and J. Pan, 1997). Next is occurrence of input error. Even if the normal distribution assumption is correct, the VaR may still be inaccurate if the variances and covariances used are incorrect. These are estimated from historical data and come with standard errors, meaning that the variance-covariance matrix used in the VaR calculation is based on estimates, some of which may be significantly off (Duffie, D. and J. Pan, 1997). Finally, the presence non-stationary variables. Another issue arises when the variances and covariances of assets change over time. This non-stationarity is common, as the underlying factors driving these numbers evolve. For example, the correlation between the U.S. dollar and the Japanese yen might shift if oil prices increase by 15%. Such changes can lead to inaccuracies in the calculated VaR (Damodaran, A. (n.d.))

Addressing Variance-Covariance Method Criticisms

Recent efforts to improve the Variance-Covariance method have focused on addressing its criticisms. Researchers have explored alternative ways to calculate VaR without assuming a normal distribution.

For instance, Hull and White (1998) propose methods to estimate VaR with non-normal distributions, allowing users to specify any probability distribution for variables, as long as transformations of these distributions fall into a multivariate normal distribution. However, these models face practical challenges, as inputs estimation for non-normal models is complex, especially with historical data. Additionally, calculating the probabilities of losses and VaR becomes more difficult with asymmetric and fat-tailed distributions compared to the normal distribution.

Enhanced Estimation Techniques

Other research efforts aim to improve estimation techniques for more accurate variance and covariance values in VaR calculation. Some suggest refinements in sampling methods and data innovations to improve future variance and covariance estimates. Others, like Engle (2001), propose statistical innovations, such as using models that account for changing standard deviations over time (heteroskedasticity) instead of assuming constant standard deviation (homoskedasticity). Engle's Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models offer better variance forecasts and, consequently, more accurate VaR measures (Damodaran).

Dealing with Non-Linear Instruments

A significant limitation of the Variance-Covariance method is its design for portfolios with a linear relationship between positions and risk. This approach fails with options, which have non-linear payoffs. To address this, researchers have developed Quadratic Value at Risk measures, also known as delta-gamma models, to estimate VaR for portfolios that include options and option-like securities, like convertible bonds. Although these quadratic measures are more complex mathematically and can lose some intuitive appeal, they provide a more accurate VaR estimation for portfolios with non-linear instruments.

2.1.2.2 Historical Simulations Methods

Historical simulations offer a straightforward method for estimating Value at Risk (VaR) for different portfolios. This technique involves generating a hypothetical time series of portfolio returns by applying actual historical data to simulate changes that would have occurred in each period (Damodaran).

Method Overview

Carrying out a historical simulation involves, gathering time series data on each market risk factor, similar to the variance-covariance method. However, instead of using this data to forecast future variances and covariances, historical simulations rely on the portfolio's historical changes to derive the necessary information to compute VaR.

Assumptions of Historical Simulation

The Historical simulations method lacks distributional assumptions. This method does not depend on any assumed return distributions. Instead, VaR is purely determined by actual historical price movements without the assumption of normality. Moreover, each day in the historical time series is given equal importance. This can be problematic if there is a variability trend, such as lower variability in earlier periods and higher variability in later periods. Finally, the method assumes that the historical period used adequately reflects the risks faced by the market in other periods, implying that historical patterns will recur.

Assessment of Historical Simulation

Historical simulations are favoured for their simplicity but have inherent weaknesses due to their underlying assumptions. Such include, dependence on Historical Data **as** Historical simulations rely heavily on past data. Unlike the Variance-Covariance approach, there is no allowance for distributional assumptions or subjective information, as is possible in Monte Carlo simulations.

Data Trends is another weakness**.** By equally weighting all data points, historical simulations may understate VaR if volatility increases during the historical period. For example, price changes from different years impact VaR equally, regardless of changing volatility trends. Additionally, new assets or market risks pose a challenge since Historical simulations struggle to handle new assets and risks because of a lack of historical data to compute VaR. This issue affects all VaR estimation methods but is particularly pronounced in historical simulations (Damodaran).

While historical simulations avoid the need for specific distributional assumptions about returns, they implicitly assume that past return distributions will be representative of future returns. This assumption is challenging to maintain in volatile markets with frequent structural shifts, making the approach less reliable in such environments.

Modifications to Historical Simulation for VaR

Some of the modifications to improve historical simulations include Recent Data Weighting: This modification to historical simulations involves assigning greater weight to more recent data, as returns in the recent past are often better predictors of near-future performance. Boudoukh, Richardson, and Whitelaw (1998) suggest a variant where recent data is given more weight using a decay factor. In this method, each return is given a probability weight based on its recency, allowing VaR to quickly reflect recent market events.

Another modification could be combining with Time Series Models: Cabedo and Moya (2003) propose enhancing historical simulation by integrating it with time series models. By fitting a time series model, like an autoregressive moving average (ARMA) model, to historical data, the parameters of this model can be used to forecast VaR. This approach blends historical trends with statistical forecasting to improve VaR estimation.

In addition to those, is volatility adjustment. Hull and White (1998) suggest updating historical data to account for changes in volatility. For assets where recent volatility is higher than historical volatility, they recommend adjusting the historical data to reflect this change. This modification ensures that the VaR calculation considers recent market conditions, providing a more accurate risk assessment (Damodaran).

These modifications address some of the criticisms of historical simulations, improving their accuracy and responsiveness to current market conditions.

2.1.2.3 Monte Carlo Simulation for VaR

Overview

Monte Carlo simulations for Value at Risk (VaR) share initial steps with the Variance-Covariance approach. Both methods start by identifying the market risks affecting the assets in a portfolio and converting individual assets into positions in standardized instruments. The key difference arises in the third step: rather than calculating variances for market risk factors, Monte Carlo simulations define probability distributions for each market risk factor and their interactions.

While assuming normal distributions simplifies parameter estimation, Monte Carlo simulations' real strength lies in their flexibility to choose different distributions and incorporate subjective judgments. Once the distributions are set, the simulation process begins. Market risk variables take on various outcomes in each run, reflecting the portfolio's value. After thousands of runs, a distribution of portfolio values is formed, which is then used to assess VaR. For instance, in 10,000 simulations, the 95th percentile VaR corresponds to the 500th lowest value, and the 99th percentile to the 100th lowest value (Damodaran).

Evaluation

Monte Carlo simulations are considered more advanced than historical simulations, though numerous users still rely on historical data for their distributional assumptions. As the number of market risk factors increases and their interactions become more complex, Monte Carlo simulations become more challenging to execute. Estimating the probability distributions for numerous market risk variables and running many simulations (tens of thousands) are necessary for accurate VaR estimates.

Compared to the Variance-Covariance approach, Monte Carlo simulations do not require unrealistic assumptions about normality in returns. Unlike historical simulations, they incorporate historical data while allowing for subjective judgments and additional information to refine forecasted distributions. Monte Carlo simulations are versatile enough to assess VaR for any portfolio, including those with options and option-like securities.

Modifications

Modifications to Monte Carlo simulations aim to address their computational intensity. These variations focus on narrowing the scope, using different techniques, and reducing the number of simulations required.

The first modification would be scenario simulation**.** A way to reduce the computational burden is to analyse several discrete scenarios. Frye (1997) suggests applying pre-specified shocks to the system, while Jamshidan and Zhu (1997) propose using principal component analysis to reduce the number of factors. Instead of allowing each risk variable to assume all possible values, they consider likely combinations to create scenarios, calculating values across these scenarios to obtain simulation results.

Monte Carlo Simulations with Variance-Covariance Modification. This modification combines the speed of the Variance - Covariance method with the flexibility of Monte Carlo simulations. Glasserman, Heidelberger, and Shahabuddin (2000) demonstrated that using approximations from the Variance-Covariance approach to guide the sampling process in Monte Carlo simulations, can achieve substantial savings in time and resources while maintaining a high level of accuracy.

The trade-off with these modifications is clear: some of the power and precision of the Monte Carlo approach are sacrificed in exchange for gains in estimation efficiency and reduced computational time (Damodaran).

Comparing Approaches

The Variance-Covariance approach, including its delta-gamma and delta- normal variations, depends on strong assumptions relating to the return distributions of standardized assets but is easy to compute when these assumptions are established. The Historical Simulation approach on the other hand, does not require such assumptions about return distributions but implicitly assumes that the historical data used is a representation of the future. The Monte Carlo Simulation approach offers the most flexibility when selecting return distributions and incorporating subjective judgments and external data. However, it is the most computationally demanding method.

2.2 EMPIRICAL LITERATURE REVIEW

The existing empirical literature on Value at Risk (VaR) has primarily focused on evaluating the performance of VaR models for specific asset classes or limited combinations of assets, rather than examining VaR's suitability for truly diversified portfolios. This has led to a gap in research, as portfolio managers and investors need reliable risk assessment tools that can effectively capture the diversification benefits and unique characteristics of multi-asset portfolios.

A significant portion of the VaR literature has concentrated on equity portfolios. Giot and Laurent (2003) examined the performance of historical simulations and GARCH-based VaR models for a portfolio of European stocks. They found that the GARCH-based approach outperformed the historical simulation method. They attributed this outperformance to the GARCH model's ability to capture time-varying volatility and leverage effects in equity returns. Similarly, Christoffersen and Jacobs (2004) evaluated various VaR models for equity portfolios in the United States. They concluded that models accounting for time-varying volatility and fat-tailed return distributions, provided more accurate VaR estimates.

Researchers have also studied the application of VaR to fixed-income portfolios. Jorion (2001) investigated the use of VaR for bond portfolios, highlighting the importance of incorporating yield curve dynamics and interest rate risk factors into the VaR calculation. Jarrow and Rudd (1982) proposed an approximate VaR evaluation method for bond portfolios, which they found to be more computationally efficient than Monte Carlo simulation.

Beyond equities and fixed-income securities, some studies have explored the application of VaR for commodity and derivative portfolios. Glasserman (2004) examined the performance of VaR models for portfolios containing options and other derivative instruments, emphasizing the need to account for non-linear payoff structures of these assets. He found that traditional VaR models often underestimate the risk of derivative-heavy portfolios, particularly during periods of high volatility. Zhang and Lin (2019) compared the VaR to Markov Regime-

Switching (MRS) models for commodity portfolios. They found that the MRS model outperforms VaR during periods of high volatility.

While these studies provide valuable into the performance of VaR models for specific assets and asset classes, they do not adequately address the use of VaR for diversified portfolio that include a wide range of asset types.

Perignon and Smith (2010) noted that VaR models may not accurately capture the diversification benefits of holding a mix of assets, as they tend to focus on the individual risk characteristics of each asset rather than the portfolio. They found that VaR estimates for diversified portfolios were often higher than the sum of the VaR estimates for individual assets, suggesting that VaR models may not fully account for the risk reduction benefits of diversification.

2.3 Gap in the Literature

The existing literature's focus on asset-specific VaR models highlights a significant gap in the research. There is a lack of comprehensive studies that evaluate the suitability of VaR to truly diversified portfolios, which is often composed of stocks, bonds, real estate, commodities, and other alternative investments. This gap is crucial, as portfolio managers and investors need reliable risk assessment tools that can effectively capture the diversification benefits and unique characteristics of multi asset portfolios. Without a thorough understanding of how VaR models perform for diversified portfolios, it is difficult to make informed decisions about risk management and asset allocation.

2.4 Conceptualization of the study

The study aims to address the research gap above by conducting a comprehensive evaluation of the VaR models and their suitability for diversified portfolios. The analysis will involve predicting the portfolio losses for diversified portfolios with varying asset class composition using the theoretical foundations, and methodologies of the VaR models already discussed in this chapter. The accuracy of the VaR models will be assessed using back-testing techniques, and the results will provide useful insight into the effectiveness of the models in portfolio risk prediction and in extension, management.

2.5 Summary of the chapter

The literature review has outlined the historical development and theoretical foundations of VaR and VaR models, highlighting their methodologies, strengths and weaknesses. By identifying the research gap and conceptualizing the study, this chapter sets the stage for the empirical analysis that follows. The subsequent chapters will build on this foundation, focusing on the application and evaluation of VaR models in managing portfolio risk.

CHAPTER THREE: METHODOLOGY

3.1 Research Design

The study will adopt a quantitative research approach to evaluate the effectiveness of different Value at Risk (VaR) models in managing risk for a diversified portfolio. The study aims to provide a comparative analysis of some of these models, assessing their performance across various asset classes and under different market conditions. By employing historical market data and robust statistical techniques, the research seeks to fill gaps in the existing literature and offer practical insights for portfolio managers.

3.2 Data

To ensure comprehensive coverage and relevance, the study will utilize historical price data from Yahoo Finance. The study will cover a significant time span from 2005 to 2023. The period captures different market conditions and includes major financial events such as the 2008 financial crisis and the Covid-19 pandemic. This period provides a comprehensive dataset that includes both stable and volatile market phases. The data will encompass equities, fixed income securities, real estate, commodities and alternatives.

The asset classes will be represented by Exchange-Traded Funds (ETFs) which are investment funds that hold a collection of securities. These include stocks, bonds, or commodities, and are traded on stock exchanges like individual stocks. ETFs are characterized by their liquidity, and diversification in that they can be bought and sold throughout the trading day at market prices, allowing greater flexibility compared to mutual funds and they provide investors with exposure to broad range of securities which helps mitigate risk associated with investing in individual stocks or bonds. In addition to this, ETFs mirror the performance of their respective asset class or sector hence their use.

3.3 Portfolio construction

Asset Class	**ETF**	**Weight**
Equities	Vanguard Total Stock Market ETF (VTI)	30%

Fixed Income	iShare Core U.S. Aggregate Bond ETF (AGG)	30%
Real Estate	Vanguard Real Estate ETF (VNQ)	15%
Commodities	SPDR Gold Shares (GLD)	15%
Alternatives	Invesco QQQ Trust (QQQ)	10%

Table 1: Exchange Traded Funds Portfolio

Rationale for Weighting

Equities (VTI) - 30%

A significant allocation to equities allows for potential growth, because stock generally recover well after market downturns. VTI (Vanguard Total Stock Market ETF) provides broad market exposure, capturing the performance of the entire U.S stock market, which is crucial for analysing the overall market response during crises.

Fixed Income (AGG) – 30%

A substantial allocation to bonds helps to stabilize the portfolio during periods of market volatility. Bonds typically perform well in risk-off environments, providing income and reducing overall portfolio risk. AGG's (iShare Core U.S. Aggregate Bond ETF) diversified bond exposure mitigates credit risk, making it suitable for risk averse investors.

Real Estate (VNQ) – 15%

Real estate can provide both income and potential appreciation, acting as a hedge against inflation. A moderate allocation to VNQ (Vanguard Real Estate ETF) allows for exposure to the real estate sector, which can behave differently than equities and bonds during economic downturns

Commodities (GLD) – 15%

Gold is frequently regarded as a safe-haven asset during periods of economic uncertainty. Including GLD (SPDR Gold Shares) in the portfolio offers protection against inflation and currency risk, enhancing the portfolio's resilience during crises.

Alternatives (QQQ) – 10%

Exposure to large-cap technology stocks through QQQ (Invesco QQQ Trust) allows the portfolio to benefit from growth trends, especially as technology companies often lead market recoveries. This allocation balances the conservative nature of the portfolio with the potential for higher returns.

This portfolio is suitable for analysis because of the following reasons: it offers a broad market coverage through the selected ETFs, which provide exposure to U.S. stocks, bonds, real estate, commodities, and tech stocks. This diverse representation captures a significant portion of the investable universe, making the portfolio an excellent candidate for a comprehensive risk analysis. The longevity of the data associated with these ETFs is another key reason. Most of them have historical data dating back to the early 2000s, allowing for performance analysis over an extended period, including critical events such as the 2008 financial crisis and the COVID-19 pandemic in 2020. Additionally, the liquidity and tradability of these ETFs make them suitable for this analysis. Being some of the largest and most liquid ETFs available, they have been actively traded even during periods of significant market stress, ensuring data reflects realistic trading conditions. Finally, the ETFs are known for having low tracking error, meaning they provide a good representation of their respective asset classes. (Fontinelle, 2024)

3.4 Value at Risk Implementation

3.4.1 Historical Simulation Model VaR

The objective is to calculate the VaR based on historical data without making any specific distribution for asset returns.

The Historical VaR is determined through several key steps. First, returns are calculated from the available data. These returns are then arranged in ascending order to create a distribution of returns, which is necessary for identifying the worst-case scenarios that align with confidence levels. In this analysis, different confidence levels will be considered. For instance, if a 95% confidence level is applied, then the focus is on the worst 5% of losses. The return corresponding to the confidence level is located on the 5th percentile from the sorted returns. The resulting figure represents the Historical VaR, indicating the maximum expected loss over the given period at the selected confidence level. (Institute, 2023)

3.4.2 Variance – Covariance Model

In this model, the objective is to calculate VaR assuming that returns are normally distributed. The standard deviation and mean of the returns are used.

