Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)
MASTER OF SCIENCE IN STATISTICAL SCIENCES
END OF SEMESTER EXAMINATION
STA 8203: TIME SERIES ANALYSIS AND FORECASTING

DATE: 27th August 2021 Time: 3 Hours

Instructions

1. This examination consists of **FOUR** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question One

1.1. Define, precisely, what it means for a time series Y_t to be an AR (1) series. [2 marks]

1.2. What condition will ensure that an AR (1) series will be stationary and causal. [2 marks]

1.3. Suppose that Y_t is a stationary, causal AR (1) series. Describe, as exactly as you can, the type of ARMA series which is produced by taking simple differences of Y_t. [5 marks]

1.4. Does the equation $Y_t = \frac{1}{2}Y_{t-1} + \frac{3}{4}Y_{t-2} + \varepsilon_t$ define a stationary, causal time series? Explain. [5 marks]

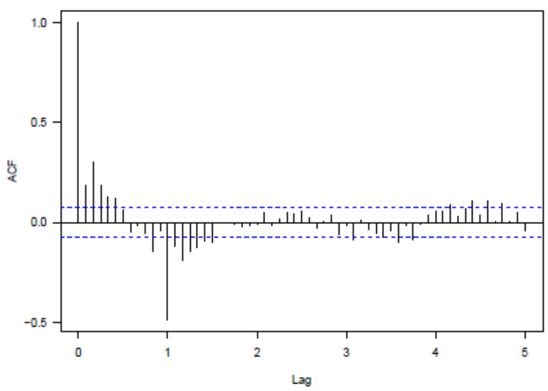
1.5. Write the general form of seasonal ARIMA model stating different parameters. [3 marks]

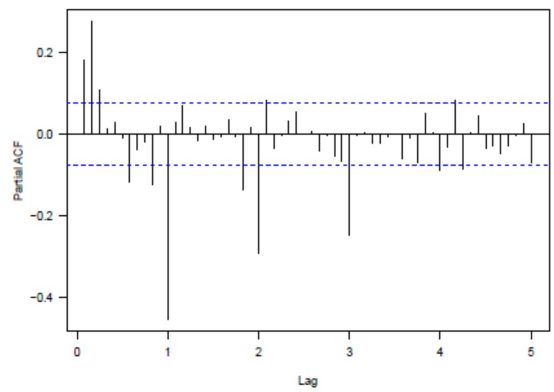
1.6. Explain three different exponential smoothing models. [3 marks]

Question Two

2.1. In order to forecast the monthly unemployment rate series data, it was decided to difference the series twice; once at lag 1 and once at lag 12. Explain why such differencing is required and explain how the decision to difference with these particular lags may have been arrived at. [4 marks]

2.2. The plots of the estimated ACF and PACF for a differenced version of the unemployment series. Give a detailed explanation of how these plots can be used to suggest a model for the differenced series and write down the model you feel is appropriate here, explaining why you think it is appropriate. [6 marks]





2.3. The following parameter estimates were computed for seasonal ARIMA model based on the original data. Use the results to answer the questions that follow.

```
ele.arima<-arima(eletricity, order=c(1,0,0), seasonal = list(order=c(2,1,0), period =12))
```

Parameter	**Estimate**	**Std. error**
AR (1)	0.2856	0.0642
SAR (1)	-0.8598	0.0639
SAR (2)	-0.2963	0.0667

(a) Write the presented autoregressive model in the form of ARIMA (p, d, q) (P, D, Q) s

[4 marks]

(b) Write down an explicit expression for the fitted model [6marks]

Question Three

3.1 The following cosine function can be used to model the seasonal pattern that might exist in the data.

$$f(t) = \alpha \times cos[(\omega t) - \theta]$$

Show how the model can be represented by both sine and cosine. [5 marks]

3.2 The volume of the average heart is 140 millilitres (ml), and it pushes out about one-half its volume (70 ml) with each beat. In addition, the frequency of a well-trained athlete heartbeat for a well-trained athlete is 50 beats (cycles) per minute. A doctor is interested in modelling the volume, V(t) in millilitres of blood in the heart as a function of time t measured in seconds using a sinusoidal function of the form

$$V(t) = A\cos\big(B(t - C)\big) + D$$

a) Considering the cosine model, determine the parameters C of the model with explanation. [2 marks]

b) Using the frequency of heart beats is 50 beats per minute, determine the value of B. Write the formula for V(t) using the values of A, B, C, and D that have been determined. [6 marks]

c) Using the model determine the amount of blood in the heart after 4 seconds after the heart was full. [2 marks]

d) The following cosine function was used to model monthly disease prevalence data

$$f(t) = \alpha \times cos[(\omega t) - \theta] + C$$

Explain the model parameters in the context of disease prevalence. [5 marks]

Question Four

4.1. Natural cubic splines are currently used in environmental epidemiology to adjust for seasonality and long-term time-trend by adding a variable or a set of variables representing time (t) into generalized linear models. Explain different terms used in such models to adjust different time series components. [10 marks]

4.2 State and explain both the variables and parameters in the general regression model for interrupted time series (ITS) [10 marks]