The VaR is determined through the following steps: The returns for each asset in the portfolio are computed, then aggregated based on the portfolio weights to obtain the overall portfolio returns. Next, the mean return of the portfolio as well as the standard deviation, which measure volatility, are computed. A confidence level is then chosen, and the corresponding z-score is obtained from the standard normal distribution. VaR is calculated using the following equation:

$$\boldsymbol{VaR = \mu + Z_{\propto}\, \sigma}$$

Where: Z is the z-score corresponding to confidence level, μ is the mean of the portfolio and σ is the volatility of the portfolio

The result is the maximum expected lass over the specified period and chosen confidence level, under the assumption of normally distributed returns. (Institute, 2023)

3.4.3 Monte Carlo Method

The objective in this model is to calculate VaR using simulated returns, allowing for more complex modelling of return distributions and capturing tail risk.

The VaR is determined through the following steps: A model is chosen for simulating future returns. In this case, the Geometric Brownian Motion is applied and accounts for drift and volatility. The mean return and the volatility are estimated from the historical data. Using these estimates, many random paths are generated based on the Geometric Brownian Motion model which follows the following equation:

$$\boldsymbol{S_{t+1} = S_t * \ exp\ (\left(\mu - \frac{1}{2}\,\sigma^2\right)\Delta t + \ \sigma\sqrt{\Delta t} * Z}$$

Where: μ is the mean of the portfolio, σ is the volatility of the portfolio, Z is a random value drawn from a standard normal distribution and Δt is the change in time.

The portfolio's returns for each simulated path are calculated aggregating the simulated returns of all assets according to their weights in the portfolio. The ending value of the portfolio is then computed, and the losses are determined by comparing the ending portfolio values with the initial value. The simulated portfolio losses are ranked in ascending order, and the VaR is identified as the loss at the desired confidence level. (Institute, 2023)

3.5 Back-testing

This is the process of validating the accuracy of the VaR estimates by comparing them with the actual historical losses. The process of back testing begins by comparing the estimated VaR with the actual losses for each period in the historical data (Prep, 2023). During this comparison, the instances where the actual loss exceeds the VaR are identified. They are referred to as exceptions. Following this, statistical testing is employed to assess the validity of the VaR model. Statistical tests such as Kupiec's log-likelihood ratio (LR) test and Christoffersen's Conditional Coverage tests are commonly used. The LR test examines how many times the VaR of a financial institution is violated over a given period, while Christoffersen's test examines whether the exceptions are randomly distributed over time, thereby ensuring that the pattern of exceptions is not clustered or indicative of model bias. The Kupiec's (1995) test statistic is determined as follows:

$$LR = -2In\left(\frac{(1-p)^{T-N}p^{n}}{\left[1-\left(\frac{N}{T}\right)\right]^{T-N}\left(\frac{N}{T}\right)^{N}}\right)$$

Where p is the probability level, T is the sample size, N is the number of expectations, and LR is the test statistic for unconditional convergence where unconditional convergence implies that attention is not paid to the independence of exception observations. Kupiec's test involves calculating the ratio between the maximum likelihoods of a given result under two competing hypotheses. (Prep, 2023). The null hypothesis (that the model performs well) will be accepted or rejected based on the p-value obtained from the LR statistic which follows a chi-square distribution with one degree of freedom.

The interpretation is based on the p-value associated with the LR statistic. If the p-value is below the chosen significance level (0.05), the null hypothesis is rejected, concluding that the model may not be adequately accurate. If the p-value is above this level, we fail to reject the null hypothesis, suggesting that the model's performance is adequate for predicting portfolio risk at the given confidence level.

3.6 Summary

This comprehensive methodology outlines the approach to evaluating the effectiveness of different VaR models in managing risk for a diversified portfolio. By integrating historical data, detailed VaR model implementation, and thorough statistical analysis, the study aims to provide valuable insights on the performance of VaR models. The findings will offer practical guidance for portfolio managers, enhancing their ability to select the most appropriate VaR models for effective risk management in diverse and dynamic market environments.

CHAPTER 4: DATA ANALYSIS

4.1 Introduction

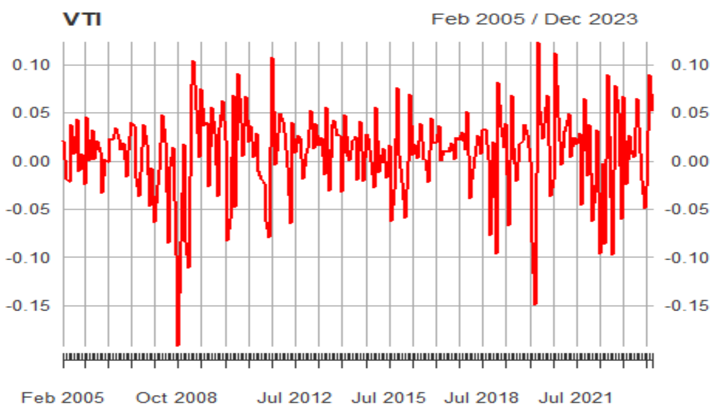
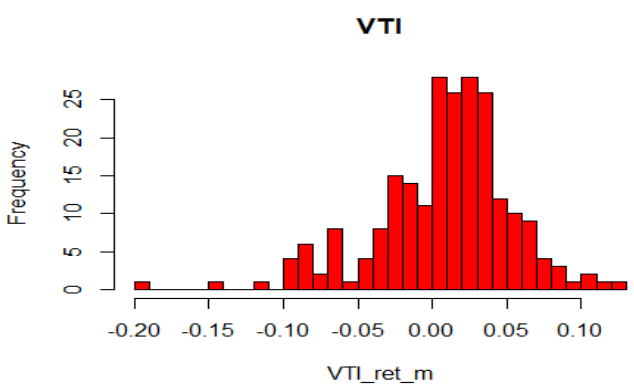
The data analysis was conducted in RStudio, beginning with data acquisition for each Exchange-Traded Fund (ETF). As previously stated, data was sourced from Yahoo Finance and spans from January 2005 to December 2023. After acquiring the data, the adjusted closing prices were isolated, converted to monthly data, and subsequently used to calculate monthly returns. Visualizations of the monthly returns for each ETF, including line graphs and histograms, were generated to illustrate the distribution of returns over the period.

Tests for normality, including Q-Q plots and the Shapiro-Wilk test, were conducted on the returns. Additionally, Q-Q plots were used to assess the possibility of a t-distribution fit. The results indicate that the normal distribution is a better fit than the t-distribution, particularly for the Variance-Covariance method and Monte Carlo Simulations Methods. Following these preparatory steps, the Value at Risk (VaR) for the portfolio was calculated using the three methods: the Variance-Covariance method, the Historical Simulations method, and the Monte Carlo Simulations method. Confidence levels of 90%,95%, and 99% were adopted, with the Variance Covariance method and Monte Carlo Simulations method assuming returns follow a normal distribution.

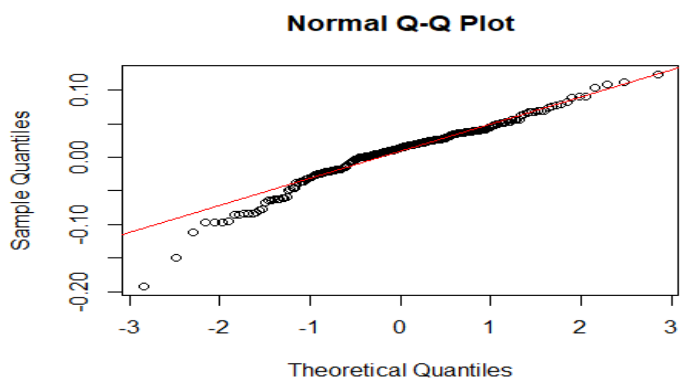
Finally, back-testing was conducted for each model to evaluate predictive accuracy. This included identifying the number of VaR breaches and calculating the Log-Likelihood Ratio Test Statistic for the Kupiec test.

4.2 Data Visualization

VTI

Monthly Returns Line Graph	Histogram
	

Q – Q Plot (Normal Distribution)



Q – Q Plot (t - Distribution)

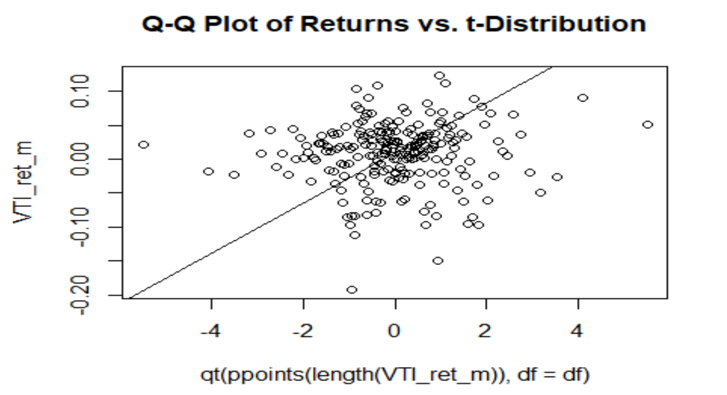
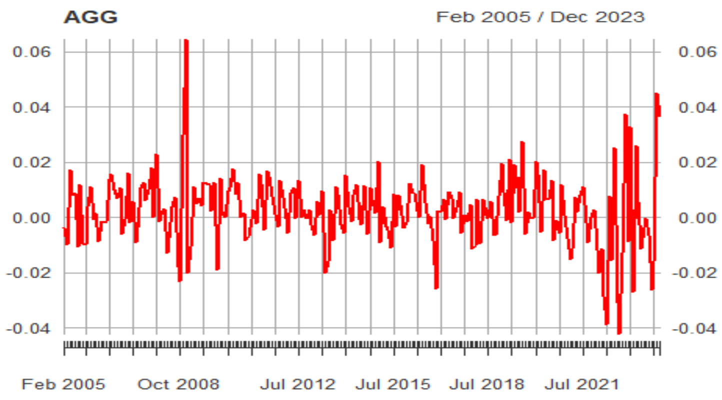


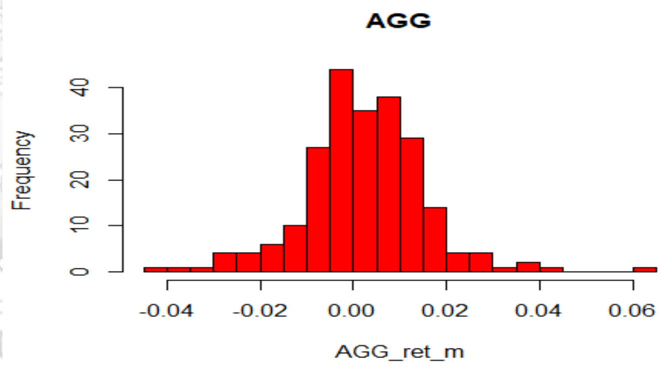
Table 2: Visualization of VTI Data

AGG

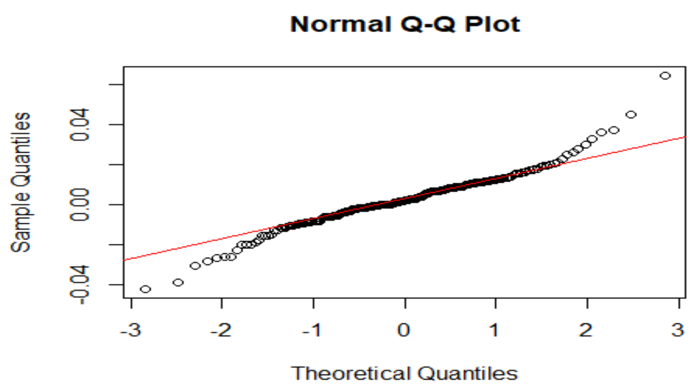
Monthly Returns Line Graph



Histogram



Q – Q Plot (Normal Distribution)



Q – Q Plot (t - Distribution)

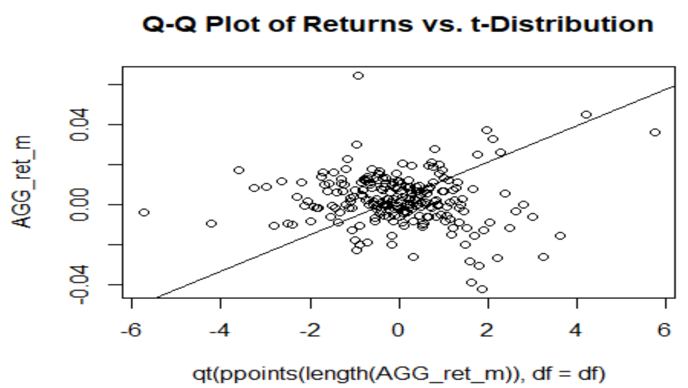


Table 3: Visualization of AGG Data

<u>**VNQ**</u>

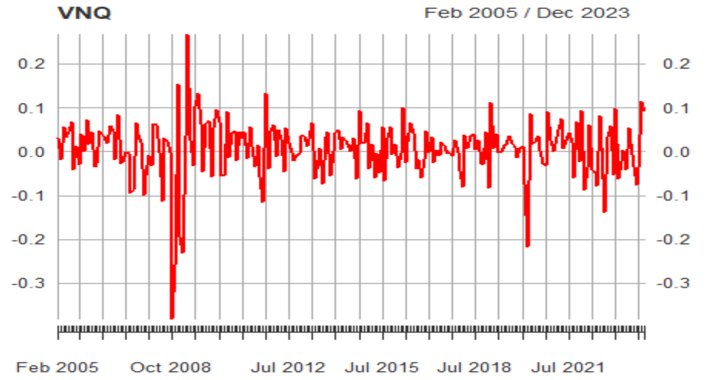
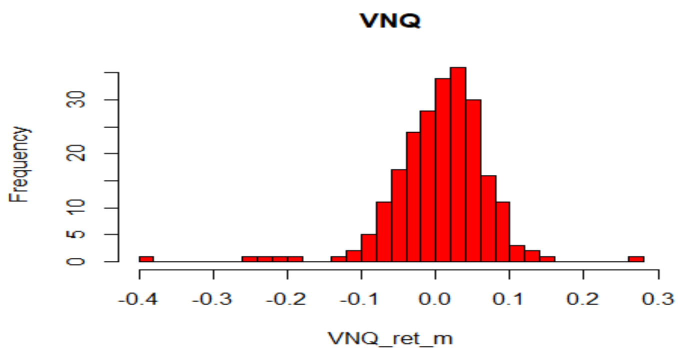
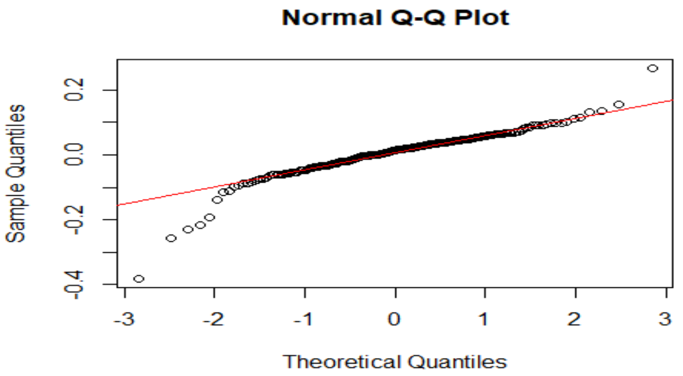
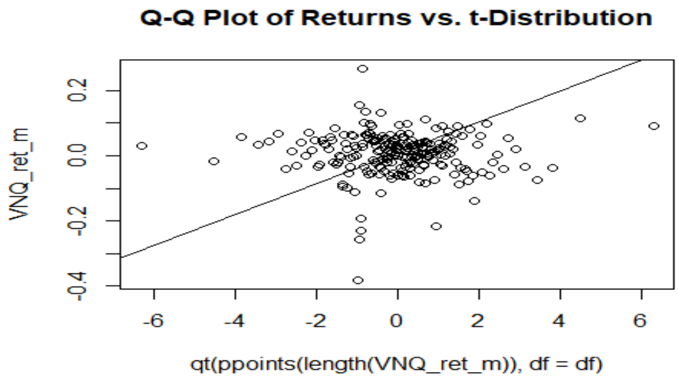
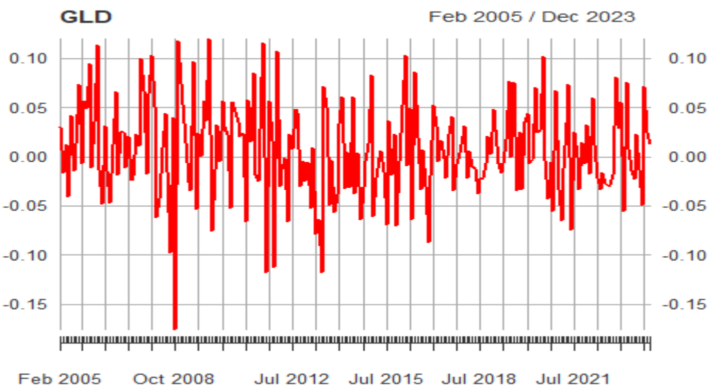
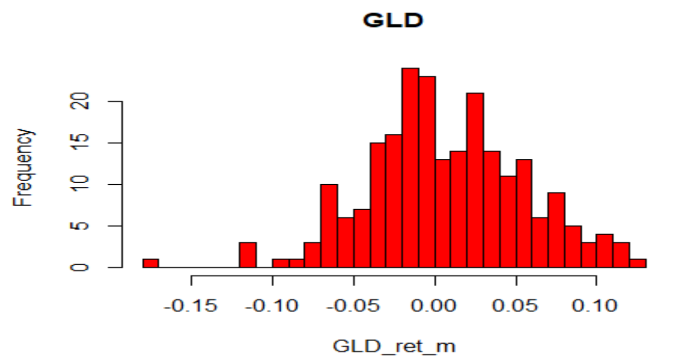
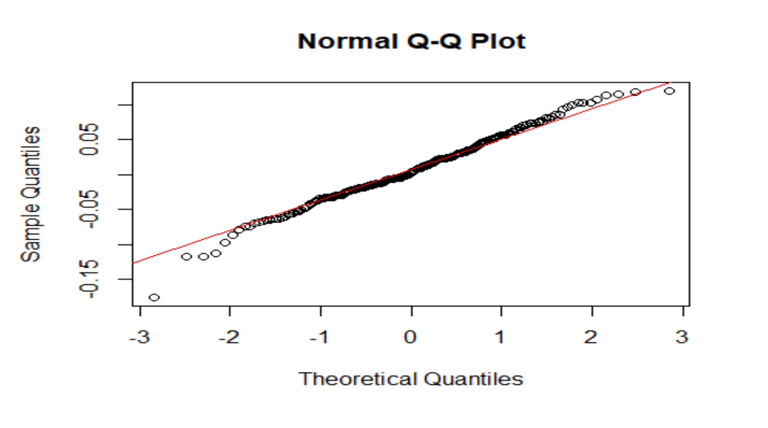
<table>
<tr><td>Monthly Returns Line Graph</td><td>Histogram</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Q – Q Plot (Normal Distribution)</td><td>Q – Q Plot (t - Distribution)</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Table 4: Visualizations of VNQ Data

<u>**GLD**</u>

<table>
<tr><td>Monthly Returns Line Graph</td><td>Histogram</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Q – Q Plot (Normal Distribution)</td><td>Q – Q Plot (t - Distribution)</td></tr>
</table>



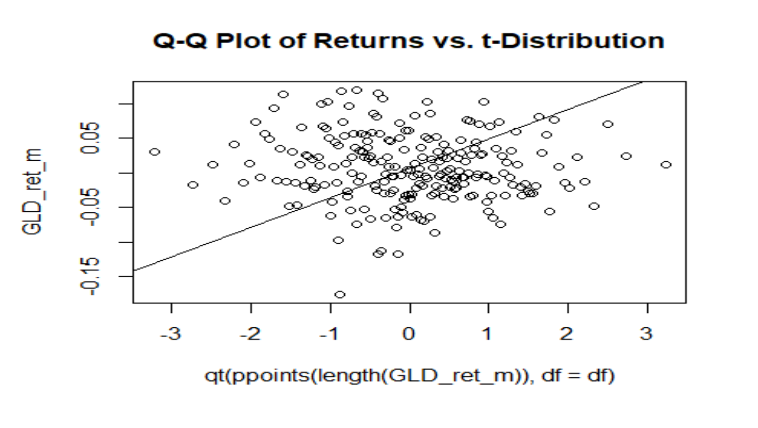


Table 5: Visualization of GLD Data

QQQ

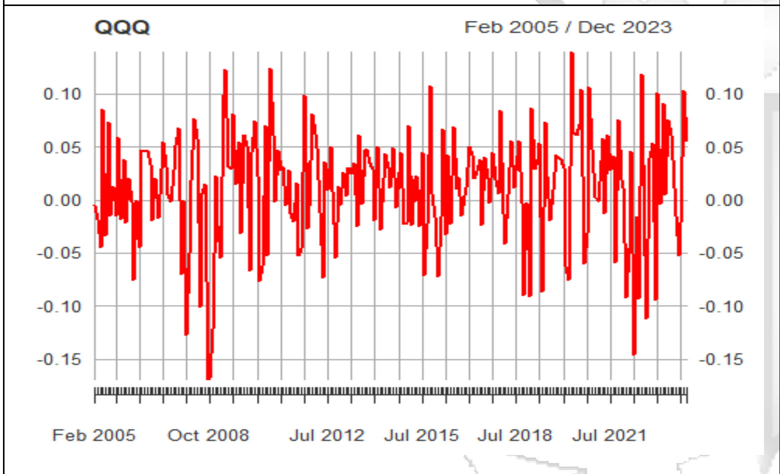
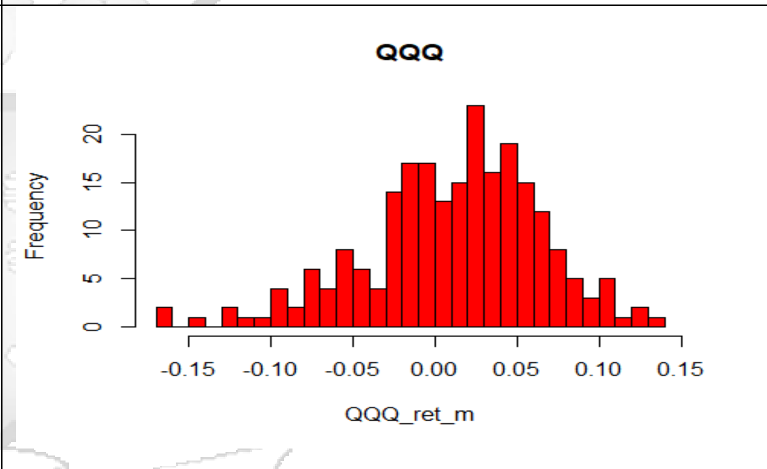
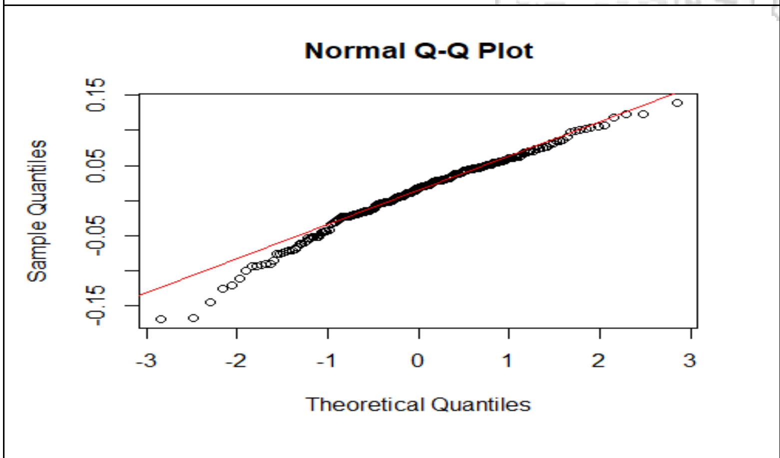
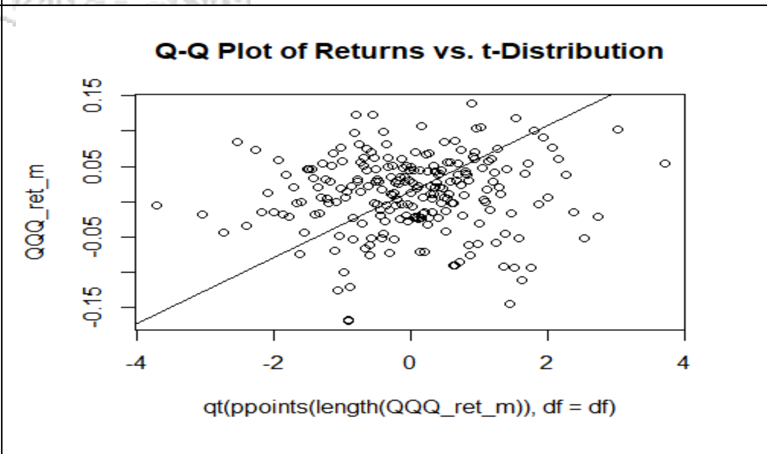
Monthly Returns Line Graph	Histogram
	
Q – Q Plot (Normal Distribution)	**Q – Q Plot (t - Distribution)**
	

Table 6: Visualization of QQQ Data

4.3 Descriptive statistics

4.3.1 Exchange Traded Funds (ETFs)

ETF	Mean Return	Standard Deviation	Skewness	Kurtosis
VTI	0.0078 (0.78%)	0.0457(4.57%)	-0.791	4.746
AGG	0.0024(0.24%)	0.0129(1.29%)	0.282	6.201
VNQ	0.0060(0.60%)	0.0662(6.62%)	-1.316	10.047
GLD	0.0067(0.67%)	0.0486(4.86%)	-0.115	3.426
QQQ	0.0112(1.12%)	0.0537(5.37%)	-0.583	3.647

Table 7: Exchange Traded Funds Descriptive Statistics

VTI

The mean return is positive, indicating an average gain over time. The standard deviation indicates a moderate volatility in returns. (Hargrave, 2024) The negative skewness indicates a tendency towards small gains with a risk of significant losses. Kurtosis greater than 3 suggests high probability of extreme returns, indicating potential for both significant gains and losses. (suvarna, 2024)

AGG

The mean return is positive but low indicating modest gains. The standard deviation implies low volatility thus stable returns. (Hargrave, 2024) The positive skewness indicates potential for larger gains with fewer losses. The high kurtosis suggests a higher likelihood of extreme outcomes, indicating some risk but generally stable performance. (suvarna, 2024)

VNQ

The mean return is also positive indicating an average gain over time. The standard deviation indicates high volatility and therefore significant fluctuations in returns. (Hargrave, 2024) Strong negative skewness indicates frequent small gains but significant loss risks. The very high kurtosis indicates a very high probability of extreme outcomes, making this ETF particularly risky and volatile. (suvarna, 2024)

GLD

The positive mean return indicates an average gain over time but lower than others listed in the portfolio. The standard deviation implies moderate returns, suggesting some fluctuations but generally stable performance compared to VNQ and VTI. (Hargrave, 2024) A slightly negative skewness indicates a nearly symmetrical distribution with a tendency for small losses. A kurtosis close to 3 suggests a distribution close to normality, indicating relatively stable returns with fewer extreme outcomes than others listed in the portfolio. (suvarna, 2024)

QQQ

This ETF has the highest mean return in the portfolio indicating strong average gains over time. Standard deviation implies moderate volatility suggesting fluctuations in returns but not as extreme as VNQ. (Hargrave, 2024) Negative skewness indicates a tendency towards more frequent small losses compared to gains. A kurtosis greater than 3 suggests a higher probability of returns compared to a normal distribution, indicating some risk but potentially rewarding performance. (suvarna, 2024)

4.3.2 Portfolio

Mean Return

The Portfolio's mean return of 0.61% indicates that, on average, the portfolio is expected to generate a modest gain over a specific period (monthly). This return reflects the weighted contributions of the individual ETFs based on their respective mean returns.

Standard Deviation

The portfolio standard deviation of 0.0923% suggests that the returns are relatively stable and exhibit low volatility. This low standard deviation indicates that the portfolio's returns are not expected to fluctuate significantly from the mean return, which is beneficial for risk-averse investors. (Hargrave, 2024)

Diversification benefits

The combination of different asset classes (equities, bonds, real estate, and commodities) helps reduce overall portfolio risk through diversification. Since individual ETFs have varying levels of risk and return characteristics, their combination in this portfolio leads to lower volatility than if one were to invest heavily in a single asset class.

Expected performance

While the mean return is positive, it is essential to consider that this level of return may not be sufficient for all investors, especially those seeking higher returns. However, for those prioritizing capital preservation and lower risk exposure, this portfolio structure aligns with such goals.

4.4 Model Results

Variance-Covariance Method

Confidence Levels	90%	95%	99%
Value at Risk	-0.03285855	-0.04389842	-0.06460739

Table 8: Value at Risk results from Variance - Covariance Method

These results highlight the portfolio's potential losses under different confidence levels. At a 90% confidence level, the VaR is approximately -3.29%, indicating that the portfolio is expected to lose no more than 3.29% of its value in a given period under normal market conditions. At 95%, the VaR increases to -4.39%, while at 99%, the VaR rises significantly to -6.46%, accounting for extreme but rare losses. This implies the portfolio is expected to lose no more than 4.39% and 6.46% at the respective confidence levels (Strike, 2023). A progression is evidence implying a relationship between confidence levels and VaR magnitudes, which is, higher confidence levels capture more extreme loss scenarios, making them suitable for risk-averse managers, whereas lower confidence levels focus on moderate losses.

Historical Simulations Method

Confidence Levels	90%	95%	99%
Value at Risk	-0.0323675	-0.04604236	-0.07699174

Table 9: Value at Risk results from Historical Simulations Method

The results from the Historical Simulations method indicate VaR values of -3.24%, -4.60%, and -7.70% at confidence levels of 90%, 95%, and 99% respectively. At 90% confidence, the portfolio is expected to lose no more than 3.24% in each period based on historical fluctuations, with a 10% probability of exceeding this loss. At 95%, VaR rises to 4.60% reflecting a loss estimate that is more conservative, while at 99%, the loss estimate rises even further to 7.70%, capturing rare and extreme market conditions (Strike, 2023). Correlation between the VaR and confidence levels is also evident in this case, as higher confidence levels account for more severe loss scenarios.

Monte Carlo Simulation

Normal Distribution

Confidence Levels	**90%**	**95%**	**99%**
Value at Risk	-0.03282918	-0.04324755	-0.06544647

Table 10: Value at Risk results from Monte Carlo Simulations Method assuming Normality

The Monte Carlo method with normality assumptions yield VaR estimates of -3.28%, -4.32%, and -6.54% for 90%, 95%, and 99% confidence levels respectively. At 90% confidence, the portfolio has a 10% probability of incurring losses that exceed 3.28%. This increases to 4.32% at 95% confidence and 6.54% confidence, emphasizing that higher confidence levels account for more severe downside risks (Strike, 2023). The progression aligns with expectations, as higher confidence levels account for and capture more extreme downside risks.

Geometric Brownian Motion

Confidence Levels	**90%**	**95%**	**99%**
Value at Risk	-0.01997305	-0.02711253	-0.04035722

Table 11: Value at Risk results from Monte Carlo Method using GBM Approach

Under the Monte – Carlo simulations using the Geometric Brownian Motion (GBM) approach, the VaR results are -1.99%, -2.71%, and -4.04% at confidence levels of 90%, 95%, and 99% respectively. At 90% confidence, the portfolio is expected to lose no more than 1.99% of its value in each period, increasing to 2.71% at 95% confidence, and 4.04% at 99% confidence. These results show that the GBM approach generally produces lower VaR estimates compared to other methods, reflecting its assumption of log-normality, where compounding returns and volatility dynamics affect risk assessment. However, this method introduces randomness in simulating paths, offering a unique perspective on potential losses compared to models assuming linear normality. The correlation between the confidence levels and the VaR values is consistent with other methods, highlighting higher loss thresholds for higher confidence levels.

4.5 Comparison Analysis

4.5.1 Back-testing results

The interpretation of the back testing results for each VaR model is based on the p-values from the Kupeic Proportion of Failure (POF) Test

Variance – Covariance

Confidence Levels	90%	95%	99%
P - Value	0.8763594	0.6229961	0.2974775

Table 12: POF test results for Variance - Covariance VaR

The results from back-testing the Variance – Covariance method indicate reliable performance across most confidence levels, with p-values exceeding the significance threshold of 0.05. At 90% confidence level, a high p-value of 0.8764 suggests the number of breaches align closely with the expected 10% tail loss while at the 95% confidence level, the p-value is 0.6230 confirming the method's effectiveness in estimating risk. However, at the 99% confidence level, the lower p-value of 0.2975 highlights a reduction in reliability. This is because the model starts to show limitations in capturing extreme tail risks (Prep, 2023). This pattern suggests that the Variance – Covariance method performs well overall, but its reliance on normality assumptions make it less robust and reliable under extreme market conditions.

Historical Simulation

Confidence Levels	90%	95%	99%
P - Value	0.9471843	0.8444527	0.6425917

Table 13: POF test results for Historical Simulations VaR

The Historical Simulation method consistently performs well across all confidence levels, demonstrating high reliability in estimating risk. At 90% confidence level, the p-value of 0.9472 confirms that the number of breaches is nearly identical to the expected 10% tail loss. Similarly, the p-value of 0.8445 at the 95% confidence level underpins the method's robustness, with violation also closely matching expectations. Even at 99% confidence level, the method maintains a strong performance, as seen from the high p-value of 0.6426 (Prep, 2023). The consistent reliability of this method, as seen from the results, can be attributed to its empirical approach, which uses historical data rather than theoretical distributions, making it well suited for capturing real-world market dynamics.

Monte Carlo Simulation using Normal Distribution

Confidence Levels	90%	95%	99%
P - Value	0.8763594	0.6229961	0.2974775

Table 14: POF test results for Monte – Carlo Simulation (Normality approach) VaR

The Monte Carlo Simulations method assuming normality yields similar results performance trends to the Variance – Covariance method, with high p-values at lower confidence levels and declining reliability at higher levels. At 90% confidence level, a high p-value of 0.8764 suggests the number of breaches align closely with the expected 10% tail loss while at the 95% confidence level, the p-value is 0.6230 confirming the method's effectiveness in estimating risk. However, at the 99% confidence level, the lower p-value of 0.2975 highlights a reduction in reliability (Prep, 2023). This trend reflects the limitations of assuming normality in simulated returns, which may not fully account for the heavy tails and extreme events present in financial markets.

Monte Carlo Simulation Using Geometric Brownian Motion

Confidence Levels	90%	95%	99%
P - Value	1.100398^{-2}	1.623064^{-5}	7.685647^{-7}

Table 15: POF test results for Monte – Carlo Simulation (GBM approach) VaR

The results from this method highlight significant underestimation of risk across all confidence levels. At 90%, the p-value of 0.0110 suggests a statistically significant deviation from the expected number of breaches, indicating poor performance. This worsens as the confidence levels decreases, with extremely low p-values of 0.000016 and 0.0000008 respectively, confirming consistent underperformance (Prep, 2023). These results suggest that the GBM-based Monte-Carlo method may not be reliable when providing VaR estimates for portfolios requiring robust risk management.

4.6 Key Take aways

Model selection should be based on market conditions. For a portfolio with returns that approximate normality, the variance-covariance and Monte Carlo (Normal Distribution) methods provide reliable estimates with minimal computational demands. However, they are more sensitive to deviations from normality.

The Historical Simulation Method is advantageous in non-normally distributed or skewed return environments, as it does not require any distributional assumptions. The back-testing supports its robustness in capturing real-world risk patterns, making it valuable for diversified portfolios and during high-volatility periods.

The Geometric Brownian Motion can model asset prices over time. However, when applied to Monte Carlo simulations, it proves to be less effective mainly due to its dependence on drift and volatility assumptions, especially when these do not hold steady. This suggests caution in applying GBM for VaR, especially for portfolios sensitive to extreme market conditions.

In summary, each model has distinct advantages and limitations, and their effectiveness depends on the specific portfolio characteristics and market conditions. Given the back-testing, Historical Simulation may offer the most reliable and adaptable estimate in this case, with Variance-Covariance and Monte Carlo with Normal Distribution serving as good alternatives if normality assumptions are reasonably met.

CHAPTER 5: CONCLUSION & RECOMMENDATIONS

5.1 Introduction

This chapter consolidates the key findings on the study on the effectiveness of Value at Risk (VaR) models in managing risk for diversified portfolios. The study sought to evaluate three prominent VaR models, Variance-Covariance, Historical simulations, and Monte Carlo Simulations, and their application to a portfolio comprising five ETFs representing different asset classes. The research set to address the objectives at the beginning of the study as well as provide insights for portfolio managers in choosing and implementing the appropriate VaR model. Overall, the findings confirm that the objectives and research questions were addressed through detailed analysis and back-testing of each model.

5.2 Summary of findings

Variance-Covariance Method

This method, which assumes normally distributed returns, provides reliable and straightforward estimates of VaR for the portfolio. Back-testing results indicated that the model accurately captured the portfolio's risk profile with an observed p-value of 0.623, suggesting the model met expectations at the 95% confidence level. This makes the method useful in environments where returns approximate a normal distribution and are relatively stable over time. (N.A, 2024)

Historical Simulation Method

This method offered a robust VaR estimate that reflect market behaviour despite not relying on any return distribution. Back-testing yielded a high p-value (0.844), indicating that the model is consistent and reliable for portfolios with historical data respective of market conditions. Historical Simulation effectively captures risk, particularly in environments where returns are skewed or non-normally distributed, as it directly reflects actual return patterns without assumptions. Hence the most suitable model for our diversified ETF portfolio. (N.A, 2024)

Monte Carlo Simulations Method

The Monte Carlo Simulation under the assumption of normal distribution provides a reliable VaR estimate that aligns with expectations in back-testing yielding a p-value of 0.623. This model's flexibility in modelling various scenarios make it valuable for stress-testing portfolio risk. However, similar to the Variance-Covariance method, it is most suitable for portfolio returns that are normally distributed. (N.A, 2024)

The Monte Carlo Simulation using the Geometric Brownian Motion demonstrated an underestimated VaR estimate for the portfolio evident in the low p-value (0.00001623), indicating a significant number of violations. This result suggests that while GBM may be valuable for modelling long-term asset price paths, it can underperform in capturing short-term risks, especially when the volatility assumption does not hold steady. This method may therefore be less suited to portfolios sensitive to extreme market conditions.

5.3 Recommendations

The first recommendation would be the application of Historical Simulation for Non-Normal returns. Portfolio managers working with diversified portfolios that exhibit skewed or non-normally distributed returns should consider using the Historical Simulation method. This model's reliance on actual return history allows it to better capture risks reflective of current portfolio behaviour and market conditions without relying on distributional assumptions. (N.A, 2024)

Next would be the application of Variance-Covariance and Monte Carlo (Normal Distribution) for stable markets. When the portfolio returns approximate a normal distribution and market conditions are stable, the Variance-Covariance and Monte Carlo methods under the normal distribution assumption can offer more reliable risk assessments with minimal computational demands. These models are advantageous for routine risk assessments and stress testing in stable environments.

Another recommendation would be caution when applying the Geometric Brownian motion in VaR estimation. While the model is used widely for long-term price modelling, portfolio managers should exercise caution in using this model for short-term risk estimation in VaR. The back-testing results indicated a tendency to underestimate tail risks, which could lead to insufficient risk coverage during periods of high volatility.

Finally, would be consideration of complementary risk measures. Given VaR's limitations in capturing extreme losses, it is recommended that portfolio managers complement VaR with additional risk measures such as Conditional Value at Risk (CvaR) or stress testing to provide a more comprehensive view of portfolio risk. (N.A, 2024)

5.4 Limitation of Study

Some of the limitations encountered in this study data constraints and model assumptions. The analysis was based on historical data from five ETFs, which, though diversified, may not fully capture the variety of asset classes or market behaviours that exists. The assumption of normality in the Variance-Covariance and Monte Carlo Simulations may not fully represent real-world market behaviours, especially in extreme events. The assumption of constant drift and volatility in the GBM model also proved restrictive, as back-testing indicated the model's limitations. Additionally, this study focused solely on VaR which does not account for extreme tail risks beyond the threshold.

5.5 Suggestions for future research

Future studies can explore additional risk measures such as CvaR, which offer insights into risk beyond the VaR threshold. This would provide a more comprehensive view of the portfolio risk, particularly during periods of high market volatility. Also, future research could apply VaR to other asset classes, alternative investments, and commodities especially in emerging markets. This would allow for deeper understanding of VaR applicability across diverse portfolios and provide valuable insights into model performance in less conventional asset categories and markets. (N.A, 2024)

Given the challenges encountered due to the normality assumption, future research should investigate alternative distributions or non-parametric methods. This would help determine whether these alternative models improve the accuracy of VaR estimates for portfolios with non-normal or extreme distributions.

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APPENDIX

R-Studio Data Analysis

Value at Risk Models

Brian Mutiso

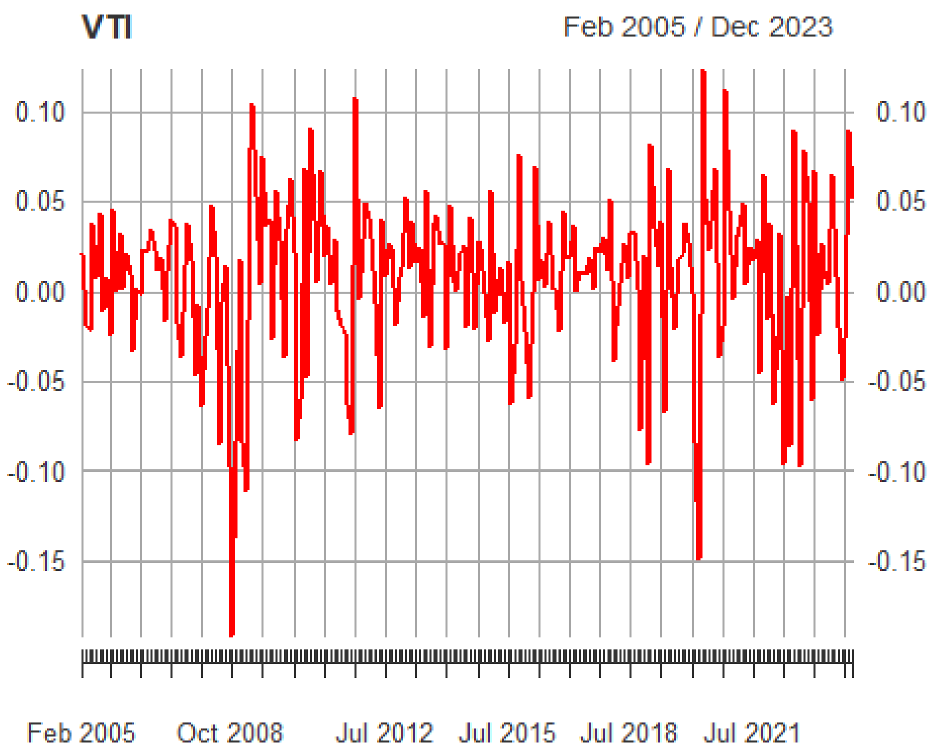
2025-01-19

```
library(quantmod)
library(PerformanceAnalytics)
```

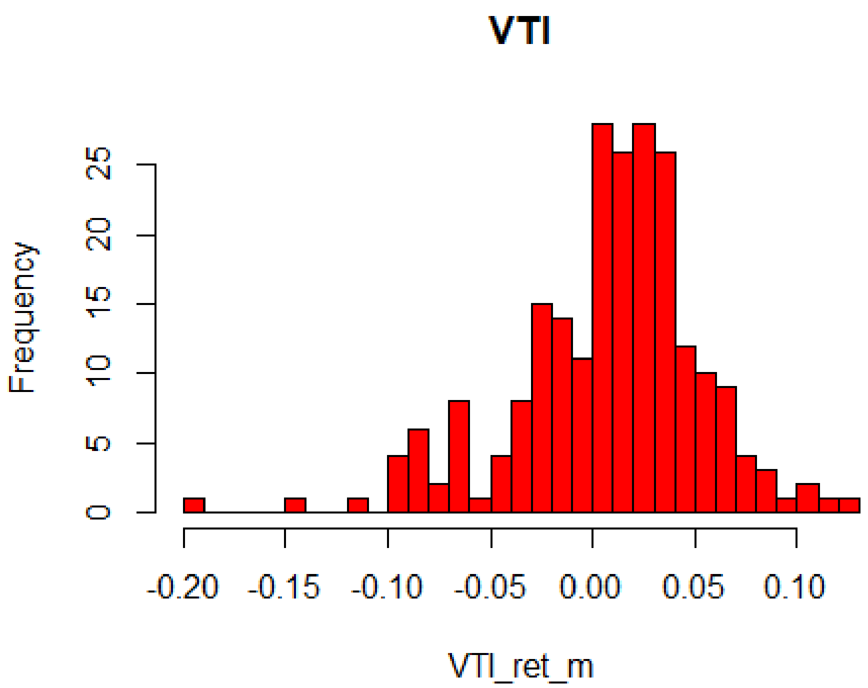
VTI

```
# VTI Daily prices
VTI_D <- getSymbols("VTI", from='2005-01-02', to='2023-12-31', warnings=FA
LSE, auto.assign = FALSE)
head(VTI_D)

##            VTI.Open VTI.High VTI.Low VTI.Close VTI.Volume VTI.Adjusted
## 2005-01-03   59.400   59.425  58.450    58.600     525000     40.58275
## 2005-01-04   58.750   58.750  57.640    57.820     244200     40.04257
## 2005-01-05   57.875   57.975  57.420    57.420     380800     39.76555
## 2005-01-06   57.550   57.895  57.430    57.695     201800     39.95599
## 2005-01-07   57.800   57.825  57.355    57.510     271200     39.82788
## 2005-01-10   57.550   58.050  57.540    57.740     584200     39.98716

# Converting to monthly
VTI_M= to.monthly(VTI_D$VTI.Adjusted, OHLC=FALSE)
colnames(VTI_M)<- "VTI"

# Computing monthly returns
VTI_ret_m<- na.omit(Return.calculate(VTI_M, method = "log"))

length(VTI_ret_m)

## [1] 227

head(VTI_ret_m)

##                   VTI
## Feb 2005  0.020636379
## Mar 2005 -0.018897160
## Apr 2005 -0.022272862
## May 2005  0.038080428
## Jun 2005  0.007553189
## Jul 2005  0.042851638
```

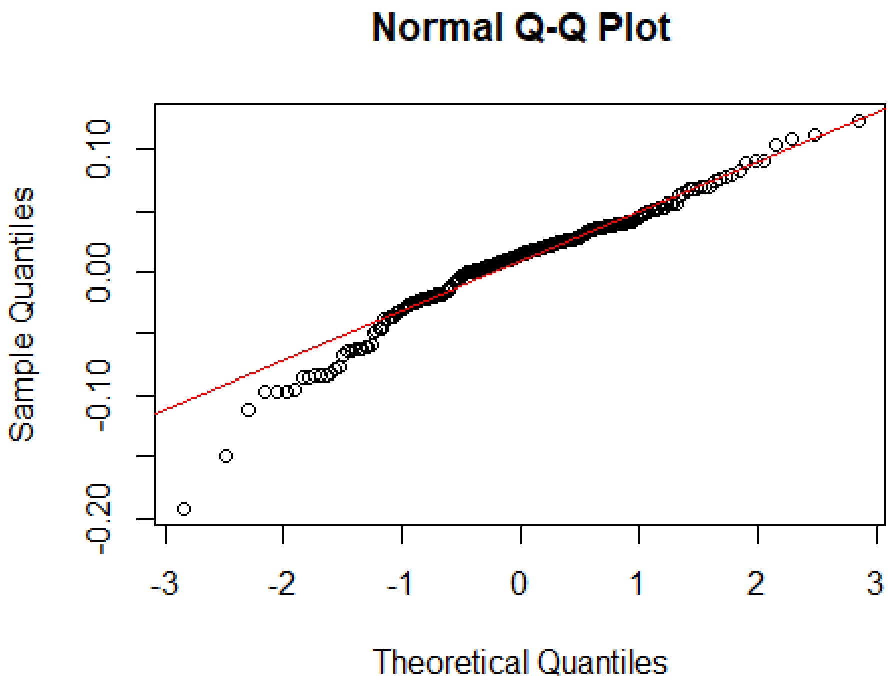
```r
# Plots
plot(VTI_ret_m, main="VTI", x.lab="Time",y.lab="Returns",type="l",col="red
")
```




```r
hist(VTI_ret_m, main = "VTI", col = "red",breaks = 25)
```



```r
# QQ plot to test for normality
qqnorm(VTI_ret_m)
qqline(VTI_ret_m, col = "red")
```

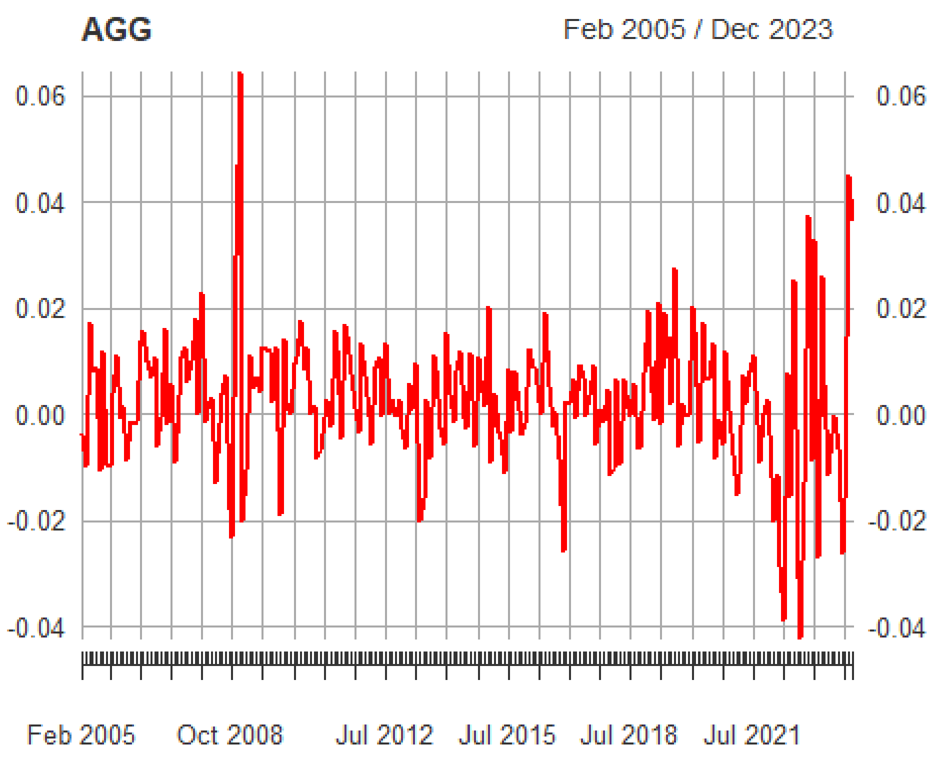



```r
# Shapiro test
shapiro.test(as.numeric(VTI_ret_m))

##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(VTI_ret_m)
## W = 0.95966, p-value = 5.117e-06
```

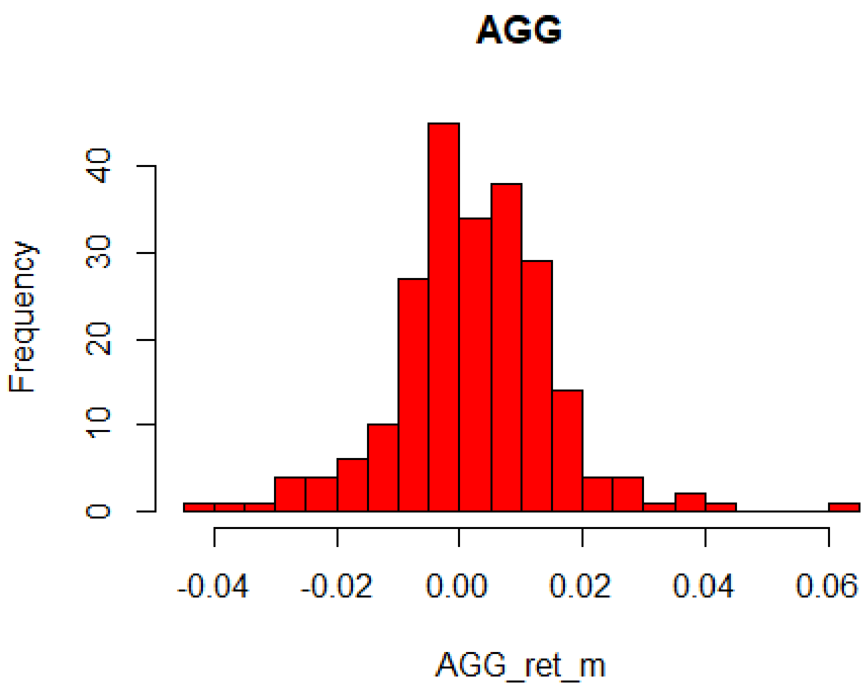
#AGG

```r
# AGG Daily prices
AGG_D <- getSymbols("AGG", from='2005-01-02', to='2023-12-31', warnings=FA
LSE, auto.assign = FALSE)
head(AGG_D)

##            AGG.Open AGG.High AGG.Low AGG.Close AGG.Volume AGG.Adjusted
## 2005-01-03   102.34   102.55  102.12    102.40     479600     54.70809
## 2005-01-04   102.46   102.50  102.07    102.30     190500     54.65463
## 2005-01-05   102.20   102.35  102.06    102.26      69900     54.63329
## 2005-01-06   102.29   102.37  102.18    102.33      75400     54.67067
## 2005-01-07   102.39   102.44  102.11    102.30     137000     54.65463
## 2005-01-10   102.32   102.32  102.10    102.25      69500     54.62795

# Converting to monthly
AGG_M= to.monthly(AGG_D$AGG.Adjusted, OHLC=FALSE)
colnames(AGG_M)<- "AGG"
```

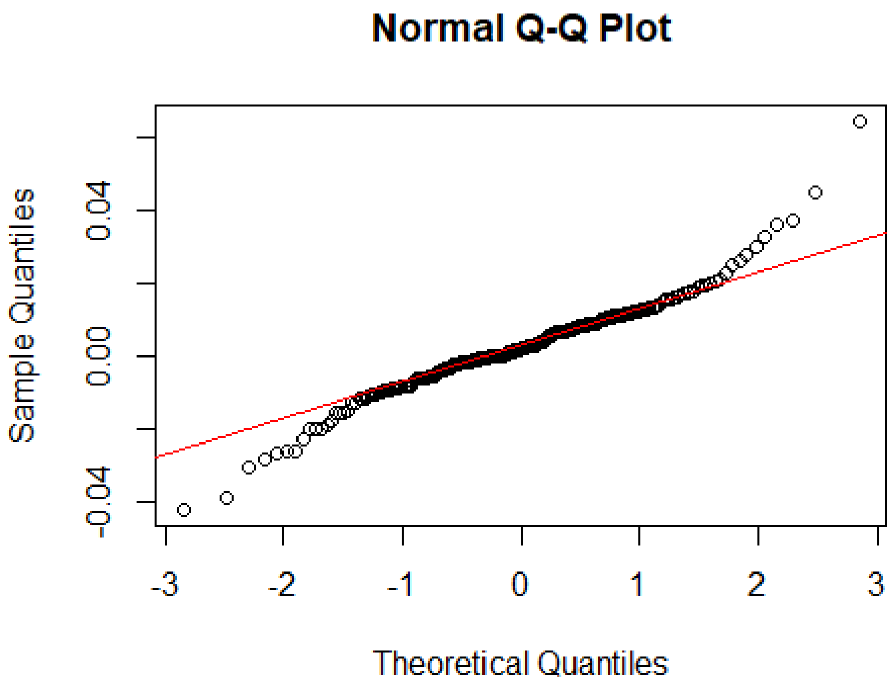
```r
# Computing monthly returns
AGG_ret_m<- na.omit(Return.calculate(AGG_M, method = "log"))

length(AGG_ret_m)

## [1] 227

head(AGG_ret_m)

##                   AGG
## Feb 2005 -0.003719731
## Mar 2005 -0.009789573
## Apr 2005  0.017080778
## May 2005  0.008241354
## Jun 2005  0.008724905
## Jul 2005 -0.010408525

# Plots
plot(AGG_ret_m, main="AGG", x.lab="Time",y.lab="Returns",type="l",col="red
")
```




```r
hist(AGG_ret_m, main = "AGG", col = "red",breaks = 25)
```




```r
# QQ plot to test for normality
qqnorm(AGG_ret_m)
qqline(AGG_ret_m, col = "red")
```




```r
# Shapiro test
shapiro.test(as.numeric(AGG_ret_m))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(AGG_ret_m)
## W = 0.95703, p-value = 2.581e-06
```

VNQ

```
# VNQ Daily prices
VNQ_D <- getSymbols("VNQ", from='2005-01-02', to='2023-12-31', warnings=FA
LSE, auto.assign = FALSE)
head(VNQ_D)
```

```
##            VNQ.Open VNQ.High VNQ.Low VNQ.Close VNQ.Volume VNQ.Adjusted
## 2005-01-03    56.75    56.75   55.50     55.89      31900     23.68541
## 2005-01-04    55.89    56.28   55.05     55.05      52500     23.32944
## 2005-01-05    55.17    55.17   52.83     53.22      77300     22.55391
## 2005-01-06    53.15    53.82   53.15     53.63      42200     22.72766
## 2005-01-07    53.87    53.88   53.27     53.51      24200     22.67681
## 2005-01-10    53.45    53.65   53.20     53.34      12500     22.60476
```

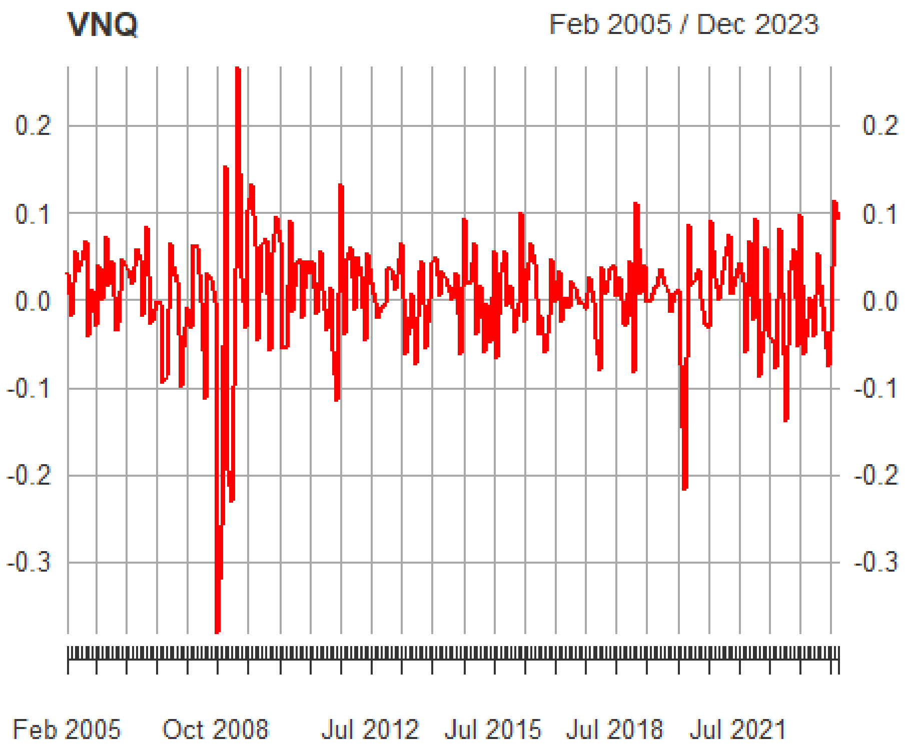
```
# Converting to monthly
VNQ_M= to.monthly(VNQ_D$VNQ.Adjusted, OHLC=FALSE)
colnames(VNQ_M)<- "VNQ"

# Computing monthly returns
VNQ_ret_m<- na.omit(Return.calculate(VNQ_M, method = "log"))

length(VNQ_ret_m)
```

```
## [1] 227
```

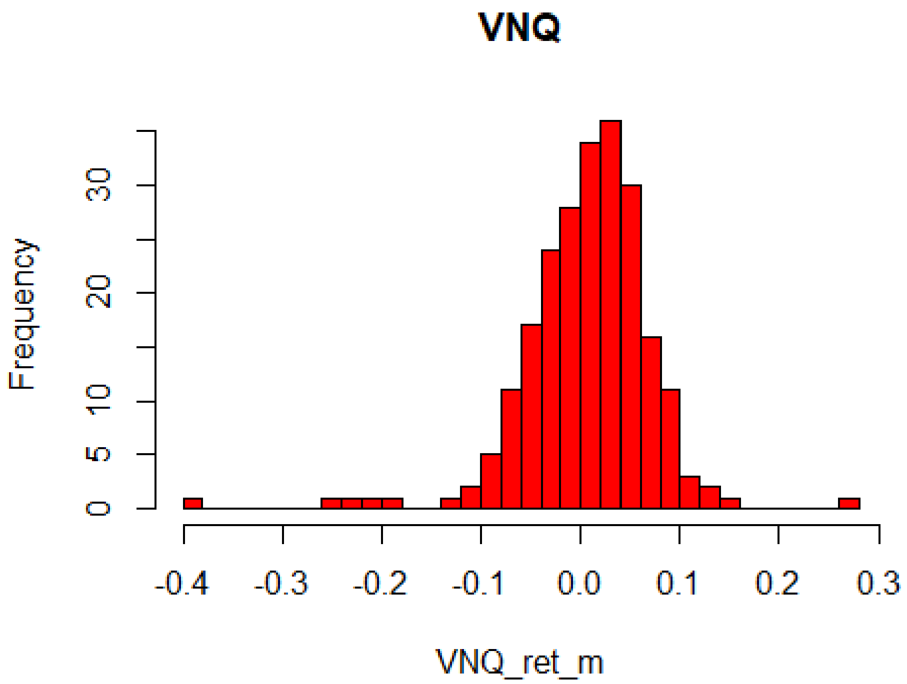
```
head(VNQ_ret_m)
```

```
##                  VNQ
## Feb 2005  0.03122285
## Mar 2005 -0.01722274
## Apr 2005  0.05609684
## May 2005  0.03460638
## Jun 2005  0.04494891
## Jul 2005  0.06894735
```

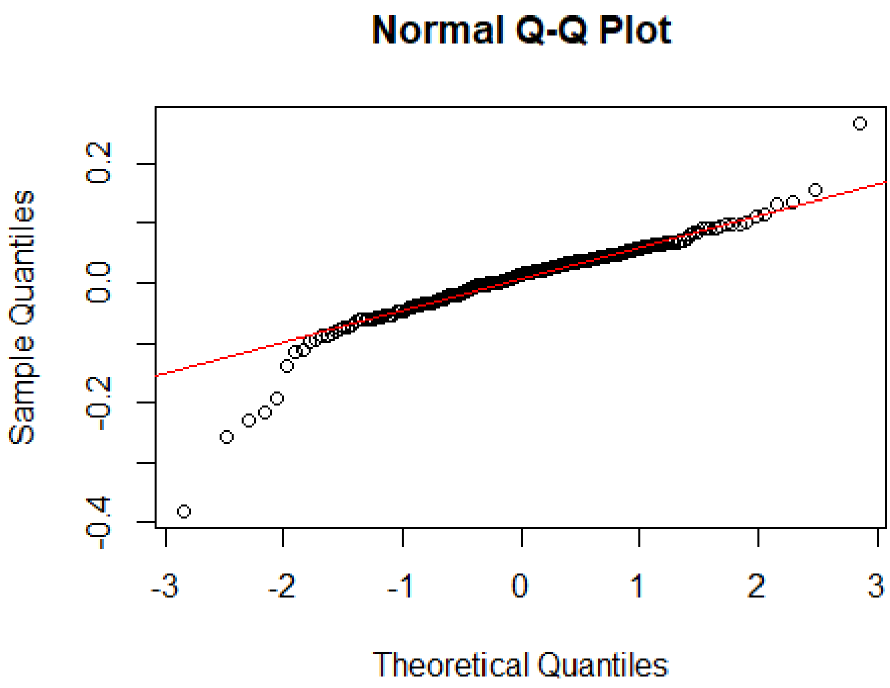
```
# Plots
plot(VNQ_ret_m, main="VNQ", x.lab="Time",y.lab="Returns",type="l",col="red
")
```




```
hist(VNQ_ret_m, main = "VNQ", col = "red",breaks = 45)
```




```
# QQ plot to test for normality
qqnorm(VNQ_ret_m)
qqline(VNQ_ret_m, col = "red")
```

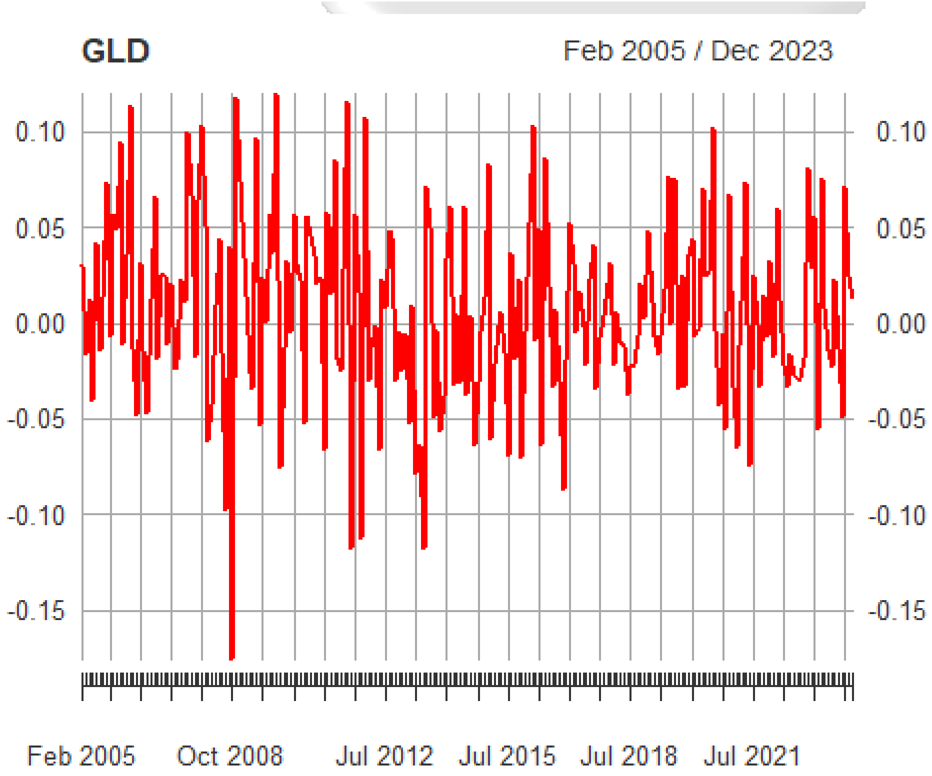



```
# Shapiro test
shapiro.test(as.numeric(VNQ_ret_m))

##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(VNQ_ret_m)
## W = 0.8987, p-value = 2.996e-11
```

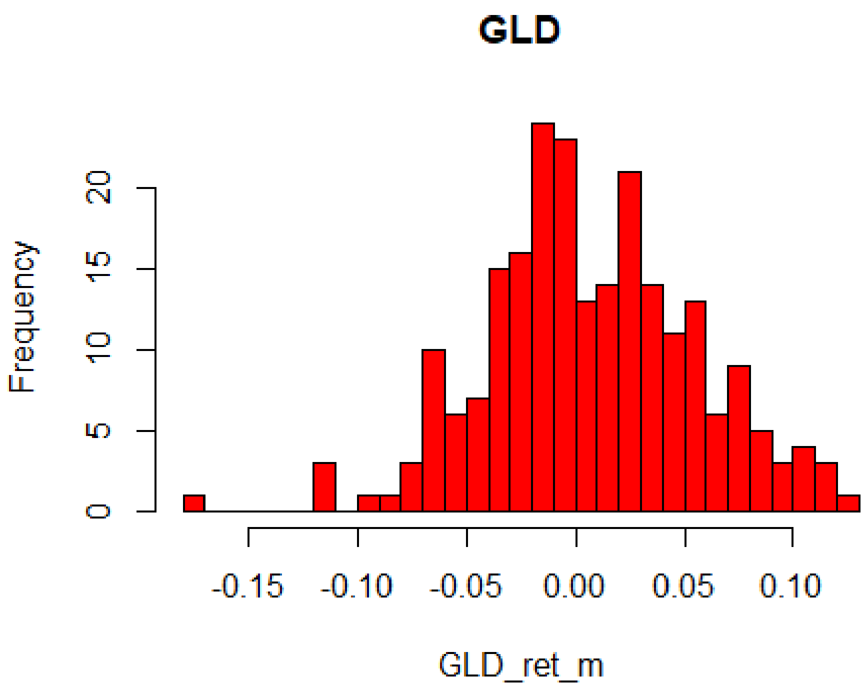
GLD

```
# GLD Daily prices
GLD_D <- getSymbols("GLD", from='2005-01-02', to='2023-12-31', warnings=FA
LSE, auto.assign = FALSE)
head(GLD_D)

##            GLD.Open GLD.High GLD.Low GLD.Close GLD.Volume GLD.Adjusted
## 2005-01-03    42.98    43.17   42.74     43.02    4750400        43.02
## 2005-01-04    42.80    42.91   42.46     42.74    3456800        42.74
## 2005-01-05    42.75    42.88   42.60     42.67    2033600        42.67
## 2005-01-06    42.48    42.56   42.07     42.15    2556400        42.15
## 2005-01-07    42.09    42.39   41.70     41.84    4492700        41.84
## 2005-01-10    41.99    42.07   41.89     41.95    1025800        41.95

# Converting to monthly
GLD_M= to.monthly(GLD_D$GLD.Adjusted, OHLC=FALSE)
colnames(GLD_M)<- "GLD"

# Computing monthly returns
GLD_ret_m<- na.omit(Return.calculate(GLD_M, method = "log"))
```

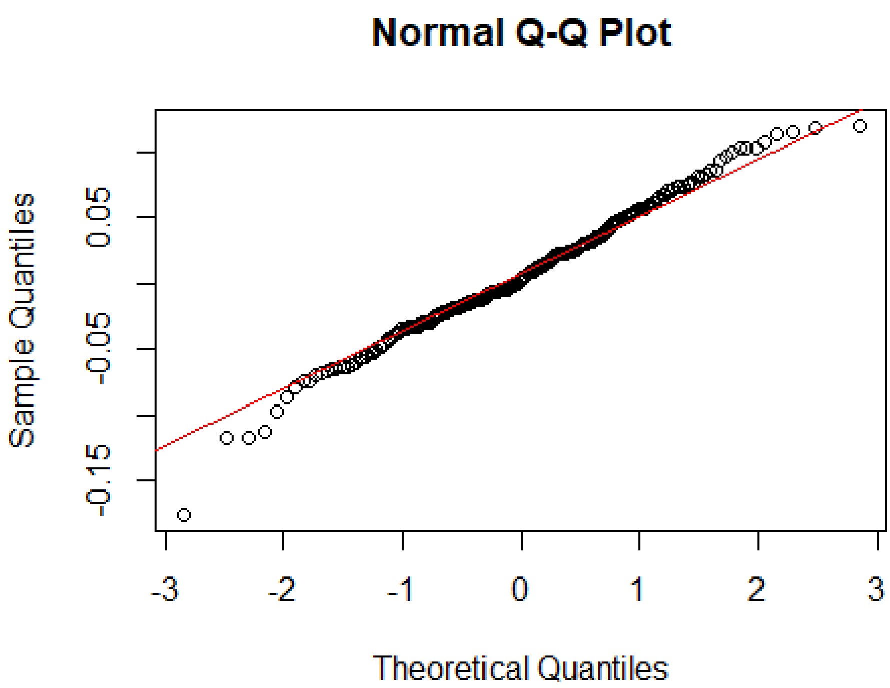
```r
length(GLD_ret_m)

## [1] 227

head(GLD_ret_m)

##                  GLD
## Feb 2005  0.03055626
## Mar 2005 -0.01644505
## Apr 2005  0.01230139
## May 2005 -0.04000526
## Jun 2005  0.04207924
## Jul 2005 -0.01437537

# Plots
plot(GLD_ret_m, main="GLD", x.lab="Time",y.lab="Returns",type="l",col="red
")
```




```r
hist(GLD_ret_m, main = "GLD", col = "red",breaks = 25)
```




```r
# QQ plot to test for normality
qqnorm(GLD_ret_m)
qqline(GLD_ret_m, col = "red")
```

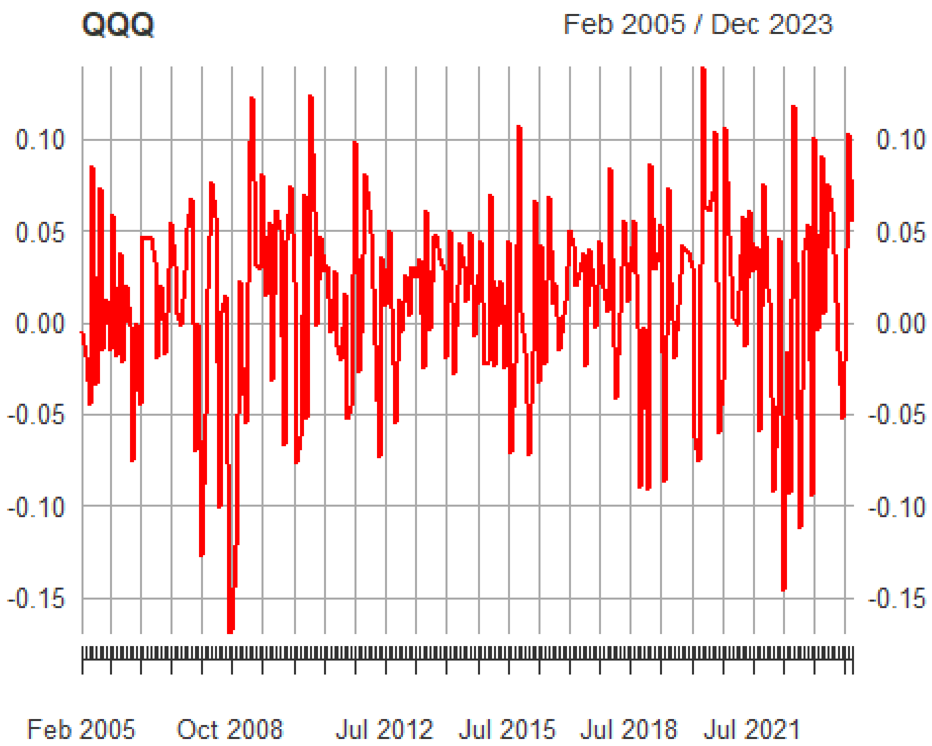



```r
# Shapiro test
shapiro.test(as.numeric(GLD_ret_m))
```

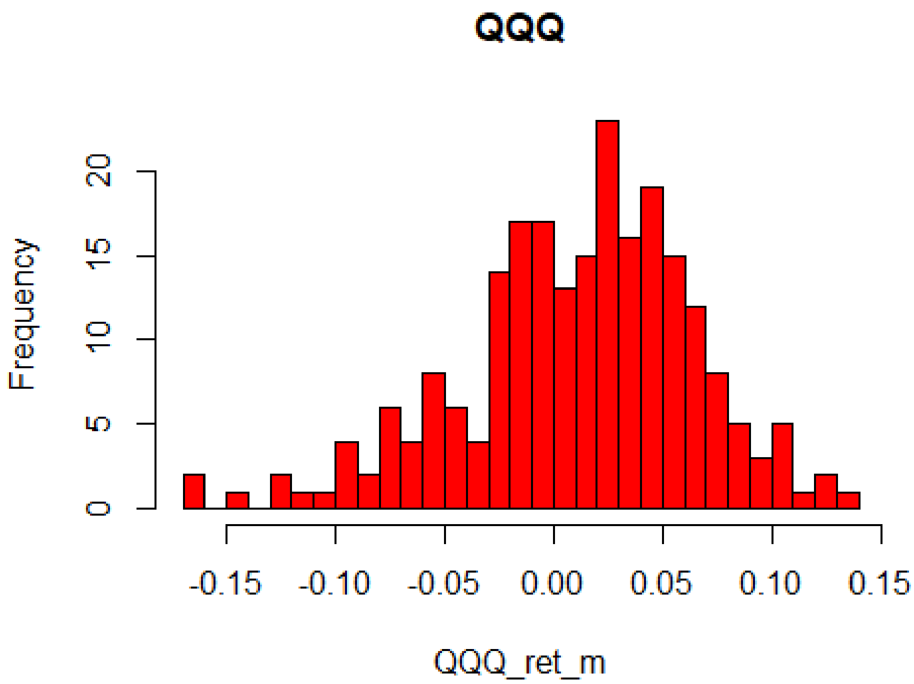
```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(GLD_ret_m)
## W = 0.9906, p-value = 0.1494
```

QQQ

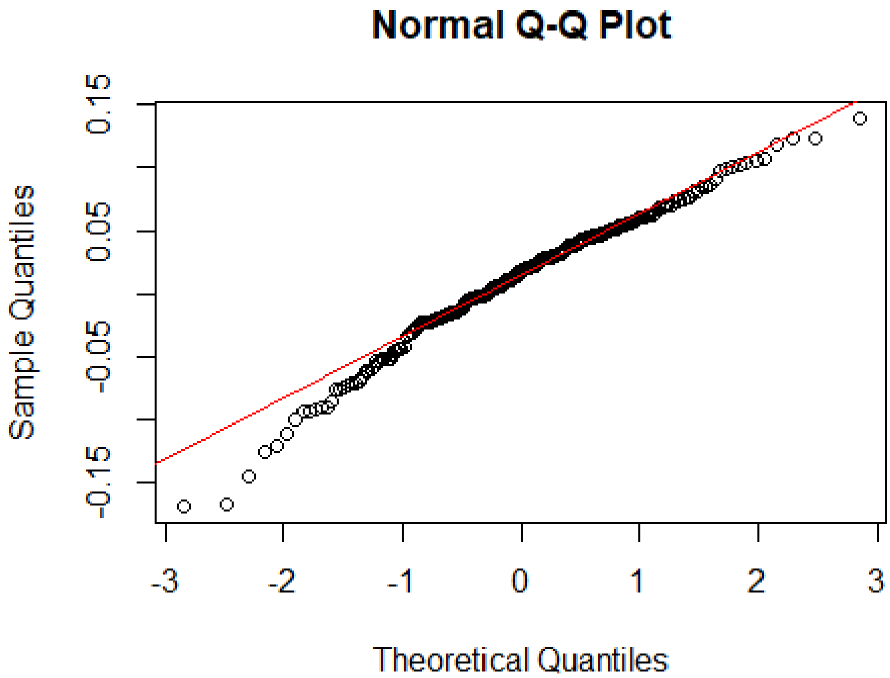
```
# QQQ Daily prices
QQQ_D <- getSymbols("QQQ", from='2005-01-02', to='2023-12-31', warnings=FA
LSE, auto.assign = FALSE)
head(QQQ_D)

##            QQQ.Open QQQ.High QQQ.Low QQQ.Close QQQ.Volume QQQ.Adjusted
## 2005-01-03    40.09    40.29   39.37     39.50  100970900     33.86627
## 2005-01-04    39.67    39.74   38.55     38.78  136623200     33.24897
## 2005-01-05    38.68    38.96   38.47     38.54  127925500     33.04320
## 2005-01-06    38.63    38.71   38.34     38.35  102934600     32.88029
## 2005-01-07    38.56    38.87   38.21     38.55  123104000     33.05176
## 2005-01-10    38.44    38.87   38.39     38.53   88764200     33.03461

# Converting to monthly
QQQ_M= to.monthly(QQQ_D$QQQ.Adjusted, OHLC=FALSE)
colnames(QQQ_M)<- "QQQ"

# Computing monthly returns
QQQ_ret_m<- na.omit(Return.calculate(QQQ_M, method = "log"))

length(QQQ_ret_m)

## [1] 227

head(QQQ_ret_m)

##                   QQQ
## Feb 2005 -0.004823801
## Mar 2005 -0.017618012
## Apr 2005 -0.044452246
## May 2005  0.084912610
## Jun 2005 -0.033810670
## Jul 2005  0.073369566

# Plots
plot(QQQ_ret_m, main="QQQ", x.lab="Time",y.lab="Returns",type="l",col="red
")
```




```r
hist(QQQ_ret_m, main = "QQQ", col = "red",breaks = 25)
```



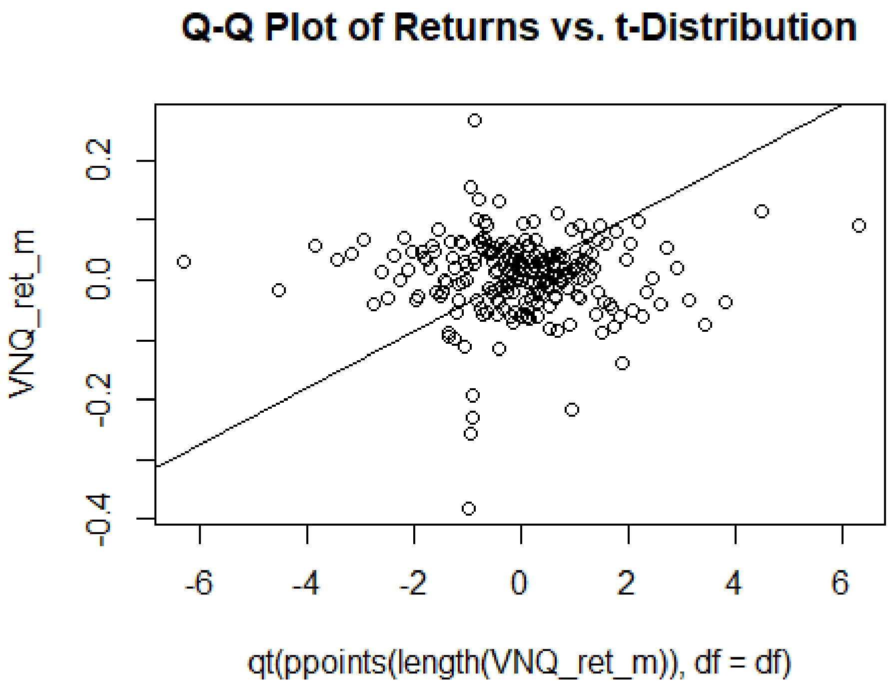

```r
# QQ plot to test for normality
qqnorm(QQQ_ret_m)
qqline(QQQ_ret_m, col = "red")
```



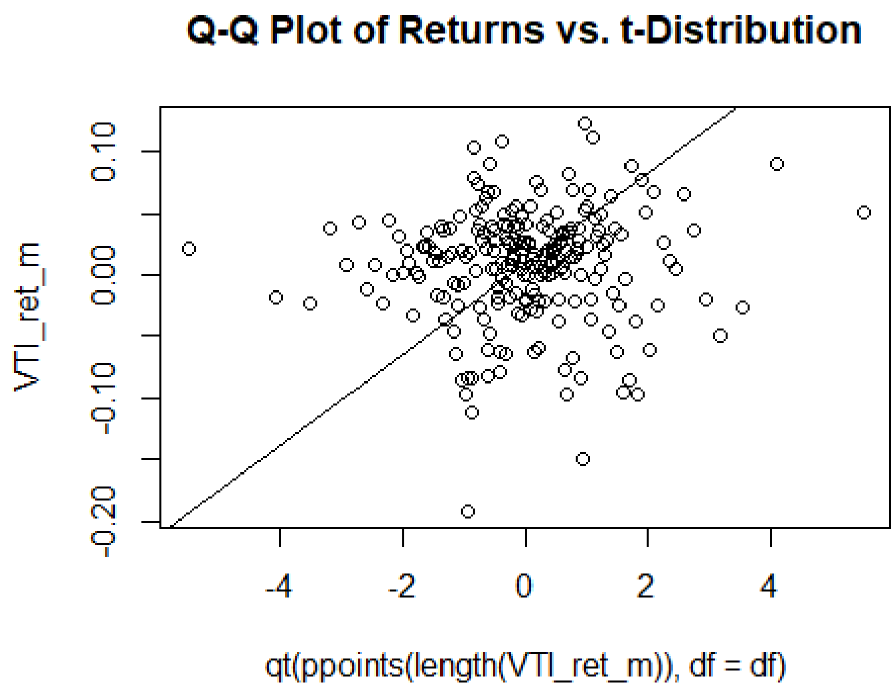

```
# Shapiro test
shapiro.test(as.numeric(QQQ_ret_m))

##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(QQQ_ret_m)
## W = 0.97812, p-value = 0.001369

# Load necessary library for skewness and kurtosis
library(moments)

# Calculate descriptive statistics for each ETF
descriptive_stats <- data.frame(
  ETF = c("VTI", "AGG", "VNQ", "GLD", "QQQ"),
  Mean_Return = sapply(list(VTI_ret_m, AGG_ret_m, VNQ_ret_m, GLD_ret_m, QQ
Q_ret_m), mean, na.rm = TRUE),
  SD_Return = sapply(list(VTI_ret_m, AGG_ret_m, VNQ_ret_m, GLD_ret_m, QQQ_
ret_m), sd, na.rm = TRUE),
  Skewness = sapply(list(VTI_ret_m, AGG_ret_m, VNQ_ret_m, GLD_ret_m, QQQ_r
et_m), skewness, na.rm = TRUE),
  Kurtosis = sapply(list(VTI_ret_m, AGG_ret_m, VNQ_ret_m, GLD_ret_m, QQQ_r
et_m), kurtosis, na.rm = TRUE)
)

# Display descriptive statistics
print(descriptive_stats)

##     ETF Mean_Return  SD_Return   Skewness  Kurtosis
## VTI VTI 0.007799043 0.04569724 -0.7913459  4.746429
## AGG AGG 0.002439605 0.01289255  0.2820501  6.200708
```

```
## VNQ VNQ 0.005970986 0.06616912 -1.3156185 10.046659
## GLD GLD 0.006653167 0.04855422 -0.1150388  3.425754
## QQQ QQQ 0.011195029 0.05369967 -0.5827559  3.646758
```

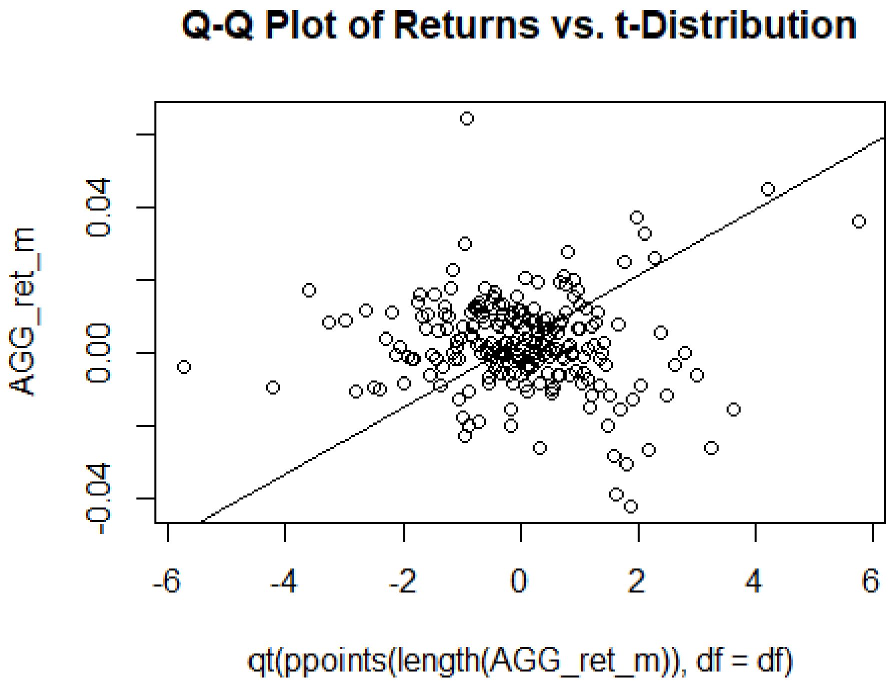
```r
library(MASS)

# Fitting  t-distribution to returns
t_fit <- fitdistr(VNQ_ret_m, "t")
df <- t_fit$estimate['df']

# Q-Q plot against a t-distribution
qqplot(qt(ppoints(length(VNQ_ret_m)), df = df), VNQ_ret_m,
       main = "Q-Q Plot of Returns vs. t-Distribution")
qqline(VNQ_ret_m, distribution = function(p) qt(p, df = df))
```



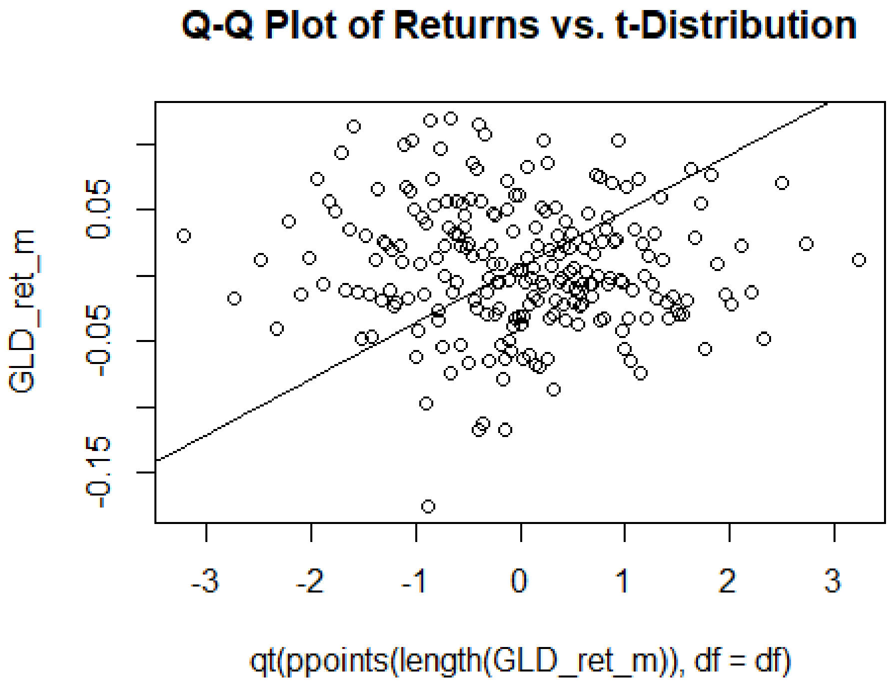

```r
# Fit t-distribution to returns
t_fit <- fitdistr(VTI_ret_m, "t")
df <- t_fit$estimate['df']

# Q-Q plot against a t-distribution
qqplot(qt(ppoints(length(VTI_ret_m)), df = df), VTI_ret_m,
       main = "Q-Q Plot of Returns vs. t-Distribution")
qqline(VTI_ret_m, distribution = function(p) qt(p, df = df))
```



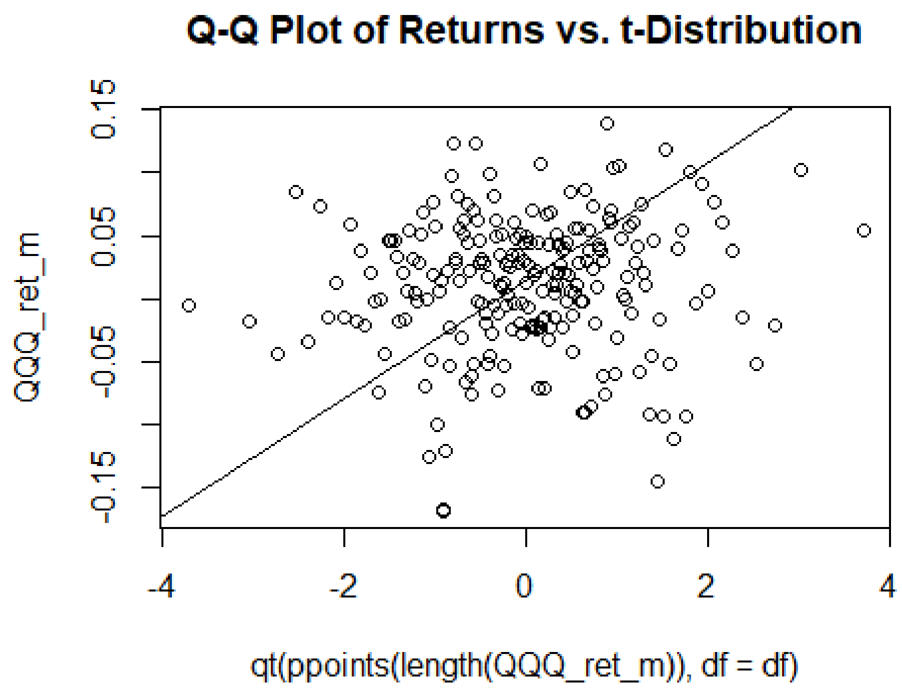

```
# Fit t-distribution to returns
t_fit <- fitdistr(AGG_ret_m, "t")
df <- t_fit$estimate['df']

# Q-Q plot against a t-distribution
qqplot(qt(ppoints(length(AGG_ret_m)), df = df), AGG_ret_m,
       main = "Q-Q Plot of Returns vs. t-Distribution")
qqline(AGG_ret_m, distribution = function(p) qt(p, df = df))
```




```r
# Fit t-distribution to returns
t_fit <- fitdistr(GLD_ret_m, "t")
df <- t_fit$estimate['df']

# Q-Q plot against a t-distribution
qqplot(qt(ppoints(length(GLD_ret_m)), df = df), GLD_ret_m,
       main = "Q-Q Plot of Returns vs. t-Distribution")
qqline(GLD_ret_m, distribution = function(p) qt(p, df = df))
```




```r
# Fit t-distribution to returns
t_fit <- fitdistr(QQQ_ret_m, "t")
df <- t_fit$estimate['df']

# Q-Q plot against a t-distribution
qqplot(qt(ppoints(length(QQQ_ret_m)), df = df), QQQ_ret_m,
       main = "Q-Q Plot of Returns vs. t-Distribution")
qqline(QQQ_ret_m, distribution = function(p) qt(p, df = df))
```



PORTFOLIO CONSTRUCTION


```
# Portfolio construction
portfolio <- cbind(VTI_ret_m,AGG_ret_m,VNQ_ret_m,GLD_ret_m,QQQ_ret_m)
head(portfolio)

##                   VTI          AGG         VNQ         GLD          QQQ
## Feb 2005  0.020636379 -0.003719731  0.03122285  0.03055626 -0.004823801
## Mar 2005 -0.018897160 -0.009789573 -0.01722274 -0.01644505 -0.017618012
## Apr 2005 -0.022272862  0.017080778  0.05609684  0.01230139 -0.044452246
## May 2005  0.038080428  0.008241354  0.03460638 -0.04000526  0.084912610
## Jun 2005  0.007553189  0.008724905  0.04494891  0.04207924 -0.033810670
## Jul 2005  0.042851638 -0.010408525  0.06894735 -0.01437537  0.073369566

# Assigned weights
weights<- c(0.3,0.3,0.15,0.15,0.10)

# Average Monthly returns
Average_returns<- c(mean(VTI_ret_m),mean(AGG_ret_m),mean(VNQ_ret_m),mean(G
LD_ret_m),mean(QQQ_ret_m))
Average_returns

## [1] 0.007799043 0.002439605 0.005970986 0.006653167 0.011195029

# Portfolio mean
Portfolio_mean <- sum (weights*Average_returns)
Portfolio_mean

## [1] 0.00608472

# Portfolio variance
# Calculate the variance-covariance matrix
```

```r
var_cov_matrix <- cov(portfolio, use = "complete.obs")

# View the variance-covariance matrix
var_cov_matrix

##              VTI          AGG          VNQ          GLD          QQQ
## VTI 0.0020882374 0.0001321067 0.0023319038 0.0002159967 0.0022297930
## AGG 0.0001321067 0.0001662179 0.0003219864 0.0002247243 0.0001629373
## VNQ 0.0023319038 0.0003219864 0.0043783523 0.0004636946 0.0022713744
## GLD 0.0002159967 0.0002247243 0.0004636946 0.0023575124 0.0001748599
## QQQ 0.0022297930 0.0001629373 0.0022713744 0.0001748599 0.0028836546

# Calculate the portfolio variance
portfolio_variance <- t(weights) %*% var_cov_matrix %*% weights

# Calculate the portfolio standard deviation (risk)
portfolio_sd <- sqrt(portfolio_variance)

# View the results
portfolio_variance

##              [,1]
## [1,] 0.0009234058

portfolio_sd

##            [,1]
## [1,] 0.03038759
```

VALUE AT RISK CALCULATION

Variance - Covariance Method

99% confidence level

```
# Defining confidence level
confidence_level1<- 0.99

# z-score for normal distribution
z_score1<- qnorm(1- confidence_level1)

# Computing Value at Risk
VaR_var_cov_99 <-  (Portfolio_mean + z_score1 * portfolio_sd)

# Results
VaR_var_cov_99

##             [,1]
## [1,] -0.06460739
```

95% confidence level

```
# Defining confidence level
confidence_level2<- 0.95

# z-score for normal distribution
z_score2<- qnorm(1- confidence_level2)

# Computing Value at Risk
VaR_var_cov_95 <-  (Portfolio_mean + z_score2 * portfolio_sd)

# Results
VaR_var_cov_95

##             [,1]
## [1,] -0.04389842
```

90% confidence level

```
# Defining confidence level
confidence_level3<- 0.90

# z-score for normal distribution
z_score3<- qnorm(1- confidence_level3)

# Computing Value at Risk
VaR_var_cov_90 <-  (Portfolio_mean + z_score3 * portfolio_sd)

# Results
VaR_var_cov_90
```

```
##                [,1]
## [1,] -0.03285855

Variance_Covariance <- c(VaR_var_cov_90,VaR_var_cov_95,VaR_var_cov_99)
Variance_Covariance

## [1] -0.03285855 -0.04389842 -0.06460739
```

Historical Method

```r
# Compute historical portfolio returns
portfolio_returns <- portfolio %*% weights

# Sort returns in ascending order
sorted_returns <- sort(portfolio_returns)
```

99% Confidence Level

```r
# Find the VaR at the (1 - confidence_level) percentile
VaR_historical_99 <- quantile(sorted_returns, probs = 1 - confidence_level
1)

VaR_historical_99

##          1%
## -0.07699174
```

95% Confidence Level

```r
# Find the VaR at the (1 - confidence_level) percentile
VaR_historical_95 <- quantile(sorted_returns, probs = 1 - confidence_level
2)

VaR_historical_95

##          5%
## -0.04604236
```

90% Confidence Level

```r
# Find the VaR at the (1 - confidence_level) percentile
VaR_historical_90 <- quantile(sorted_returns, probs = 1 - confidence_level
3)

VaR_historical_90

##         10%
## -0.03236795

Historical_Methods<- c(VaR_historical_90,VaR_historical_95,VaR_historical_
99)
Historical_Methods

##         10%          5%          1%
## -0.03236795 -0.04604236 -0.07699174
```

Monte Carlo Simulation

Using a normal distribution

```
set.seed(123)  # For reproducibility

# Number of simulations
num_simulations <- 10000

# Simulate returns based on the covariance matrix and mean returns
simulated_returns <- mvrnorm(num_simulations, mu = colMeans(portfolio), Si
gma = cov(portfolio))

# Calculate portfolio returns for each simulation
simulated_portfolio_returns <- simulated_returns %*% weights
```

99% Confidence Level

```
# Compute the VaR as the (1 - confidence_level) percentile
VaR_monte_carlo_99 <- quantile(simulated_portfolio_returns, probs = 1 - co
nfidence_level1)

VaR_monte_carlo_99

##          1%
## -0.06544647
```

95% Confidence Level

```
# Compute the VaR as the (1 - confidence_level) percentile
VaR_monte_carlo_95 <- quantile(simulated_portfolio_returns, probs = 1 - co
nfidence_level2)

VaR_monte_carlo_95

##          5%
## -0.04324755
```

90% Confidence Level

```
# Compute the VaR as the (1 - confidence_level) percentile
VaR_monte_carlo_90 <- quantile(simulated_portfolio_returns, probs = 1 - co
nfidence_level3)

VaR_monte_carlo_90

##         10%
## -0.03282918

Monte_Carlo_Normal <- c(VaR_monte_carlo_90,VaR_monte_carlo_95,VaR_monte_ca
rlo_99)
Monte_Carlo_Normal
```

```
##          10%          5%          1%
## -0.03282918 -0.04324755 -0.06544647
```

Using Geometric Brownian Motion

```
# Defining Parameters
set.seed(123)  # For reproducibility

num_simulations <- 10000 # Number of Monte Carlo simulations
num_periods <- 12        # Number of periods to simulate (12 months)
Portfolio_prices <- cbind(VTI_M, AGG_M, VNQ_M, GLD_M, QQQ_M)
head(Portfolio_prices)

##               VTI      AGG      VNQ   GLD      QQQ
## Jan 2005 39.85560 54.97517 21.91400 42.22 32.06577
## Feb 2005 40.68662 54.77106 22.60901 43.53 31.91146
## Mar 2005 39.92498 54.23749 22.22295 42.82 31.35417
## Apr 2005 39.04556 55.17187 23.50522 43.35 29.99093
## May 2005 40.56111 55.62844 24.33289 41.65 32.64878
## Jun 2005 40.86863 56.11591 25.45158 43.44 31.56336

S0 <- colMeans(Portfolio_prices) # Initial prices of the ETFs (Mean of the
Historical Prices)
S0

##       VTI       AGG       VNQ       GLD       QQQ
##  97.74760  80.99223  50.60013 123.01022 126.91444

# Calculating mean returns and standard deviation (volatility) for each ET
F
mu <- colMeans(portfolio) # Mean monthly returns for each ETF
sigma <- apply(portfolio, 2, sd) # Volatility (standard deviation) for eac
h ETF
mu

##         VTI         AGG         VNQ         GLD         QQQ
## 0.007799043 0.002439605 0.005970986 0.006653167 0.011195029

sigma

##        VTI        AGG        VNQ        GLD        QQQ
## 0.04569724 0.01289255 0.06616912 0.04855422 0.05369967

# Initializing an array to store simulated prices and returns
simulated_prices <- array(0, dim = c(num_periods, length(S0), num_simulati
ons))
simulated_portfolio_returns <- matrix(0, nrow = num_periods - 1, ncol = nu
m_simulations)  # To store portfolio returns

# Running the Simulations Using GBM
for (i in 1:num_simulations) {
  simulated_prices[1, , i] <- S0  # Initial prices at time t = 0

  for (t in 2:num_periods) {
    Z <- rnorm(length(S0))  # Random shock for each ETF
    simulated_prices[t, , i] <- simulated_prices[t - 1, , i] * exp((mu - 0
.5 * sigma^2) + sigma * sqrt(1) * Z)
  }
```

```
  # Compute returns from prices (log returns)
  simulated_returns <- diff(log(simulated_prices[, , i]))  # (num_periods-
1, length(S0))

  # Compute portfolio returns for the current simulation
  simulated_portfolio_returns[, i] <- simulated_returns %*% weights  # Ret
urns as (num_periods-1, 1)
}

# Flatten the simulated portfolio returns into one large vector
all_simulated_portfolio_returns <- as.vector(simulated_portfolio_returns)
```

99% confidence level

```
# Calculate the VaR at the specified confidence level

VaR_monte_carlo_GBM_99 <- quantile(all_simulated_portfolio_returns, probs
= 1 - confidence_level1)

# Print the single VaR value
VaR_monte_carlo_GBM_99

##          1%
## -0.04035722
```

95% confidence level

```
# Calculate the VaR at the specified confidence level

VaR_monte_carlo_GBM_95 <- quantile(all_simulated_portfolio_returns, probs
= 1 - confidence_level2)

# Print the single VaR value
VaR_monte_carlo_GBM_95

##          5%
## -0.02711253
```

90% confidence level

```
# Calculate the VaR at the specified confidence level

VaR_monte_carlo_GBM_90 <- quantile(all_simulated_portfolio_returns, probs
= 1 - confidence_level3)

# Print the single VaR value
VaR_monte_carlo_GBM_90

##         10%
## -0.01997305

Monte_Carlo_GBM <- c(VaR_monte_carlo_GBM_90,VaR_monte_carlo_GBM_95,VaR_mon
te_carlo_GBM_99)
Monte_Carlo_GBM
```

```
##         10%          5%          1% 
## -0.01997305 -0.02711253 -0.04035722
```

BACK TESTING

Variance-Covariance Method

99% Confidence level

```r
# Identifying violations
violations_var_cov1<- portfolio_returns < VaR_var_cov_99[1 , 1]

# Counting the number of violations
num_violations_var_cov1 <- sum(violations_var_cov1)

# Kupeic POF Test
Y1 <- num_violations_var_cov1
# Significance level
alpha1<- 1 - confidence_level1
#total number of observations
N <- length(portfolio_returns)
# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_var_cov1 <- -2 * log(((1 - alpha1)^(N - Y1) * alpha1^Y1) /
                                ((1 - Y1/N)^(N - Y1) * (Y1/N)^Y1))

# Comparing to Chi-square critical value
p_value_var_cov1 <- 1 - pchisq(LR_POF_var_cov1, df = 1)
p_value_var_cov1

## [1] 0.2974775
```

95% Confidence Level


```r
# Identifying violations
violations_var_cov2<- portfolio_returns < VaR_var_cov_95[1 , 1]

# Counting the number of violations
num_violations_var_cov2 <- sum(violations_var_cov2)

# Kupeic POF Test
Y2 <- num_violations_var_cov2
# Significance level
alpha2<- 1 - confidence_level2
#total number of observations
N <- length(portfolio_returns)
# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_var_cov2 <- -2 * log(((1 - alpha2)^(N - Y2) * alpha2^Y2) /
                                ((1 - Y2/N)^(N - Y2) * (Y2/N)^Y2))

# Comparing to Chi-square critical value
p_value_var_cov2 <- 1 - pchisq(LR_POF_var_cov2, df = 1)
p_value_var_cov2

## [1] 0.6229961
```

90% Confidence Level

```
# Identifying violations
violations_var_cov3<- portfolio_returns < VaR_var_cov_90[1 , 1]

# Counting the number of violations
num_violations_var_cov3 <- sum(violations_var_cov3)

# Kupeic POF Test
Y3 <- num_violations_var_cov3
# Significance level
alpha3<- 1 - confidence_level3
#total number of observations
N <- length(portfolio_returns)
# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_var_cov3 <- -2 * log(((1 - alpha3)^(N - Y3) * alpha3^Y3) /
                                ((1 - Y3/N)^(N - Y3) * (Y3/N)^Y3))

# Comparing to Chi-square critical value
p_value_var_cov3 <- 1 - pchisq(LR_POF_var_cov3, df = 1)
p_value_var_cov3

## [1] 0.8763594

pvalue_Variance_Covariance<- c(p_value_var_cov3,p_value_var_cov2,p_value_v
ar_cov1) # 90%,95%,99%
pvalue_Variance_Covariance

## [1] 0.8763594 0.6229961 0.2974775
```

Historical Simulation

99% Confidence Level

```
# Identifying violations

violations_historical1 <- portfolio_returns < VaR_historical_99

# Counting the number of violations
num_violations_historical1 <- sum(violations_historical1)

# Calculating the expected number of violations
expected_violations1<- length(portfolio_returns)*(1 - confidence_level1)

# Kupeic POF Test
N <- length(portfolio_returns) #total number of observations
X1 <- num_violations_historical1 # Number of violations
alpha1<- 1 - confidence_level1 # Significance level

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_historical1 <- -2 * log(((1 - alpha1)^(N - X1) * alpha1^X1) /
                                ((1 - X1/N)^(N - X1) * (X1/N)^X1))

# Compare to chi-square critical value (with 1 degree of freedom)
p_value_historical1 <- 1 - pchisq(LR_POF_historical1, df = 1)
p_value_historical1

## [1] 0.6425917
```

95% Confidence Level

```
# Identifying violations

violations_historical2 <- portfolio_returns < VaR_historical_95

# Counting the number of violations
num_violations_historical2 <- sum(violations_historical2)

# Calculating the expected number of violations
expected_violations2<- length(portfolio_returns)*(1 - confidence_level2)

# Kupeic POF Test
N <- length(portfolio_returns) #total number of observations
X2 <- num_violations_historical2 # Number of violations
alpha2<- 1 - confidence_level2 # Significance level

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_historical2 <- -2 * log(((1 - alpha2)^(N - X2) * alpha2^X2) /
                                ((1 - X2/N)^(N - X2) * (X2/N)^X2))

# Compare to chi-square critical value (with 1 degree of freedom)
p_value_historical2 <- 1 - pchisq(LR_POF_historical2, df = 1)
p_value_historical2
```

```
## [1] 0.8444527
```

90% Confidence Level

```
# Identifying violations

violations_historical3 <- portfolio_returns < VaR_historical_90

# Counting the number of violations
num_violations_historical3 <- sum(violations_historical3)

# Calculating the expected number of violations
expected_violations3<- length(portfolio_returns)*(1 - confidence_level3)

# Kupeic POF Test
N <- length(portfolio_returns) #total number of observations
X3 <- num_violations_historical3 # Number of violations
alpha3<- 1 - confidence_level3 # Significance level

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_historical3 <- -2 * log(((1 - alpha3)^(N - X3) * alpha3^X3) /
                                ((1 - X3/N)^(N - X3) * (X3/N)^X3))

# Compare to chi-square critical value (with 1 degree of freedom)
p_value_historical3 <- 1 - pchisq(LR_POF_historical3, df = 1)
p_value_historical3

## [1] 0.9471843

pvalue_Historical_Methods <- c(p_value_historical3,p_value_historical2,p_v
alue_historical1) # 90%,95%,99%
pvalue_Historical_Methods

## [1] 0.9471843 0.8444527 0.6425917
```

Monte Carlo Simulations

Using Normal Distribution

99% Confidence Level

```
# Identifying violations
violations_monte_carlo_norm1 <- portfolio_returns < VaR_monte_carlo_99

# Counting the number of violations
num_violations_monte_carlo_norm1 <-sum(violations_monte_carlo_norm1)

# Kupeic POF Test
U1 <- num_violations_monte_carlo_norm1

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo_norm1 <- -2 * log(((1 - alpha1)^(N - U1) * alpha1^U1) /
                                  ((1 - U1/N)^(N - U1) * (U1/N)^U1))

# Comparing the Cji-square critical value
p_value_monte_carlo_norm1 <- 1  - pchisq(LR_POF_monte_carlo_norm1, df = 1)
p_value_monte_carlo_norm1

## [1] 0.2974775
```

95% confidence

```
# Identifying violations
violations_monte_carlo_norm2 <- portfolio_returns < VaR_monte_carlo_95

# Counting the number of violations
num_violations_monte_carlo_norm2 <-sum(violations_monte_carlo_norm2)

# Kupeic POF Test
U2 <- num_violations_monte_carlo_norm2

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo_norm2 <- -2 * log(((1 - alpha2)^(N - U2) * alpha2^U2) /
                                  ((1 - U2/N)^(N - U2) * (U2/N)^U2))

# Comparing the Cji-square critical value
p_value_monte_carlo_norm2 <- 1  - pchisq(LR_POF_monte_carlo_norm2, df = 1)
p_value_monte_carlo_norm2

## [1] 0.6229961
```

90% confidence level

```
# Identifying violations
violations_monte_carlo_norm3 <- portfolio_returns < VaR_monte_carlo_90

# Counting the number of violations
num_violations_monte_carlo_norm3 <-sum(violations_monte_carlo_norm3)

# Kupeic POF Test
```

```
U3 <- num_violations_monte_carlo_norm3

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo_norm3 <- -2 * log(((1 - alpha3)^(N - U3) * alpha3^U3) /
                                   ((1 - U3/N)^(N - U3) * (U3/N)^U3))

# Comparing the Cji-square critical value
p_value_monte_carlo_norm3 <- 1  - pchisq(LR_POF_monte_carlo_norm3, df = 1)
p_value_monte_carlo_norm3

## [1] 0.8763594

pvalue_Monte_Carlo_Normal<- c(p_value_monte_carlo_norm3,p_value_monte_carl
o_norm2,p_value_monte_carlo_norm1)
pvalue_Monte_Carlo_Normal

## [1] 0.8763594 0.6229961 0.2974775
```

Using GBM

99% Confidence level

```
# Identifying violations
violations_monte_carlo1 <- portfolio_returns < VaR_monte_carlo_GBM_99

# Counting the number of violations
num_violations_monte_carlo1 <-sum(violations_monte_carlo1)

# Kupeic POF Test
W1 <- num_violations_monte_carlo1

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo1 <- -2 * log(((1 - alpha1)^(N - W1) * alpha1^W1) /
                                   ((1 - W1/N)^(N - W1) * (W1/N)^W1))

# Comparing the Cji-square critical value
p_value_monte_carlo1 <- 1  - pchisq(LR_POF_monte_carlo1, df = 1)
p_value_monte_carlo1

## [1] 7.685647e-07
```

95% Confidence level

```
# Identifying violations
violations_monte_carlo2 <- portfolio_returns < VaR_monte_carlo_GBM_95

# Counting the number of violations
num_violations_monte_carlo2 <-sum(violations_monte_carlo2)

# Kupeic POF Test
W2 <- num_violations_monte_carlo2

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo2 <- -2 * log(((1 - alpha2)^(N - W2) * alpha2^W2) /
                                   ((1 - W2/N)^(N - W2) * (W2/N)^W2))
```

```r
# Comparing the Cji-square critical value
p_value_monte_carlo2 <- 1  - pchisq(LR_POF_monte_carlo2, df = 1)
p_value_monte_carlo2

## [1] 1.623064e-05
```

90% Confidence level

```r
# Identifying violations
violations_monte_carlo3 <- portfolio_returns < VaR_monte_carlo_GBM_90

# Counting the number of violations
num_violations_monte_carlo3 <-sum(violations_monte_carlo3)

# Kupeic POF Test
W3 <- num_violations_monte_carlo3

# Log-Likelihood Ratio Test Statistic for Kupiec Test
LR_POF_monte_carlo3 <- -2 * log(((1 - alpha3)^(N - W3) * alpha3^W3) /
                                  ((1 - W3/N)^(N - W3) * (W3/N)^W3))

# Comparing the Cji-square critical value
p_value_monte_carlo3 <- 1  - pchisq(LR_POF_monte_carlo3, df = 1)
p_value_monte_carlo3

## [1] 0.01100398

pvalue_Monte_Carlo_GBM<- c(p_value_monte_carlo3,p_value_monte_carlo2,p_val
ue_monte_carlo1)
pvalue_Monte_Carlo_GBM

## [1] 1.100398e-02 1.623064e-05 7.685647e-07
```